STAT 330. Midterm 1 - Question 2
(October 7 - 9, 2020)

Name: **Student ID:**

Q2.[20 points] Consider the experiment of flipping an *uneven* coin twice independently. Suppose the probability of getting a head with one flip is 1/3. Answer the following questions.

[5] (i) Write down the sample space of the experiement.

[5] (ii) Let X_1 be the number of heads from the first flip; X_2, the second flip. Give the joint pmf of (X_1, X_2).

[5] (iii) Let Y be the number of heads from the experiement: $Y = X_1 + X_2$. Obtain $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$.

[5] (iv) If $W = X_1X_2$, what is $\mathrm{Cov}(Y, W)$?

Solution (i) $S = \{(H,H), (H,T), (T,H), (T,T)\}$

(ii) All the possible values of X_1, X_2 are 0, 1.

The following table gives the joint pmf of X_1 and X_2:

x_1 \ x_2	1	0
1	$(\frac{1}{3})(\frac{1}{3})$	$(\frac{1}{3})(\frac{2}{3})$
0	$(\frac{2}{3})(\frac{1}{3})$	$(\frac{2}{3})(\frac{2}{3})$

$p(x_1, x_2)$ for $x_1, x_2 = 0, 1$.

$p(1,1) = \frac{1}{9}$

$p(1,0) = p(0,1) = \frac{2}{9}$

$p(0,0) = \frac{4}{9}$

(iii) $\because Y = X_1 + X_2$, $E(X_1) = E(X_2) = 1(\frac{1}{3}) + 0(\frac{2}{3}) = \frac{1}{3}$

$E(X_1^2) = E(X_1)$

$V(X_1) = V(X_2) = 1(\frac{1}{3}) - (\frac{1}{3})^2 = \frac{1}{3}(\frac{2}{3}) = \frac{2}{9}$

$\therefore E(Y) = 2/3$, $V(Y) \overset{X_1 \perp\!\!\!\perp X_2}{=} 2(\frac{2}{9}) = 4/9$

(iv) $\mathrm{Cov}(Y, W) = E(YW) - E(Y)E(W)$

$E(W) = E(X_1)E(X_2) = \frac{1}{9}$, $E(YW) = E(X_1^2X_2 + X_1X_2^2)$

$= 2E(X_1^2)E(X_2) = \frac{2}{9}$

$\Rightarrow \mathrm{Cov}(Y, W) = \frac{2}{9} - \frac{2}{3}(\frac{1}{9})$

$= 4/27$