ANOTHER ARGUMENT AGAINST VAGUE OBJECTS*

THE view that the world might actually *be* vague, as opposed to being vaguely described and as opposed to not having sufficient information to determine whether some proposition is true or false, has not received a favorable reception. Bertrand Russell[1] claims that it "is a case of the fallacy of verbalism—the fallacy that consists in mistaking the properties of words for the properties of things"; Michael Dummett[2] says that "the notion that things might actually *be* vague, as well as being vaguely described, is not properly intelligible"; and Gareth Evans[3] presents a logic-based argument against it. Despite the fact that most agree with the anti-vagueness-in-reality conclusion, the argument has generated considerable criticism; but the criticism has been diffuse in the sense that the commentators do not always agree on the very structure of the argument nor on the possible objections to Evans's assumptions. Evans said (I use λx rather than $\hat{x}$ for abstraction):

> Let a and b be singular terms such that the sentence '$a = b$' is of indeterminate truth value, and let us allow for the expression of the idea of indeterminacy by the sentential operator, '∇'. Then we have:
> (1) $\nabla\ (a = b)$
> (1) reports a fact about b which we express by ascribing to b the property '$\lambda x[\nabla\ (x = a)]$'
> (2) $\lambda x[\nabla\ (x = a)]b$
> But we have
> (3) $\sim\nabla\ (a = a)$
> and hence:
> (4) $\sim\lambda x[\nabla\ (x = a)]a$
> But by Leibniz's law, we may derive from (2) and (4):
> (5) $\sim\ (a = b)$

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[1] "Vagueness," *Australasian Journal of Psychology and Philosophy*, I, 2 (1923): 84–92.

[2] "Wang's Paradox," *Synthèse*, XXX, 3/4 (1975): 301–324.

[3] "Can There Be Value Objects?" *Analysis*, XXXVIII, 4 (1978): 208. N. Salmon, *Reference and Essence* (Princeton: University Press, 1979), pp. 243ff, esp. fn. 24, produces a proof, somewhat different from Evans's, to the same conclusion (that vagueness of objects is impossible). He also thinks that Evans's proof is correct. These arguments have been widely discussed in the literature.

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contradicting the assumption, with which we began, that the identity statement '$a = b$' is of indeterminate truth value.

Now *this* argument, phrased in terms of "truth values," would seem to read ∇ as meaning "has some truth value other than 0 or 1." A little later, however, Evans remarks, "if Δ [the dual of ∇, meaning 'is definite'—FJP] determines a logic at least as strong as S_5," then we can further derive $\Delta\ (a \neq b)$ from the supposition that $\nabla\ (a = b)$. So the question arises: Does Evans intend his Δ and ∇ to be modal operators or "truth-value indicating" operators? In either case—and our commentators are divided on which he intended—there seem to be difficulties with the argument.

It is my opinion that these two understandings of ∇ and Δ correspond to two radically different conceptions of that to which vagueness is due. A modal interpretation of vagueness takes each statement ϕ as being either true or false at any given index (world), but claims that prefixing, say ∇, to such a statement orders one, in effect, to look at some related (accessible) index to determine whether ϕ is true or false there before $\nabla\phi$ can be judged true or false at the initial index. Such an outlook does not seem to make $\nabla\phi$ say that ϕ is vague "in a realistic sense," or that ϕ is vague in the sense of "the world being vague"—after all, by hypothesis, ϕ *is* either true or false. Rather, such an account would seem most happily to find a home as an account of epistemic vagueness: ϕ is either true or false, but we do not know which, either because we lack the information necessary to determine it or because we are not clear as to which proposition ϕ expresses. But it nonetheless does have a truth value. Contrasted to this, a *truth-value interpretation* of $\nabla\phi$ actually says that ϕ is neither true nor false, and therefore that "the world" takes on this indeterminacy. It should also be recalled that Evans is explicitly interested in the claim that "the world might actually *be* vague, as well as being vaguely described." So, the truth-value interpretation of Δ and ∇ would appear to be more suited to his overall aim than a modal-operator interpretation (despite what were perhaps his own intuitions[4]).

There are a number of principles concerning ∇ and Δ which one might consider reasonable in interpreting them as "definite" and

[4] David Lewis informs me that Evans had written him a letter confirming that ∇ and Δ were intended as modal operators, and that it was a slip when, in the paper, he mentioned that p followed from Δp; for p is allowed to be definitely false as a way of making Δp true.

"indefinite" operators.[5] Evans himself mentions only two basic ones, (*duals*) and (*refl*), but also seems to suppose that definiteness and indefiniteness are contraries, (*contra*):

(duals) $\nabla\phi = df \sim \Delta \sim \phi$ (that ∇ and Δ are *duals*)

(refl) $\Delta\ (\alpha = \alpha)$, for all α. (the reflexivity of identity is definite)

(contra) $\Delta\phi \leftrightarrow \sim\nabla\phi$ (that no sentence is both definite and indefinite)

Whichever of the two interpretations one takes of Δ and ∇, the structure of the argument is:

suppose (i) $\nabla\ (a = b)$

then (ii) $\lambda x[\nabla\ (a = x)]b$

equivalent to (i) by lambda abstraction

but (iii) $\sim\lambda x[\nabla\ (a = x)]a$

from (refl) by lambda abstraction and (contra)

ergo (iv) $a \neq b$ from (ii)–(iii) Leibniz's law

One objection to Evans's argument is that there is no contradiction here: $\nabla\ (a = b)$ can happily coexist with $a \neq b$, contrary to Evans's assertion that this is a "contradicting [of] the assumption, with which we began, that the identity statement '$a = b$' is . . . indeterminate. . . ." At best, or so it is claimed (see John Heintz and Peter van Inwagen; the point is implicit also in John Broome and Terrance Parsons[6]), the argument proves

(v) $\nabla\ (a = b) \rightarrow a \neq b$ from (i)–(iv) by conditional proof

Perhaps, but it seems not to have been also noted that from this we get the following:

(vi) $a = b \rightarrow \sim\nabla\ (a = b)$ contrapositive of (v)

(vii) $a = b \rightarrow \Delta\ (a \neq b)$ (vi) and (duals)

So, if in fact a and b are identical, then they are definitely distinct. If one holds the principle

[5] For a survey, see my "The Not-So-Strange Modal Logic of Indeterminacy," *Logique et Analyse*, XXVII, 108 (1984): 415–422.

[6] Heintz, "Might There be Vague Objects?", paper read to Canadian Phil. Assn, Halifax (1981); van Inwagen, "How to Reason About Vague Objects," *Philosophical Topics*, XVI, 1 (1988): 255–284; Broome, "Indefiniteness in Identity," *Analysis*, XLIV, 1 (1984): 6–12; and Parsons, "Entities Without Identity," *Philosophical Perspectives*, I, 1 (1988): 1–19.

(T) $$\Delta\phi \rightarrow \phi$$

(as P. F. Gibbins[7] and Broome do), then we can infer from (vii) and (T)—letting ϕ be $a \neq b$

(viii) $$a = b \rightarrow a \neq b$$

from which the troublesome

(ix) $$a \neq b$$

follows. But this result holds for any a and b (recall that it is no longer conditional upon ∇ $(a = b)$), in particular it holds for a and a:

(x) $$a \neq a$$

which everyone thinks is contradictory. (And, if not, then from (refl) and (T), we could derive an even more explicit contradiction.) It seems to me that this shows that allowing (i)–(iv) to stand already shows that the believer in vagueness-in-reality is in trouble. What is needed is to attack directly the (i)–(iv) argument.

Surely the most common attack is to focus on the use of lambda abstraction, perhaps in conjunction with Leibniz's law. This line of attack on the argument takes one of two forms. The first form points out that, generally speaking, one cannot "substitute into" so-called opaque contexts, and that treating ∇ and Δ as modal operators is as likely to generate such contexts as any other modal operators. Surely, so it is argued, we should not expect to say that a has all the modal properties that b does, even when '$a = b$' is true, unless the names a and b are "rigid designators" (which are defined so as to have all modal properties in common, if they are identical). This subline of attack locates the difficulty with Evans's argument in its passage from steps (ii) and (iii) to step (iv); or, more precisely, in the statement of Leibniz's law—the proper statement of the law would say (in its contrapositive form): "if property ϕ is predicated of the singular term 'a' and the property $\sim\phi$ of the singular term 'b', then if 'a' and 'b' are both rigid designators, it follows that $a \neq b$." And the correlative claim about vague identity would be that the only identity statements that can be vague are those wherein at least one of the singular terms is not rigid. So, on this line of thought, what Evans's argument proves is that you cannot have statements of vague identity when both terms are rigid. An approach similar to this for the modal

[7] "The Strange Modal Logic of Indeterminacy," *Logique et Analyse*, XXV, 100 (1982): 443–446.

argument is given by David Wiggins,[8] where the point is put in terms of the scope of definite-description operators with regards to ∇ and Δ.

The second form this objection might take is to deny that every lambda expression designates a ("real") property; in particular, it is claimed that not all modalized formulas designate a "real" property. Leibniz's law is then stated in its second-order form: $a = b \leftrightarrow (\forall F)(Fa \leftrightarrow Fb)$, where the quantifier ranges over "real" properties. According to this view, two objects are distinct just in case they disagree on a "real" property. And, to complete the picture, lambda abstracts formed with Δ and ∇ do not designate "real" properties; therefore, Evans's argument goes awry (see Richmond Thomason[9] and van Inwagen). Where does it go awry? There are two ways to answer the question: one could say that such lambda abstracts are ill-formed, since they do not indicate "real" properties [this would be to deny the legitimacy of moving from (i) to (ii) and of moving from (refl) to (iii)]; or one could allow their well-formedness, but deny that Leibniz's law holds in such cases [here one denies that (iv) can follow from (ii) and (iii); for actually to apply Leibniz's law, one must first prove that the abstracts or predicates in question "really" do designate properties].

However plausible this rebuttal to Evans's argument might seem on first reading, it owes all its force to the analogy with modal logic and related concepts ("opaque contexts," "modalized properties," "real properties," "rigid designation," etc.). But this is not the only way to understand Evans's argument; indeed, as I remarked before, it is not even a correct way to look at it, if the argument is to be taken as being directed against "vagueness in reality." For the believer in vagueness-in-reality is committed to a many-valued logic, and therefore the Δ and ∇ operators must be understood as truth-value indicating operators. This is the only way of viewing the logic as "talking

[8] "On Singling Out An Object," in Philip Pettit and John McDowell, *Subject, Thought and Context* (New York: Oxford, 1986), pp. 171–182, esp. pp. 176/7; see also H. Noonan, "Vague Objects," *Analysis*, XLII, 2 (1982): 3–6, where he talks of a singular term "vaguely denoting" an object. Wiggins takes Evans's argument to prove something different from (v), viz., $a = b \rightarrow \Delta\,(a = b)$. This becomes our (v) if it is assumed that a statement is definite just in case its negation is. About this conclusion, Wiggins says it shows that "there is no future in the supposition that one could say that a was b but refuse to affirm that it was definitely b" (p. 175). It is not clear to me how Wiggins thinks he has reconstructed Evans's argument here, except perhaps by ignoring the requirement that we start with the supposition that '$a = b$' is vague.

[9] "Identity and Vagueness," *Philosophical Studies*, XLII, 3 (1982): 329–332.

directly about reality" rather than about our statements or our knowledge. In reconstructing this argument—Evans's real argument against vagueness-in-reality, we will avoid using Δ and ∇ (because of their connections to modal logic—see the series of papers in the late 1960s in *Logique et Analyse* by Hugh Montgomery and Richard Routley where these symbols were used as "contingency" and "non-contingency" operators). Instead we will use the more common J operators.[10] For each truth value i, there is a sentence operator J_i which, when applied to a sentence, has the effect of claiming that the sentence has exactly the truth value i. If the value of ϕ is i, then $J_i\phi$ takes the value 1 ("completely true"); otherwise, $J_i\phi$ takes the value n ("completely false"). From these basic operators, more complex ones (e.g., "has at most the value i," or, "is either 'most true' or 'most false'," etc.) can be defined, should one wish.[11] Since we will concentrate at the outset on a three-valued logic, we have $J_1\phi$, $J_2\phi$, and $J_3\phi$. If ϕ takes the value 2, for example, then $J_2\phi$ takes the value 1 and each of $J_1\phi$ and $J_3\phi$ takes the value 3. The intuition here is that $J_1\phi$ means that ϕ is definitely true, $J_2\phi$ means that ϕ is indeterminate, and $J_3\phi$ means that ϕ is definitely false. In such a setting, Evans's argument is:

suppose	(i′)	$J_2(a = b)$
then	(ii′)	$\lambda x[J_2(a = x)]b$
but	(iii′)	$\sim\lambda x[J_2(a = x)]a$
ergo	(iv′)	$a \neq b$

Under the assumption that conditional proof works as usual, we would then derive

(v′) $$J_2(a = b) \rightarrow a \neq b$$

analogously with before. And, as before, the choices seem to be either to deny that λ abstraction is legitimate or to deny that Leibniz's law applies to such predicates. Another option comes with nonclassical truth values, however: deny the validity in general of Leibniz's law. To the person who wishes to deny the legitimacy of λ

[10] See B. Rosser and A. Turquette, *Many-Valued Logics* (Amsterdam: North Holland, 1952).

[11] Note that such definitions can be used in the other direction also. If the language has, for example, constants for each of the truth values and a connective that operates like our $\leftrightarrow$ (below), then the J-operators can be defined. Indeed, there are many different ways to be able to have J-operators in the language by such definitions. I merely insist that the language be rich enough to "talk about" vagueness—so it must have the resources to define the J-operators.

abstraction, it should be pointed out that the present logic is completely extensional. Each formula, and its parts, is evaluated "in the actual world." (That is, to find out the semantic value of any given formula in a given row of a truth table, one need only look at that row, not at any other rows.) This means that the grounds for denying λ abstraction which might be used in the modal case are taken away. Among other things, it follows that one can "substitute into" any part of any formula and preserve truth, regardless of whether it has λ abstracts over J operators. In general, if a and b are names in this language and '$a = b$' is true, then any predicate that can be formulated in the language will apply truly to a just in case it applies truly to b. This is part of what is meant by saying that the language is truth-functional. There are no "rigid designators," no "modal properties," no distinction between "predicates that do and predicates that do not designate real properties," or the like. So this evasion, that might be brought to bear in treating the modal version of Evans's argument, cannot be correctly wielded in the many-valued case. This is van Inwagen's objection and one of Parsons's objections to Evans's argument. Parsons remarks that Evans's argument shows us "that we cannot extend property abstraction to formulas containing indeterminacy operators." I have just tried to show that such an objection is groundless, and not available to the believer in a many-valued account of vagueness.

Parsons and Broome have made detailed suggestions about how we should interpret $\rightarrow$ (and Leibniz's law) in their favorite three-valued logics. I find myself less than enthralled with their suggestions, but rather than discuss their proposals, I shall content myself with providing a somewhat different argument against vagueness-in-reality. The overall structure of my position is: a many-valued logic is the only legitimate logic that a vagueness-in-nature theorist can use. Such a theorist is required to be able to "speak about" vagueness and therefore must have at his disposal some device such as our J operators. Further, whatever principles the theorist uses must agree with classical logic on the classical values. Finally, there are some principles governing the use of J operators, Leibniz's law, the reflexivity of identity, and the interaction of J operators with quantifiers; I argue these principles to be correct, and anyway most "vagueness-in-nature" theorists believe them. I shall first lay out all these principles and then provide an argument that shows that no three-valued logic obeying the principles can admit "vagueness-in-reality." Not every three-valued logic is susceptible to my argument. The ones that are have the following properties: they have the resources to "talk about

vagueness" (either directly by having J operators or by having other operators with which the J operators can be defined), and they have sufficient connectives in the language to allow us to define certain other connectives, here called &, $\vee$, $\rightarrow$, $\sim$, $\leftrightarrow$, which in turn operate as specified below and which operate classically on the classical values (1 and 3).[12] But this is a very large group of three-valued logics; and I would say that any logic that did not obey these restrictions should not be taken seriously as an account of our reasoning about vagueness. Then I shall show how a simple generalization of certain of the J principles will prove that no finitely many-valued logic obeying certain simple restrictions can accomodate "vagueness-in-nature." (That is, my basic argument does not turn on any such concept as "higher-order vagueness.")

We start with some principles governing J operators. The first principle is "uniqueness of semantic value" (USV), saying that every formula takes exactly one of the three semantic values:

$$\text{(USV)} \quad (J_1\Phi \vee J_2\Phi \vee J_3\Phi) \,\&\, \sim(J_1\Phi \,\&\, J_2\Phi) \,\&\, \sim(J_1\Phi \,\&\, J_3\Phi) \,\&\, \sim(J_2\Phi \,\&\, J_3\Phi)$$

Second, we interpret the semantic value 1 as "completely true" and therefore insist that, if $J_1\Phi$ is true, then ϕ cannot be "completely false," i.e., that

$$(J_1) \qquad J_1\phi \rightarrow \phi$$

is an axiom. Note that (J_1) does not prejudge whether ϕ is or is not "completely true," only that, if it is "completely true," then it is "true". Third, the $\leftrightarrow$ connective obeys this rule of inference:

(E) if $(\phi \leftrightarrow \psi)$ is semantically valid, then so is $(J_i\phi \leftrightarrow J_i\psi)$, for any $1 \leq i \leq 3$

In fact, (E) is no restriction on the logic at all. It amounts to assuming that, if a biconditional formula is provable, then any bicondi-

[12] Just as in the case with J operators, where it was not required that the language have them as primitives but only that they be definable, here too we do not insist that the language have these sentential connectives—only that they be definable with the resources that the language does have. By 'operate classically on classical values' I of course mean that '&' operates like the classical 'and' on the classical values, that '$\vee$' operate like the classical 'or' on the classical values, and so forth.

tional formula expressing an instance of the diagonal is also provable. (A biconditional formula "expresses an instance of the diagonal" iff both sides have the same truth value. Any logic that makes the $\leftrightarrow$ true when both sides have the same truth value obeys this constraint.)

With regard to identity, the logic is to have two properties: first,

(refl′) $$J_1\alpha = \alpha$$

is an axiom; and, second, it must have a version of Leibniz's law in it which entails all the instances that this one entails: that, if a and b are identical, then, for each property, it cannot happen that one has it and the other lacks it, that is,

(LL) $$a = b \leftrightarrow (\forall F)(Fa \leftrightarrow Fb)$$

And, finally, with regard to the interaction between quantifiers and J operators, the only principle to be used is this:

(J_2-$\forall$) $$J_2(\forall x)Fx \rightarrow \sim(\exists x)J_3Fx$$

which can be read, roughly, as "if it is true to degree 2 that everything is F, then there cannot be anything of which it is completely false that it is F." Whatever the difficulties are in giving an explicit and complete statement of the interrelationships among the J operators and the quantifiers, this much at least seems clear: if there were an object for which it was definitely false that it was F, then it cannot be vague/indeterminate/undecided whether everything is F—on the contrary, it would be completely false that everything is F.[13]

One lemma needs to be mentioned, which follows from (USV) and the fact that the J operators are two-valued. Recall that, if any J operator is applied to any sentence, its value will be either 1 or 3. Therefore, putting a J_2 in front of a sentence that already has a main J operator will yield the value 3. More generally, if each sentential part of a formula ϕ is already in the scope of some J operator, prefixing a J_2 operator to ϕ will yield the value 3; that is, its negation

[13] The use to which I put this principle involves the second-order quantifier $(\forall F)$; but (J_2-$\forall$) applies to such quantifiers just as much as it does to first-order ones. Whatever reasons one thinks it holds in the first-order case can be carried over to the second-order case. The principle is independent of the order of the quantifier.

will yield the value 1 (since negation operates classically on the classical values):

(Lemma) $\sim J_2\Phi$ (equivalently, $J_3\Phi$), if all sentences in ϕ are already in the scope of some J operator

Now let us look at the argument. Suppose an object a is vague. According to the vagueness-in-reality theorist, this implies that there will be a true vague identity statement, that is, a statement of the form $J_2\ (a = b)$.

a. $J_2\ (a = b)$	Assumption
b. $a = b \leftrightarrow (\forall F)(Fa \leftrightarrow Fb)$	(LL)
c. $J_2\ (a = b) \leftrightarrow J_2\ (\forall F)(Fa \leftrightarrow Fb)$	from (b) and (E)
d. $J_2\ (\forall F)(Fa \leftrightarrow Fb)$	from (a), (b) $\leftrightarrow$ elimination
e. $\sim(\exists F)\ J_3\ (Fa \leftrightarrow Fb)$	from (d) and (J_2-$\forall$)
f. $(\forall F)[J_1(Fa \leftrightarrow Fb) \vee J_2(Fa \leftrightarrow Fb)]$	from (e) and (USV)
g. $J_1\ (J_1\ (a = a) \leftrightarrow J_1\ (a = b)) \vee J_2\ (J_1\ (a = a) \leftrightarrow J_1\ (a = b))$	from (f), instantiate ($\forall F$) to $(\lambda x)\ J_1\ (a = x)$ and λ convert
h. $J_1\ (J_1\ (a = a) \leftrightarrow J_1\ (a = b))$	from (g), (lemma), disj.syll.
i. $J_1\ (a = a) \leftrightarrow J_1\ (a = b)$	from (h) and (J_1)
j. $J_1\ (a = b)$	from (i) and (refl′), $\leftrightarrow$ elim.
k. $\sim J_2\ (a = b)$	from (j) and (USV)

Now, each step is legitimate here. All propositional inferences ($\leftrightarrow$ elimination, disjunctive syllogism) were done on classical values and all other steps were applications of the principles listed, in which I include the legitimacy of using the indicated instance of the quantifier in step (g), as argued earlier. Although "ϕ and $\sim\phi$" is not in general an impossible state of affairs in a three-valued logic, here (k) and (a) really are jointly impossible, since they both have classical values.

A structurally identical argument can be made within any finitely many-valued logic (so long as it has only one designated value). The only differences are that certain of the background principles need to be altered: given an n-valued logic (n being "most false"), we have

(USV′) $(J_1\Phi \vee J_2\Phi \vee \cdot\ \cdot\ \cdot \vee J_n\Phi)\ \&\ \sim(J_i\Phi\ \&\ J_k\Phi)$, for $i \neq k$

(E′) if $(\phi \leftrightarrow \psi)$ is semantically valid, then so $(J_i\phi \leftrightarrow J_i\psi)$, for any $1 \leq i \leq n$

(J_i-$\forall'$) $J_i(\forall\ x)Fx \rightarrow \sim(\exists\ x)J_nFx$, for any $i \neq n$

(lemma′) $\sim J_i\phi$ (for any i other than 1 and n), if all sentences in ϕ are already in the scope of some J operator

which are all clearly as justified in an n-valued logic as their correspondents were in the three-valued case. The principles

(J_1) $J_1\phi \rightarrow \phi$

(refl′) $J_1\alpha = \alpha$

(LL) $a = b \leftrightarrow (\forall\ F)(Fa \leftrightarrow Fb)$

remain the same. The argument now is: suppose an object a is vague, then the vagueness-in-reality theorist is committed to $J_i\ (a = b)$ for some $1 < i < n$.

a. $J_i\ (a = b)$	Assumption
b. $a = b \leftrightarrow (\forall\ F)(Fa \leftrightarrow Fb)$	(LL)
c. $J_i\ (a = b) \leftrightarrow J_i\ (\forall\ F)(Fa \leftrightarrow Fb)$	from (b) and (E′)
d. $J_i\ (\forall\ F)(Fa \leftrightarrow Fb)$	from (a), (b) $\leftrightarrow$ elimination
e. $\sim(\exists\ F)J_n\ (Fa \leftrightarrow Fb)$	from (d) and (J_i-$\forall'$)
f. $(\forall\ F)[J_1\ (Fa \leftrightarrow Fb) \vee \ldots \vee J_{n-1}\ (Fa \leftrightarrow Fb)]$	from (e) and (USV′)
g. $J_1(J_1\ (a = a) \leftrightarrow J_1\ (a = b)) \vee \ldots \vee J_{n-1}\ (J_1\ (a = a) \leftrightarrow J_1\ (a = b))$	from (f), instantiate $(\forall\ F)$ to $(\lambda x)J_1\ (a = x)$ and λ convert
h. $J_1\ (J_1\ (a = a) \leftrightarrow J_1\ (a = b))$	from (g) and (lemma′)
i. $J_1\ (a = a) \leftrightarrow J_1\ (a = b)$	from (h) and (J_1)
j. $J_1\ (a = b)$	from (i) and (refl′)
k. $\sim J_i\ (a = b)$	from (j) and (USV′)

Finally, the argument also is valid in any infinite-valued logic in which we can formulate it (and which has principles similar to those listed above). The restriction about being able to formulate it amounts to requiring at most a denumerable number of truth values, or else the language will not have the ability to form the required number of J operators. All the principles listed above seem true in any plausible infinite-valued logic, including the (J_i-$\forall$) principle. [Often in infinite-valued logics the semantic value of $(\exists\ x)Fx$ is defined as the limit of the values of $F\alpha$, for all α; and this means there need be no particular α for which $F\alpha$ takes that value. This, in turn, means that there is no easy way to state a general relationship

between J formulas with embedded quantifiers and those where the quantifier is outside the scope of the J operator. But, since we require only an analogue of (J_i-$\forall$), such difficulties will not arise for our argument.[14]]

These arguments are similar to Evans's original one, with the exception of making it explicit that we are operating in a many-valued environment and making explicit precisely what principles are involved. The conclusion is: no standard logic, not even a standard many-valued logic, can admit vague objects, if that is taken (as our theorists wish) to include the possibility of making vague identity statements, under pain of explicit, two-valued contradiction. Parsons has objected to this conclusion on the grounds that "it proves too much." If this argument is correct, he says, then a similar argument "can be used to refute the existence of indeterminacy in truth value of ordinary claims." I agree that the conclusion follows (under the plausible requirements laid out above), but draw a different conclusion than the one implicit in Parsons. Rather than show the impossibility of vagueness, or indeterminateness, I think it shows the incorrectness of analyzing these concepts by means of a many-valued logic. Instead, vagueness ought to be viewed as a semantic notion and investigated by means of different evaluation techniques laid atop a classical logic.[15] But this is just to deny that there is any such thing as vagueness-in-reality.

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[14] For a different set of objections to such "fuzzy logics," see C. Morgan and my "Some Notes on Fuzzy Logic," *Linguistics and Philosophy*, I, 1 (1977): 87–121.

[15] About the penultimate sentence of the text, Sylvan wishes me to emphasize that a relevance logic might be used instead of a classical logic. But we agree on the point that vagueness is a semantic notion to be laid atop the logic.