What to do today (Dec 2, 2020)?

1. Introduction

2. Probability and Distribution (Chp 1-3)

3. Essential Topics in Mathematical Statistics

4. Further Topics, Selected from Chp 7-11

- ▶ *4.1 Nonparametric and Robust Statistics (Chp 10.1-4, 10.8-9)*

- ▶ **4.2 Bayesian Procedures (Chp 11.1)**
    - ▶ *4.2.1 Prior and Posterior Distributions*
    - ▶ *4.2.2 Bayesian Point Estimation*
    - ▶ *4.2.3 Bayesian Interval Estimation and Testing*
    - ▶ **4.2.4 Additional Topics in Bayesian Statistics**

Plan for the rest of the term

4.2 Bayesian Procedures (Chp 11)

- **Prior and Posterior Distributions**

  $X_1, \ldots, X_n$ are iid obs on $X \sim f(x|\theta)$.

  - The likelihood function of θ with the data $\mathbf{x}$ is
    $L(\theta|\mathbf{x}) = \prod_{i=1}^{n} f(x_i|\theta)$.
  - Suppose $\Theta \sim h(\theta)$, a **prior distribution**.
  - The **posterior disttribution** with data $\mathbf{x}$ is
    $k(\theta|\mathbf{x}) \propto L(\theta|\mathbf{x})h(\theta)$.

- **Bayesian Point Estimation**

  Provided with a loss function $\mathcal{L}(\theta, \delta(\mathbf{x}))$ measures the difference between θ and $\delta(\mathbf{x})$,

  $$\hat{\theta}(\mathbf{x}) = argmin_{all\ \delta(\mathbf{x})}\left(E\left[\mathcal{L}(\Theta, \delta(\mathbf{x}))\right] \right).$$

  - When the loss function is the squared difference
    $\mathcal{L}(\theta, \delta(\mathbf{x})) = (\delta(\mathbf{x}) - \theta)^2$,

  $$\hat{\theta}(\mathbf{x}) = E(\Theta|\mathbf{x}) = \int \theta k(\theta|\mathbf{x})d\theta.$$

- **Bayesian Interval Estimation and Testing**
    - **Interval Estimation**. Interval $\big(u(\mathbf{x}), v(\mathbf{x})\big)$ is chosen such that

    $$1 - \alpha = P\big[u(\mathbf{x}) < \Theta < v(\mathbf{x})\big|\mathbf{x}\big] = \int_{u(\mathbf{x})}^{v(\mathbf{x})} k(\theta|\mathbf{x})d\theta.$$

    (*credible inverval*)

    - **Testing Procedure**. To test on $H_0 : \theta \in \Omega_0$ vs $H_1 : \theta \in \Omega_1$:

    Accept H_0 if $P\big(\Theta \in \Omega_0|\mathbf{x}\big) \geq P\big(\Theta \in \Omega_1|\mathbf{x}\big)$; otherwise, reject H_0.

Comments:

- Need to choose an appropriate *prior* distn.

- The (classical) bayesian procedures assume the prior distn is fully known. Often considered $\Theta \sim h(\theta|\phi)$ with unknown ϕ $\implies$ *Empirical Bayesian Statistics*.

- Bayesian procedures are often computationally intensive. $\implies$ many modern statistical procedures to overcome the difficulty ... ...

4.2.4 Additional Topics in Bayesian Statistics

- **More Bayesian terminology**

  Consider a population distribution $f(\cdot|\theta), \theta \in \Omega$

  - A class of prior pdfs for θ is a **conjugate family of distribution** for $f(\cdot|\theta)$ if the posterier pdf of θ is in the same family of distributions as the prior.

  - A **noninformative prior** is a prior that treats all values of θ the same.

- **Modern Bayesian statistics**
  - empirical Bayesian approaches, hierarchical Bayesian models, ...

  - Gibbs sampler, Markov Chain Monte Carlo (MCMC), ...

What have we studied?

1. **Introduction**

2. **Probability and Distribution (Chp 1-3)**
   - 2.1 Probability (Chp1.1-4)
   - 2.2 Random Variable and Distribution (Chp1.5-10)
   - 2.3 Multivariate Distribution (Chp2)
   - 2.4 Some Special Distributions (Chp3)

3. **Essential Topics in Mathematical Statistics (Chp 4-6)**
   - 3.1 Elementary Statistical Inferences (Chp 4)
   - 3.2 Consistency and Limiting Distributions (Chp 5)
   - 3.3 Maximum Likelihood Methods (Chp 6)

4. **Further Topics, Selected from Chp 7-11**
   - 4.1 Nonparametric and Robust Statistics (Chp 10.1-4, 10.8-9)
   - 4.2 Bayesian Statistics (Chp 11.1)

What will we do next?

To Prepare for Final Exam

- ▶ (A). Logistics for the Final Exam

- ▶ (B). Summary of the Covered Material

What will we do next?

- ▶ Homework 6 is due today.

- ▶ No class and no tutorial from next week (Dec 7-11).

  - ▶ A summary of the material covered in STAT 330 will be posted in both the course's webpage and canvas page.

  - ▶ I'm going to have office hours during 10:30-12:30 PST on both Dec 7 Monday and Dec 9 Wednesday, via the Zoom link: (https://sfu.zoom.us/j/81821922544; Password: 278478)

- ▶ Final Exam on Fri Dec 17 during 8:00-18:00.

Logistics for the Final Exam

- ▶ Time: You may access the exam questions during Thursday Dec 17 8:00-18:00 PST and use a 4hr window to write the exam.
- ▶ Open-book exam: collaboration in any form is prohibited.
- ▶ Material to be covered: all the material covered by the lectures; Textbook Chp 1-6, 10.1-4, 10.8-9, 11.1
- ▶ Office hours during the exam period:
  - ▶ Dec 7 Mon and Dec 9 Wed: 10:30-12:20 PST (https://sfu.zoom.us/j/81821922544; Password: 278478)
  - ▶ Dec 14 Mon, Dec 15 Tue, and Dec 16 Wed: 15:00-17:00 PST (https://sfu.zoom.us/j/83111428821; Password: 617805)
- ▶ Review the final exam grading during 10:00-13:00 Jan 8 Fri 2021 via the Zoom link:

(https://sfu.zoom.us/j/89120489336; Password: 242547)

Thank you

... ... and good luck on the final exam!