Stat 285. Homework 1 (review) (Due on Monday Jan 10 by 5:00pm, 2023)
Please submit the homework via the course's canvas page.

1. Toss a coin three times.

(i) List the sample space.

(ii) Let $A = \{\text{getting exactly one head}\}$ and $B = \{\text{the first is head}\}$. Find (a) P($A$) and P($B$), and (b) P($A \bigcup B$) and P($A \bigcap B$).

(iii) Answer the following questions: (a) Are A and B mutually exclusive? Why? (b) Are A and B independent? Why?

2. Suppose that the probability of suffering a side effect from a certain flu vaccine is 0.003 per person.

(i) Find the probability that 3 out of 100 randomly selected people having the vaccine suffer the side effect.

(ii) People having the vaccine are checked until 1 person suffering the side effect is found. What is the probability that the total number of people checked is 50?

(iii) If $1,000$ people have the flu vaccine, find an approximate probability that less than 5 people suffer the side effect.

3. Suppose 911 calls received over time in a region follow a Poisson process. Let X be the number of 911 calls within a ten-minute time period in the region with mean 3. Let Y equal the time (in minutes) to the first 911 call within the time period.

(i) What is the distribution of Y? Give its probability density function.

(ii) Give $E(X)$ and $Var(X)$.

(iii) Give $E(Y)$ and $Var(Y)$.

(iv) Check whether $P(Y \geq 2)$ and $P(Y \geq 6|Y \geq 4)$ are the same. Explain your finding.

4. Suppose X_1, X_2, X_3 are mutually independent and have the same distribution with p.d.f. $f(x) = 3e^{-3x}$, $0 < x < \infty$ (the exponential distribution). Find

(i) $P(1.0 < X_1 < 2.5, 2.2 < X_2 < 3.5)$ and $P(1.0 < X_1 < 2.5, 2.2 < X_3 < 3.5|X_2 < 4)$;

(ii) Give $E(X_1)$ and $Var(X_1)$.

(iii) $E(2X_1 + X_2 - 4X_3)$ and $Var(2X_1 + X_2 - 4X_3)$;

(iv) $E[X_1(X_2 - 1/3)^2]$ and $E[X_1(X_2 - 1/3)^2(X_3 - 1/3)^2]$.

5. A candymaker produces mints that have a label weight of 10 grams. Assume that the distribution of the weights of these mints is $N(10, 1)$.

(i) Let X be the weight of a single mint selected at random from the production line. Find $P(X < 9.9)$ and $P(9.9 < X < 10.2)$.

(ii) During a particular shift 100 mints are selected at random and weighed. Let $\bar{X}_{100}$ equal the sample mean of the 100 mints selected, i.e., $\bar{X}_{100} = \frac{1}{100}(X_1 + X_2 + \ldots + X_{100})$. Find $P(9.9 < \bar{X}_{100} < 10.2)$.

(iii) Let Y be the number of the mints that weigh less than 9.9 within the 100 selected ones, find $P(Y \leq 20)$.