What to do today (Apr 4, 2023)?

Part 1. Introduction and Review (Chp 1-5)
Part 2. Basic Statistical Inference (Chp 6-9)
Part 3. Important Topics in Statistics (Chp 10-13)
Part 4. Further Topics (Selected from Chp 14-16)

§4.1 Distribution-Free Procedures (Chp15.1, 15.2)
§4.1.1 Basic Concepts
§4.1.2 Nonparametric Testing Procedures

§4.2 Quality Control Methods (Chp16.1, 16.2, 16.3)
§4.2.1 Introduction
§4.2.2 Examples of Control Charts

Some Logistics.

- Review Quiz 2 Question.
- Homework 11 has been assigned. It's due on Monday Apr 10.
- April 11 is the last day of classes in spring 2023.
- The final exam is scheduled for Apr 22 15:30.

§4.2 Quality Control Methods (Chp16.1, 16.2, 16.3)

§4.2.1 Introduction

Why to consider quality control? What is a quality control method?

Statistical Process Control: with the goal to make a process stable over time and keep it stable unless planned changes are made.

Statistical Control

- A variable that continues to be described by the same distribution when observed over time is said to be **in statistical control**, or simply in control.
- A process in control has only common cause variation.

  When the normal functioning of the process is disturbed by some unpredictable event, special cause variation is added to the common cause variation.

Control charts are statistical tools that monitor a process and alert us when the process has been disturbed so that it is now out of control.

There is a strong analogy between control charts and hypothesis testing.

- *Null Hypothesis:* The process is in control.
- *Type I error:* An in-control process yields a point outside the control limits (an out-of-control signal).
- *Type II error:* An out-of-control process produces a point inside the control limits (an in-control signal).

§4.2.2A Control Charts for Process Location

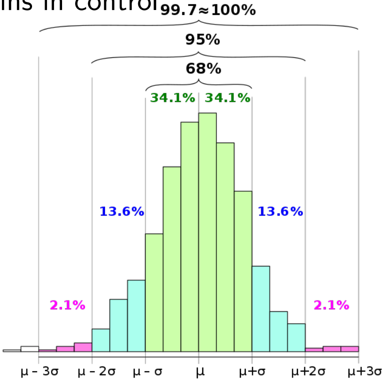
(i) The $\bar{X}$ Chart Based on Known μ and σ

- Process Monitoring Conditions
  - A quantitative variable $X \sim N(\mu, \sigma)$.
  - The process has been operating in control for a long period, so that the process mean μ and sd σ are known as long as the process remains in control.

- Making a Control Chart (e.g. 3σ Chart)
  - Draw a solid center line at μ, and draw dashed control limits at $\mu \pm 3\sigma/\sqrt{n}$
  - Take samples of size n from the process and plot the means of these samples against the order in which they were taken.

- $\bar{X}$ charts produce an **out-of-control** signal if
  - One-point out: A single point lies outside the 3σ control limits.
  - Run: The $\bar{X}$ chart shows 9 consecutive points above the center line or 9 consecutive points below the center line.

- Rationale of the $\bar{X}$ charts for process monitoring: Since $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$, the control limits mark off the range of variation in sample means that we expect to see when the process remains in control



68-95-99.7 rule: From Wikipedia, the free encyclopedia

(ii) The $\bar{X}$ Chart Based on Unknown μ and σ

- Provided k independent calculated sample means and sample standard deviations with size n, $(\bar{x}_1, s_1), \ldots, (\bar{x}_k, s_k)$
  - $\hat{\mu} = \bar{\bar{x}} = \sum_{l=1}^{k} \bar{x}_l \big/ k$
  - Since $E(S) = a_n \sigma$ with $a_n = \sqrt{2}\Gamma(n/2) \big/ \sqrt{n-1} \big/ \Gamma\big((n-1)/2\big)$, $\hat{\sigma} = \bar{s} \big/ a_n$ with $\bar{s} = \sum_{l=1}^{k} s_l \big/ k$

the UCL/LCL:

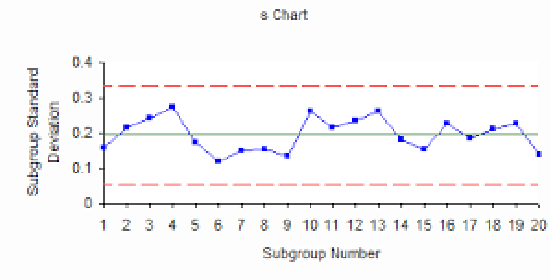
$$\hat{\mu} \pm 3\frac{\hat{\sigma}}{\sqrt{n}} \implies \bar{\bar{x}} \pm 3\frac{\bar{s}}{a_n\sqrt{n}}$$

§4.2.2B Control Charts for Process Variation

(i) The S Chart

- Suppose k independently selected samples are available, each with n observations on $X \sim N(\mu, \sigma^2)$.
  The sample standard deviations are $s_1, \ldots, s_k$ with $\bar{s} = \sum_{l=1}^{k} s_l / k$.
- Since $Var(S) = \sigma^2 - (a_n\sigma)^2 = \sigma^2(1 - a_n^2)$, using $\hat{\sigma} = \bar{s}/a_n$,

$$CL = \bar{s}; \quad UCL/LCL = \bar{s} \pm 3\bar{s}\frac{\sqrt{1 - a_n^2}}{a_n}$$



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(ii) The R Chart

$r_1, \ldots, r_k$ are k sample ranges and $\bar{r} = \sum_{l=1}^{k} r_l \big/ k$.

$$CL = \bar{r}; \quad UCL/LCL = \bar{r} \pm 3\frac{\bar{r} c_n}{b_n}$$

with b_n and c_n constants upon given n.

§4.2.2C Other Types of Control Charts

- e.g. p chart: control charts for sample proportions

$$CL = p; \quad UCL/LCL = p \pm 3\sqrt{\frac{p(1-p)}{n}}$$

“*Statistics is the science of learning from data.*”

- By processing/summarizing the data: tabulating/plotting
- By making inferences with the data, to understand uncertainties using the limited information

How do we go beyond the data?

Statistical Thinking (“The Basic Practice of Statistics”, 6th Edn, by Moore et al.)

- Data are numbers with a context.
- Where the data come from matters.
- Always look at the data.
- Beware the lurking variable.
- Variation is everywhere.
- Conclusions are not certain.

What will we do next?

Material Studied Upto Now

- Part 1. Introduction and Review (Chp 1-5)
- Part 2. Basic Statistical Inference (Chp 6-9)
- Part 3. Important Topics in Statistics (Chp 10-13)
- Part 4. Further Topics (Selected from Chp 14-16: 15.1-2; 16.1-3)

Review for Final Exam on Tue Apr 11

Review A. Outline What Has Been Studied

Review B. Working on A Few Examples

Final Exam on Apr 22 (50% in the Final Evaluation)