What have we studied in STAT 330?

(*They will be covered in the final exam.*)

1. **Introduction**

2. **Probability and Distribution (Chp 1-3)**
   - 2.1 Probability (Chp1.1-4)
   - 2.2 Random Variable and Distribution (Chp1.5-10)
   - 2.3 Multivariate Distribution (Chp2)
   - 2.4 Some Important Distributions (Chp3)

3. **Essential Topics in Mathematical Statistics (Chp 4-6)**
   - 3.1 Elementary Statistical Inferences (Chp 4)
   - 3.2 Consistency and Limiting Distributions (Chp 5)
   - 3.3 Maximum Likelihood Methods (Chp 6)

4. **Further Topics, Selected from Chp 7-11**
   - 4.1 Nonparametric and Robust Statistics (Chp 10.1-4, 10.8-9)
   - 4.2 Bayesian Statistics (Chp 11.1)

1. Introduction

- "*Statistics is the science of learning from data.*"
  - By processing/summarizing the data: tabulating/plotting
  - By making inferences with the data
    $\Longrightarrow$ go beyond the data: to understand uncertainties using the limited information

- The methods we studied before, say, from STAT-270 and/or STAT-285, don't always work for us all the time.
  - How to choose an appropriate approach from the available ones?
  - How to develop an appropriate approach when needed?

- STAT-330 *Introduction to Mathematical Statistics*
  - To provide a systematic and in-depth coverage of the material in STAT-270 and STAT-285.
  - In general, to provide the required theoretical training in studying statistics further.

2.1 Probability (Chp1.1-4)

- the definitions of probability;
- the three basic probability rules (given in the axiom deftn by Kolmogorov), the commonly used induced rules;
- conditional probability;
- independent events

2.2 Random Variable and Distribution (Chp1.5-10)

- random variable;
- distribution: pmf, pdf, cdf;
- expectation and related such as variance, mgf, and some important (eg Markov, Jensen) inequalities.

2.3 Multivariate Distribution (Chp2)

- two random variables: joint/margianl distn, covariance
- conditional distn, independence
- several random variables: function of variables

2.4 Some Important Distributions (Chp3)

- discrete distn: uniform, binomial, negative binomial, hypergeometric, Poisson, ...
- continuous distn: uniform, normal, exponential, beta, ...
- multinomial, multivariate normal ...
- χ^2-distn, t-distn, F-distn, ...
- distn of a function of rvs: Jacobian of the transformation

3.1 Elementary Statistical Inferences (Chp4)

- random sample (iid observations);
- statistic; order statistics;
- point vs interval estimation; examples;
- hypothesis testing: hypotheses, test statistic, rejection region (or p-value);
- simulation and bootstrap

3.2 Consistency and Limiting Distributions (Chp5)

- modes of convergence;
- convergence in prob, convergence in distn;
- Weak Law of Large Numbers (WLLN), Central Limit Theorem (CLT);
- applications in statistics: Δ-method, moment generating function technique

3.3 Maximum Likelihood Methods (Chp6)

- likelihood function and MLE: interpretation;
- how to compute MLE: eg, EM algorithm
- properties of MLE: consistency, asymptotic normality, asymptotic efficient, invariance
- Wald-test, likelihood ratio test (LRT);
- a duality between CI and hypothesis test
- MLE for multiple parameters (parameter vector);
- test on hypotheses of parameter vectors.

4.1 Nonparametric and Robust Statistics (Chp 10.1-4, 10.8-9)

- the definitions of location models, population median, sample median
- the Sign test, the Signed-Rank test, the MWW test
- confidence intervals for the median
- Kendall's τ and its estimator, Spearman's ρ and its estimator
- sensitivity curve

4.2 Bayesian Procedures (Chp 11.1)

- Bayes' Theorem: $P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|\bar{A})P(\bar{A})}$
- Bayesian Framework: $\Theta \sim h(\theta)$, a **prior distn**. The **posterior distn** given data $\mathbf{x}$ is

$$k(\theta|\mathbf{x}) = \left[\frac{L(\theta|\mathbf{x})}{\int L(\theta|\mathbf{x})h(\theta)d\theta}\right]h(\theta) \propto L(\theta|\mathbf{x})h(\theta).$$

- Point Estimation: $\hat{\theta}(\mathbf{x}) = argmin_{all\ \delta(\mathbf{x})}\left(E\left[\mathcal{L}\big(\Theta, \delta(\mathbf{x})\big)\right]\right)$.
  - eg. if $\mathcal{L}\big(\theta, \delta(\mathbf{x})\big) = (\delta(\mathbf{x}) - \theta)^2$, $\Longrightarrow \hat{\theta}(\mathbf{x}) = E(\Theta|\mathbf{x})$
- Interval Estimation : A credible inverval $\big(u(\mathbf{x}), v(\mathbf{x})\big)$ is chosen such that

$$1 - \alpha = P\big[u(\mathbf{x}) < \Theta < v(\mathbf{x})\big|\mathbf{x}\big] = \int_{u(\mathbf{x})}^{v(\mathbf{x})} k(\theta|\mathbf{x})d\theta.$$

- Testing; To test on $H_0 : \theta \in \Omega_0$ vs $H_1 : \theta \in \Omega_1$: Accept H_0 if $P\big(\Theta \in \Omega_0|\mathbf{x}\big) \geq P\big(\Theta \in \Omega_1|\mathbf{x}\big)$; otherwise, reject H_0.

Thank you for your participation,

... ... good luck on the final exam!