

STAT 330. Midterm 1 - Question 3
(October 7 - 9, 2020)

Name: _____

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Q3.[20 points] Suppose the joint pdf of (X, Y) is $f(x, y) = \begin{cases} 3e^{-(x+3y)}, & 0 \leq x, y < \infty \\ 0, & \text{otherwise.} \end{cases}$ Note that $E(X) = 1$ and $E(Y) = 1/3$. Answer the following questions.

- [5] (i) Obtain $\text{Cov}(X, Y)$.
- [5] (ii) Find the marginal pdf of X .
- [5] (iii) Obtain $P(X \leq 1, Y > 1/3)$.
- [5] (iv) Obtain $E(3XY + X + 3Y)$ and $E(XY^3 + 2019Y^2 | Y = 1)$.

Solution.

(i) $\because f(x, y) = \underbrace{f_X(x)}_{\begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}} \underbrace{f_Y(y)}_{\begin{cases} 3e^{-3y}, & y \geq 0 \\ 0, & y < 0 \end{cases}}$

$\therefore X \perp\!\!\!\perp Y \Rightarrow \text{Cov}(X, Y) = 0$

(ii) The marginal pdf of X is then $f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

(iii) $\because X \perp\!\!\!\perp Y \therefore P(X \leq 1, Y > \frac{1}{3}) = P(X \leq 1)P(Y > \frac{1}{3})$

$\Rightarrow P(X \leq 1, Y > \frac{1}{3}) = (1 - e^{-1})e^{-1}$

$\int_0^1 e^{-x} dx = 1 - e^{-1}$
 $e^{-1} = \int_{\frac{1}{3}}^{+\infty} 3e^{-3y} dy$

(iv) ① $E(3XY + X + 3Y)$

$= 3 \underbrace{E(X)}_1 \underbrace{E(Y)}_{\frac{1}{3}} + E(X) + 3E(Y) = 3$

② $E(XY^3 + 2019Y^2 | Y=1) = E(X \cdot 1^3 + 2019 \cdot 1^2)$

$= 2020$