STAT 330. Midterm 1 - Quesiton 1
(October 7 - 9, 2020)

Name: **Student ID:**

Q1.[10 points] Choose to work on two out of the following three questions.

[5] (i) Assume that two events A and B are mutually exclusive and independent. Prove that $\mathrm{P}(B) = 0$ if $\mathrm{P}(A) = 1/9$.

[5] (ii) Which of that the following functions is *not* a cumulative distribution function (cdf)? Why?

$$F_1(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \le x < \infty \end{cases} \quad \text{and} \quad F_2(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < \infty. \end{cases}$$

[5] (iii) Let $X \ge 0$ and $Y \ge 0$ be the lifetimes of the two brakes on an airplane. Suppose $\mathrm{E}(X) = \mathrm{E}(Y) = \theta > 0$. If the lifetime of the airplane's brake system is $T = \max(X, Y)$, prove that the expected lifetime of the brake system is between θ and 2θ, that is, $\theta \le \mathrm{E}(T) \le 2\theta$.

Q1 (i) Proof. $\because A \perp\!\!\!\perp B \quad \therefore P(AB) = P(A)P(B) = \frac{1}{9}P(B)$.

on the other hand, $\because$ A and B are mutually exclusive

$\therefore P(AB) = 0$

$\Longrightarrow \frac{1}{9}P(B) = 0$. Thus $P(B) = 0$

Q1 (ii) Solution. $F_2(x)$ is not a cdf since $\lim\limits_{x \to +\infty} F_2(x) = +\infty \neq 1$.

Q1 (iii) Proof. $\because X \le T = \max(X, Y) \le X + Y$

$\therefore \mathrm{E}(X) \le \mathrm{E}(T) \le \mathrm{E}(X+Y)$

θ $\quad$ 2θ

Thus, $\theta \le \mathrm{E}(T) \le 2\theta$.