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SORTAL QUANTIFICATION AND RESTRICTED QUANTIFICATION

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The use of ordinary first-order quantification theory as a method of displaying the 'logical form' of everyday language has by now become familiar to most philosophers in the analytic tradition. So familiar has this tool become, that I believe many otherwise open-minded philosophers have relegated certain alternatives to it to the waste-basket without giving them a fair trial. The usual complaint about the alternatives is that they "simply come down to classical quantification theory, once one strips away the difference in symbolism". Such is the verdict I wish to refute.[1]

I shall begin by mentioning some features of Wallace's 'sortal quantification theory' that do not hold in ordinary quantification theory. Wallace claims that, according to his intuitions about English, sortal predicates (1) follow on the heels of quantifier words to give the subject matter of most sentences (p. 12), (2) by picking out the subject matter of a sentence, they give the entities that enter into the truth conditions of that sentence (p. 13), (3) by picking out the subject matter of a sentence, they give a clear sense to confirmation by instances. It is claimed now, that these features are mirrored in sortal quantification but not in classical quantification theory; and further, that proving this claim amounts to demonstrating that the former is not merely a minor variant on the latter.

It is obvious that intuition (1) is not reflected in unrestricted quantification theory: the predicates of this pattern of canonical notation are in no syntactical way connected with the quantifiers, and a brief consideration of the semantical presuppositions of unrestricted quantification theory will show that there is no semantical connection either. Consider a sentence of English from the point of view of unrestricted quantification theory:

(i) All pigs are pink.

Intuitively this sentence says of pigs that they all are pink. It says nothing about any other object – it does not say that they are pink, non-pink, or

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even that the predicate 'pink' can be meaningfully applied to them. It is 'about' and only 'about' pigs.[2] A representation of (i) in unrestricted quantification theory is

(ii) $(x)(Gx \supset Kx)$.

But if we consider how this is interpreted, we find that the subject matter of our sentence is no longer just pigs, but everything in the domain – everything that the bound variables can take as values. But this is more like saying

(iii) Everything is such that if it is a pig, it is pink.

One can argue as to whether (iii) makes sense at all (I suppose Wallace would say it did not), but whatever sense it makes is surely different from that of (i).

Since 'pig' in English is a sortal (which I have represented by 'G'), (i) becomes, in Wallace's notation,

(iv) $(Ax)[G](Kx)$.

The quantifier phrase (up to the closing square bracket) is supposed to reflect our intuition (1) in the canonical notation. To see if this intuition is really reflected we must look at the characterization of (logical) *truth*, i.e., we must look at how intuition (2) is reflected. Wallace (p. 13) says

> Applying the definition of truth for a given language to a sentence of that language gives the truth-conditions for the sentence. It is reasonable to say that the sentence is *about* the entities – other than 'machinery' entities like assignments, variables, formulas – which are involved in its truth conditions.

He procedes to make this 'involved' precise. An assignment involves a certain entity if and only if it has that entity as a value for some argument. The truth conditions for a sentence involve an entity if and only if they bring into play an assignment that involves that entity. Now, the truth conditions of unrestricted quantification theory call into play *every* assignment and therefore involve every entity that the bound variables can take as values. Thus the truth conditions of (ii) involve cats, elephants, planets, numbers, etc., whereas our intuition (2) tells us it should only involve pigs. The present question is: can there be any way sortal quantification theory can give a characterization of truth such that the only entities involved in the truth conditions for (iv) are pigs? Wallace says in his last sentence: "The truth-conditions for a given sentence (of sortal quantification theory) *involve* a given entity if and only if an assignment that involves that entity falls under some sortal predicate which occurs as a subject of a sentence in the truth conditions". If we look at the definition of truth given by Wallace on pp. 12–13, we see that since a sentence is true if and only if the empty assignment satisfies it, the only entities that enter into the truth conditions of (iv) are those that fall under the sortal which is the subject of the sentence, namely pigs. We should note that no analogous claim for unrestricted quantification theory can be formulated.

Intuitively, to confirm (i) by instances, one need only look at pigs and see whether or not they are pink. We also intuitively believe that whatever confirms one sentence will equally confirm a logically equivalent sentence. Taken together, these two intuitions pose the 'paradox of confirmation' for unrestricted quantification theory, since

(v) $(x)(-Kx \supset -Gx)$

is equivalent to (ii), and yet (v) would seem to be confirmed by looking at non-pink things and seeing whether or not they are non-pigs. As long as one holds to these two intuitions about confirmation, there is no way out of the 'paradox' for unrestricted quantification theory.[3] However, given the characterization of 'entities entering into the truth conditions of (iv)', we see that these two intuitions are quite well accommodated by sortal quantification theory. Since only pigs enter into the truth conditions of (iv), we need only see whether each of them satisfies 'Kx', i.e., see of them whether or not they are pink. The reason for the difference here between sortal quantification theory and unrestricted quantification theory is, of course, that the English sentence for which (v) is the canonical representation in unrestricted quantification theory has no canonical counterpart in sortal quantification theory, since 'non-pink thing' is not a sortal, and *a fortiori* cannot be equivalent to (iv). Thus sortal quantification theory does not countenance the 'paradox of confirmation'.

Of course, some apparent alternatives to ordinary quantification theory do turn out to be nothing more than classical quantification theory, once the notational differences are done away with. The most widely-known such variant is what is traditionally called 'restricted quantification theory'.[4] Herein we can 'restrict' the variables of quantification to just those that satisfy a certain formula. So, for example, if we wish to 'restrict' our variables of quantification to those entities which satisfy Φ, we might say of these Φ's that they satisfy ψ thusly:

(vi) $(\xi x \Phi)\, \Psi$.

Restricted quantification theory, when presented simply, as done here, or when presented completely, as in Hailperin, is acknowledged to be merely a variant on unrestricted (ordinary) quantification theory (see Hailperin pp. 19, 29). This is so because (a) there is an automatic syntactical transform from (vi) to

(vii) $(y)\,(\Phi' \supset \Psi')$

of ordinary quantification theory, and (b) the truth conditions and confirmatory instances of (vi) and (vii) are identical. This, then, is the reason (or one of the reasons) that restricted quantification theory is, but sortal quantification theory is not, a mere notational variant on ordinary quantification theory.

I suggest that what Bacon (cited in Note 1 above) and others who believe sortal quantification theory to be somehow like restricted quantification when the difference in symbolism is stripped away have in mind is merely this: if a sentence of English is represented in sortal quantification theory as

(viii) $(Ax)\,[G]\,(Kx)$

then it would be represented in restricted quantification theory as

(ix) $(\xi x G x)\,(K \xi x G x)$.

But this is of course not to say that the two theories are the same. If it did, then any two theories which treated of the same subject matter would be identical – e.g., propositional logic and predicate logic.

I have said nothing about Quine's charge about Belnap's new logic of quantification. It is, however, a simple matter to follow the same argument as presented in this paper and show that Belnap's logic has semantical presuppositions that cannot be mirrored in restricted quantification theory. I hope, therefore, that philosophers will stop making the elementary mistake attributed to Bacon above, and evaluate each new 'pattern of canonical notation' on its ability to mirror features of natural languages. At least one should be aware that not every 'restriction' in a quantification theory is a restriction in the normal sense.

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NOTES

[1] It is difficult to find this charge in print. I have heard it made in conversation many times, but the only two places in print that I have seen it are: W. V. Quine, 'Abstract of Comments on Belnap's Paper', *Noûs*, 1970, p. 12, where he says that Belnap is offering "restricted quantification along the lines of Peano" (Neul Belnap's paper is 'Conditional Assertion and Restricted Quantification', *Noûs*, 1970, pp. 1–12); and John Bacon, 'A Simple Treatment of Complex Terms', *Journal of Philosophy*, 1965, pp. 328–331, where he says that his theory of restricted quantification "allows for easy subsumption" of Wallace's sortal quantification (John Wallace's paper is 'Sortal Predicates and Quantification', *Journal of Philosophy*, 1965, pp. 8–13). Most future references will be to these papers.

[2] Such claims might be objected to by those committed to some theory or other (e.g., ordinary quantification theory). Such people should realize that their objections are due to their *theory*, whereas I am here talking about pre-theoretical intuitions one has toward English. Further, studies of pronominalization tend to support the intuition mentioned in that text. E.g., from (i) we could go on: 'They are also dirty'. This last sentence, which everyone will admit is 'about' and only 'about' all the pigs, has as its subject matter exactly what (i) has.

[3] Of course one can try to develop some stronger form of 'equivalent' (say 'intensional isomorphism') or some sense where non-pink non-pigs 'really' do confirm 'All pigs are pink'. But this is to adjust our intuitions to fit the theory, not adjusting the theory to fit out intuitions.

[4] As developed by Theodore Hailperin, 'A Theory of Restricted Quantification', *JSL* **22**:19–35, 113–129 (1957).