Q1.1. $\bar{U}_n \sim N(\mu_U, \frac{\sigma_U^2}{n})$

Q1.2. $X \sim N(\mu_x, \sigma_x^2)$ with $\mu_x = \mu_U - \mu_V$

$\sigma_x^2 = \sigma_U^2 + \sigma_V^2 - 2\sigma_{UV}$

Q1.3 (i) $\because E(\bar{X}_n) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \mu$

$\therefore$ $\bar{X}_n$ is unbiased

(ii) $\because \bar{U}_n \xrightarrow{P} \mu_U$, $\bar{V}_n \xrightarrow{P} \mu_V$ by WLLN

$\therefore \bar{X}_n = \bar{U}_n - \bar{V}_n \xrightarrow{P} \mu_U - \mu_V = \mu_x$

That is, $\bar{X}_n$ is consistent.

Q1.4. Yes, S_x^2 is a consistent estimator of σ_x^2.

Note that $S_x^2 = \frac{n}{n-1}\left(\frac{1}{n}\sum X_i^2 - \bar{X}^2\right)$

$\frac{n}{n-1} \to 1$, $\frac{1}{n}\sum X_i^2 \xrightarrow{P} E(X^2)$, $\bar{X}^2 \xrightarrow{P} (EX)^2$

$\Rightarrow S_x^2 \xrightarrow{P} E(X^2) - (EX)^2 = \sigma_x^2$.