

**Is Compositionality Formally Vacuous?**

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“We prove a theorem stating that any semantics can be encoded as a compositional semantics, which means that, essentially, the standard definition of compositionality is formally vacuous,” says Zadrozny (1994: 329). We disagree.

Consider the following very simple language  $L$ , that consists of only five strings. It is generated by a grammar with only three symbols (A, B, C) in its alphabet and one syntactic operation (concatenation).<sup>1</sup> We explicitly indicate this syntactic concatenation by the symbol  $\bullet$ . The language  $L$  consists of just the strings A, B, C,  $C\bullet A$ ,  $C\bullet B$ . The semantics for  $L$  is given by a meaning function,  $m$ . We will not bother to state specifically what the meanings are for the various expressions of  $L$ , but will just say what are the conditions on the meaning which determine that the semantics is non-compositional: Each of the five strings in the language has a meaning; furthermore

- (1) a.  $m(A) = m(B)$
- b.  $m(C\bullet A) \neq m(C\bullet B)$

Even though we have not said *what* the meanings of the various expressions of  $L$  are, we have required that any such specific semantics must be non-compositional because the meaning of  $C\bullet A$  couldn't be a function just of the meanings of C and of A; for if it were, then the meaning of  $C\bullet A$  would be the same as that of  $C\bullet B$ , since the meanings of A and of B are the same.<sup>2</sup>

Yet Zadrozny claims that this semantics can be encoded compositionally. What's going on?

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<sup>1</sup> Zadrozny considers only grammars having concatenation as their sole syntactic rule. He clearly believes that the points he makes will carry over to grammars with more intricate syntactic rules and with a rich non-terminal vocabulary. But his article does not expand on this, and we will merely follow the sort of examples he considers.

<sup>2</sup> Of course, if  $C\bullet A$  and  $C\bullet B$  were not complex terms they could compositionally be given meanings that obey our restrictions. They could just be treated as atomic elements and assigned basic meanings. (A kind of vacuous compositionality). But given that they are syntactically complex terms of  $L$ , they then violate the condition that their meanings be a function only of the meanings of the component parts.

Here's what.<sup>3</sup> For each meaningful string of  $L$ , Zadrozny associates a (distinct) function. This function is defined as applying only to the one expression, plus to any of the other functions defined for the other expressions of  $L$ , so long as the syntactic concatenation of those expressions is allowed in the language. (For our simple language, we allow the function associated with  $C$  to apply to the function associated with  $A$  and also to the function associated with  $B$ ). For this language we will have the set  $F$  consisting of five functions:  $f_1, f_2, f_3, f_4, f_5$ , such that:

- (2)a.  $f_1(A) = m(A)$ ;  $f_1$  is undefined for  $B, C, C \bullet A, C \bullet B$
- b.  $f_2(B) = m(B)$ ;  $f_2$  is undefined for  $A, C, C \bullet A, C \bullet B$
- c.  $f_3(C) = m(C)$ ;  $f_3$  is undefined for  $A, B, C \bullet A, C \bullet B$
- d.  $f_4(C \bullet A) = m(C \bullet A)$ ;  $f_4$  is undefined for  $A, B, C, C \bullet B$
- e.  $f_5(C \bullet B) = m(C \bullet B)$ ;  $f_5$  is undefined for  $A, B, C, C \bullet A$
- f.  $f_3$  is defined for  $f_1$ :  $f_3(f_1) = f_4$
- g.  $f_3$  is defined for  $f_2$ :  $f_3(f_2) = f_5$
- h. none of the functions is defined for any other arguments

The existence of the set  $F$  of functions is guaranteed by the existence of the meanings presumed by  $m$ , although in the general case we will need a non-well-founded set theory to guarantee the existence of arbitrary such functions, since if an expression can be syntactically concatenated with itself this would give rise to a function having itself in its own domain and range.

From these five functions we can define another function,  $\mu$ :

- (3)  $\mu(A) = f_1$
- $\mu(B) = f_2$
- $\mu(C) = f_3$
- $\mu(C \bullet A) = f_4$
- $\mu(C \bullet B) = f_5$

Now note that from these definitions we have

- (4)  $\mu(C \bullet A) = f_4 = f_3(f_1) = \mu(C)(\mu(A))$
- $\mu(C \bullet B) = f_5 = f_3(f_2) = \mu(C)(\mu(B))$

And this shows that  $\mu$  is compositional. And from the theorems that guarantee that this can always be done, Zadrozny concludes that we can always construct a compositional meaning function that will encode any semantics we wanted to encode.

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<sup>3</sup> Our exposition departs somewhat from Zadrozny's account, although this difference does not affect the main point of his Theorem 1 and Corollary 2. (Our example follows the main point of his Corollary 2).

We don't think this is the correct interpretation of the situation. How can it be, since the semantics of the language is *by definition* non-compositional! Rather than Zadrozny's conclusion, here's how we see the situation. We are given a set,  $M$ , of four meanings:

- (5)      Meanings  
 $m_1 (= m(A) \text{ and } m(B))$   
 $m_3 (= m(C))$   
 $m_4 (= m(C \bullet A))$   
 $m_5 (= m(C \bullet B))$

Zadrozny's proof, applied to this example, would have us consider the set,  $F$ , of five functions associated with  $M$ :

- (6)      Meanings                       $F$   
 $m_1$                                        $f_1$   
     $f_2$   
 $m_3$                                        $f_3$   
 $m_4$                                        $f_4$   
 $m_5$                                        $f_5$

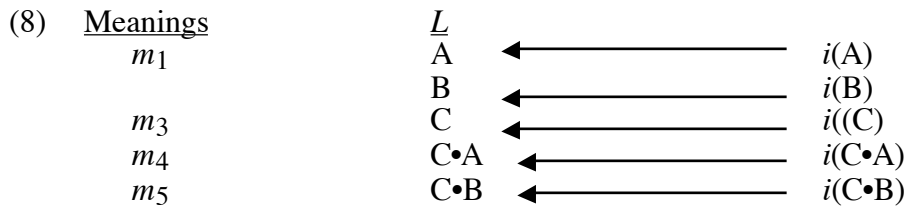
It will be recalled that  $f_1 \dots f_5$  are *functions*. These functions are not themselves meanings, but rather yield meanings when applied to an appropriate element of the language:  $f_1(A)=m_1$ ,  $f_2(B)=m_1$ ,  $f_4(C \bullet A)=m_4$ , etc. Note that all these functions, even  $f_1$  and  $f_2$ , are *distinct* functions, for they have different items in their domain ( $f_1$  is not defined for B nor is  $f_2$  defined for A).

Now we add the function  $\mu$  to the picture:

- (7)      Meanings                       $F$   
 $m_1$                                        $f_1$  ←  $\mu(A)$   
     $f_2$  ←  $\mu(B)$   
 $m_3$                                        $f_3$  ←  $\mu(C)$   
 $m_4$                                        $f_4$  ←  $\mu(C \bullet A)$   
 $m_5$                                        $f_5$  ←  $\mu(C \bullet B)$

If the functions in  $F$  were the meanings relevant to the language, then  $\mu$  would be a compositional meaning function for the language. But they aren't, because the meaning of A is the same as the meaning of B, but  $f_1$  is not the same as  $f_2$ . After all,  $f_1(A)$  yields the meaning of A, and that's the same as the meaning of B, yet  $f_1$  is not even defined for B. The members of  $F$  are functions from the expressions of the language to meanings, but they are not themselves meanings.

Zadrozny has given us a way to map compositionally each expression of the language with something that is characteristically associated with its meaning, by using  $\mu$  and the elements of  $F$ . But there is no reason to believe that this is to give a compositional account of the meanings of the expressions of the language. In fact, his detour through complex functions is not necessary for the sort of case Zadrozny discusses. Consider the following modification of Zadrozny's method, where we replace the set of  $F$ 's with the set of well-formed expressions of the language and replace the function  $\mu$  with an identity function,  $i$ . We then have this picture:



Note that  $i$  is a compositional function as well:  $i(C \bullet A) = i(C) \bullet i(A)$ . Furthermore note that each member of L is characteristically associated with the meaning of some item of L. And so we might conclude, like Zadrozny, that we have given a way to compositionally map each expression of the language with something that is characteristically associated with its meaning, by using  $i$  and the elements of the language L. Yet what we have done certainly is not the same as giving a compositional semantic theory for L. Thus establishing the existence of  $\mu$  is no more significant to establishing compositional semantics than is establishing the existence of  $i$ .

There may be a temptation to think that this “identity mapping” (associated with each expression is that expression itself) is nevertheless a theory of meaning, perhaps a purely syntactic theory of meaning, in that the item associated with each word forms a string that mirrors exactly the syntax of L and we are then invited to associate whatever meaning we want with any complex expression. But this is a mistake.

Zadrozny's theory is not a theory of meaning. It is perhaps a theory of “pointers to meanings”, and it is compositional; but it is not true that the meaning, in the original intuitive sense, has been used to construct a compositional theory of meaning. It is not true that we have here encoded the original semantics as a compositional semantics. The meanings of the complex

sentences are not functions of the meanings of the words – of course they couldn't be, since the initial semantics was by definition not compositional! – they are instead items that are characteristically associated with meanings. As we said, this is perhaps a compositional theory of *something*. But not of meaning.

Perhaps Zadrozny is postulating yet another level of analysis where compositionality is *guaranteed* to hold? Perhaps so; but this level is not meaning in whatever our initial sense was, and so it is not true that our semantics has been “encoded as a compositional semantics.” And since Zadrozny has not shown that any semantics can be encoded as a compositional semantics, he has not shown that “the standard definition of compositionality is formally vacuous.”

## References

Zadrozny, W. (1994) “From Compositional to Systematic Semantics” *Linguistics and Philosophy* **17**: 391-428.