#### What have we studied in STAT 330?

(They will be covered in the final exam.)

- 1. Introduction
- 2. Probability and Distribution (Chp 1-3)
  - 2.1 Probability (Chp1.1-4)
  - 2.2 Random Variable and Distribution (Chp1.5-10)
  - 2.3 Multivariate Distribution (Chp2)
  - 2.4 Some Important Distributions (Chp3)

#### 3. Essential Topics in Mathematical Statistics (Chp 4-6)

- 3.1 Elementary Statistical Inferences (Chp 4)
- 3.2 Consistency and Limiting Distributions (Chp 5)
- 3.3 Maximum Likelihood Methods (Chp 6)

#### 4. Further Topics, Selected from Chp 7-11

- 4.1 Nonparametric and Robust Statistics (Chp 10.1-4, 10.8-9)
- 4.2 Bayesian Statistics (Chp 11.1)

#### 1. Introduction

- "Statistics is the science of learning from data."
  - By processing/summarizing the data: tabulating/plotting
  - ► By making inferences with the data ⇒ go beyond the data: to understand uncertainties using the limited information
- The methods we studied before, say, from STAT-270 and/or STAT-285, don't always work for us all the time.
  - How to choose an appropriate approach from the available ones?
  - How to develop an appropriate approach when needed?
- **STAT-330** Introduction to Mathematical Statistics
  - To provide a systematic and in-depth coverage of the material in STAT-270 and STAT-285.
  - In general, to provide the required theoretical training in studying statistics further.

## 2.1 Probability (Chp1.1-4)

- 2.1.1 Introduction
- 2.1.2 Set Theory
- 2.1.3 Definition of Probability
- 2.1.4 Conditional Probability and Independence
- the definitions of probability;
- the three basic probability rules (given in the axiom deftn by Kolmogorov), the commonly used induced rules;
- conditional probability;
- independent events

#### 2.2 Random Variable and Distribution (Chp1.5-10)

- 2.2.1 Basic Concepts
- 2.2.2 Discrete Random Variable
- 2.2.3 Continuous Random Variable
- 2.2.4 Expectation and Related
- random variable;
- distribution: pmf, pdf, cdf;
- expectation and related such as variance, mgf, and some important (eg Markov, Jensen) inequalities.

#### 2.3 Multivariate Distribution (Chp2)

2.3.1 Basic Concepts with Two Random Variables2.3.2 Conditional Distribution and Expectation2.3.3 Extension to Several Random Variable

- two random variables: joint/margianl distn, covariance
- conditional distn, independence
- several random variables: function of variables

#### 2.4 Some Important Distributions (Chp3)

- 2.4.1 Discrete Distributions
- 2.4.2 Continuous Distributions
- 2.4.3 Multivariate Distributions
- 2.4.4 Distributions Induced from Others
- discrete distn: uniform, binomial, negative binomial, hypergeometric, Poisson, ...
- ▶ continuous distn: uniform, normal, exponential, beta, ...
- multinomial, multivariate normal ...
- $\chi^2$ -distn, *t*-distn, *F*-distn, ...
- distn of a function of rvs: Jacobian of the transformation

#### 3.1 Elementary Statistical Inferences (Chp4)

- 3.1.1 Sampling and Statistics
- 3.1.2 Confidence Interval
- 3.1.3 Order Statistics
- 3.1.4 Hypothesis Testing
- 3.1.5 Statistical Simulation and Bootstrap
- random sample (iid observations);
- statistic; order statistics;
- point vs interval estimation; examples;
- hypothesis testing: hypotheses, test statistic, rejection region (or p-value);
- simulation and bootstrap

## 3.2 Consistency and Limiting Distributions (Chp5)

3.2.1 Convergence in Probability

3.2.2 Convergence in Distribution

modes of convergence;

- convergence in prob, convergence in distn;
- Weak Law of Large Numbers (WLLN), Central Limit Theorem (CLT);
- applications in statistics: Δ-method, moment generating function technique

#### 3.3 Maximum Likelihood Methods (Chp6)

3.3.1 Maximum Likelihood Estimation

3.3.2 Likelihood-Based Tests

3.3.3 EM Algorithm

- likelihood function and MLE: interpretation;
- how to compute MLE: eg, EM algorithm
- properties of MLE: consistency, asymptotic normality, asymptotic efficient, invariance
- Wald-test, likelihood ratio test (LRT);
- a duality between CI and hypothesis test
- MLE for multiple parameters (parameter vector);
- test on hypotheses of parameter vectors.

# 4.1 Nonparametric and Robust Statistics (Chp 10.1-4, 10.8-9)

- 4.1.1 Location Models
- 4.1.2 Sample Median and the Sign Test
- 4.1.3 Signed-Rank Test and MWW Test
- 4.1.4 Measures of Association
- 4.1.5 Robust Statistics Concepts

- the definitions of location models, population median, sample median
- the Sign test, the Signed-Rank test, the MWW test
- confidence intervals for the median
- Kendall's au and its estimator, Spearman's ho and its estimator
- sensitivity curve

#### 4.2 Bayesian Procedures (Chp 11.1)

- ► Bayes' Theorem:  $P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\overline{A})P(\overline{A})}$
- Bayesian Framework: Θ ~ h(θ), a prior distn. The posterior distn given data x is

$$k( heta|\mathbf{x}) = \left[rac{L( heta|\mathbf{x})}{\int L( heta|\mathbf{x})h( heta)d heta}
ight]h( heta) \propto L( heta|\mathbf{x})h( heta).$$

Point Estimation: 
$$\hat{\theta}(\mathbf{x}) = \operatorname{argmin}_{all \ \delta(\mathbf{x})} \left( E \left[ \mathcal{L}(\Theta, \delta(\mathbf{x})) \right] \right).$$
eg. if  $\mathcal{L}(\theta, \delta(\mathbf{x})) = (\delta(\mathbf{x}) - \theta)^2, \implies \hat{\theta}(\mathbf{x}) = E(\Theta | \mathbf{x})$ 

Interval Estimation : A credible inverval (u(x), v(x)) is chosen such that

$$1 - \alpha = P[u(\mathbf{x}) < \Theta < v(\mathbf{x}) | \mathbf{x}] = \int_{u(\mathbf{x})}^{v(\mathbf{x})} k(\theta | \mathbf{x}) d\theta.$$

► Testing; To test on  $H_0 : \theta \in \Omega_0$  vs  $H_1 : \theta \in \Omega_1$ : Accept  $H_0$  if  $P(\Theta \in \Omega_0 | \mathbf{x}) \ge P(\Theta \in \Omega_1 | \mathbf{x})$ ; otherwise, reject  $H_0$ .

# Thank you for your participation, ... ... good luck on the final exam!

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