What to do today (Nov 4, 2020)?

1. Introduction

2. Probability and Distribution (Chp 1-3)

3. Essential Topics in Mathematical Statistics

3.1 Elementary Statistical Inferences (Chp 4)

3.2 Consistency and Limiting Distributions (Chp 5)

- **3.2.1 Convergence in Probability**
- **3.2.2 Convergence in Distribution**

3.3 Maximum Likelihood Methods (Chp 6)

4. Further Topics, Selected from Chp 7-11

Example 3.6 Suppose $X_1, \ldots, X_n$ is a random sample from the uniform distn $U(0, \theta)$.

- $Y_n = \max(X_1, \ldots, X_n)$ can be a "good estimator" of θ.
- Furhter, let $W_n = n(\theta - Y_n)$. $W_n \xrightarrow{D} W$, which follows the exponential distn $NE(\theta)$.

3.2.2 Convergence in Distribution: Some Related Material

Definition. A sequence of rvs $\{X_n\}$ is **bounded in probability** if $\forall\epsilon > 0$, there exists a constant B_ϵ and integer N_ϵ such that

$$n \geq N_\epsilon \Longrightarrow P\big(|X_n| \leq B_\epsilon\big) \geq 1 - \epsilon.$$

- If $X_n \xrightarrow{D} X$, $\{X_n\}$ is bounded in probability.
- If $\{X_n\}$ is bounded in probability and $Y_n \xrightarrow{P} 0$, then $X_n Y_n \xrightarrow{P} 0$.

3.2.2 Convergence in Distribution: Some Related Material

Theorem. (Δ-Method) Assume the sequence of rvs $\{X_n\}$ satisfies $\sqrt{n}(X_n - \theta) \xrightarrow{D} N(0, \sigma^2)$. If the function $g(x)$ is differentiable at θ and $g^{'}(\theta) \neq 0$, then

$$\sqrt{n}\big(g(X_n) - g(\theta)\big) \xrightarrow{D} N\Big(0, \sigma^2\big[g^{'}(\theta)\big]^2\Big).$$

Proof:

3.2.2 Convergence in Distribution: Some Related Material

Theorem. Consider the sequence of rvs $\{X_n\}$ with the mgf of X_n as $M_{X_n}(t)$ for $-h < t < h$, and a rv X with mgf $M_X(t)$ for $-h < t < h$. If $\lim_{n\to\infty} M_{X_n}(t) = M_X(t)$ for $-h < t < h$, then $X_n \xrightarrow{D} X$.

Example for its application: *to show "$B(n,\theta) \approx Poisson(n\theta) \approx N(n\theta, n\theta(1-\theta))$ when $n >> 1$".*

What will we do next?

1. *Introduction*

2. *Probability and Distribution (Chp 1-3)*

3. **Important Topics in Mathematical Statistics**

   - *3.1 Elementary Statistical Inferences (Chp 4)*
   - *3.2 Consistency and Limiting Distributions (Chp 5)*
   - **3.3 Maximum Likelihood Methods (Chp 6)**

Midterm 2 during Wednesday Nov 11-13 2020

4. *Further Topics, Selected from Chp 7-11*