Lecture 5: Comparison by Rate of Return

The fourth of the five strategies for comparison of investment strategies is *rate of return*. The attraction of this method is that it provides an effective interest rate, which can be compared with the rate that your resources could earn if invested in other projects, or in external institutions. Its disadvantages are that it is slightly more difficult to calculate, and there are a couple of (avoidable) pitfalls in applying it.

Internal Rate of Return

We define the *internal rate of return (IRR)* of a project as the interest rate at which the present worth of all the incomes and expenditures associated with the project would be zero. Mathematically,

IRR = i,

where *i* satisfies

(Sum from n=0 to n=N)*F_n(P/F,i,n)=0*

For example, if I lend you $100 now and you repay me $150 two years from now, my effective rate of return is given by

100=150(P/F,i,2)

So

i = square_root_of (150/100) - 1 = 22%

(This compares very favourably with what I could get from a bank.)

In most cases, the equation setting present worth equal to zero is too complicated to solve explicitly. There are then several alteratives: you can use trial and error; you can plot a graph; or, if you are fortunate enough to have a calculator with a `solve' function, you can ask the calculator to solve it for you.

This approach can, in some cases, lead to the first of the two pitfalls mentioned above. Consider the case of a strategy that leads to an immediate income of $3,000, but which in two years time obliges us to pay out $10,000, and over the subsequent four years generates an income of $2,000/year. We want to calculate the IRR for this investment.

The math is too difficult to solve explicitly, so we construct a graph. For an interest rate of 0, the present worth of the investment is simply the total of the incomes and payouts, which is $1,000. As the interest rate rises, the present value of the annuity drops. At $i=9.4\%$, the present value of the investment becomes zero, so one solution is that the IRR = 9.4%. (This is not a bad rate, so we might consider investing.) We continue to construct the graph, and find that the present worth drops to a minimum, then climbs again, crossing the axis at an interest rate of about 50%. So the problem has two solutions: IRR = 9.4% and IRR = 50%. The first rate of return is not bad, and the second is unbelievably good. But which represents reality?

To answer this, we have to figure out what is happening in practice. The reason the graph curves up again is that, as the interest rate rises, the present value of the $10,000 payout two years hence becomes insignificant compared with the $3,000 we receive initially. That is, we expect to receive so much income from investing the initial $3,000 at a high interest rate that we can afford to pay out $10,000 two years from now. But this isn't going to happen unless there's some external agency offering us 50% interest on an investment. No such agency exists, and we can't call one into existence simply by extending a graph. So the second solution to the equation is spurious.

External Rate of Return

How can we avoid such pitfalls in the general case? We have to avoid assuming that income received from the project can earn interest at unrealistically high rates. So we bring in a realistic figure for the interest we can earn by re-investing income, $k\%$, say. Now, we re-arrange the cash flows for the project, replacing all income by its future worth at the project's end, calculated at an interest rate k, and comparing this future worth with the future worth of all expenditures on the project, calculated at a (presently unknown) interest rate i^*. This i^* is the *external rate of return (ERR)* of the project.

Mathematically, we have to solve:

Sum_from_n=0_to_N Income_in_year_(N-n) (F/P,k,n) = Sum Payouts_in_year_(N-n) (F/P,i,n)*

for i^*.

Since using the ERR never gives multiple answers, while the IRR does sometimes give multiple answers, and the ERR is moreover easier to calculate, you'd think that anyone with half a brain would use the ERR. However, this is not so; the IRR is actually the more commonly used method. One reason for this is that the multiple-answers problem rarely occurs in practice.

Making a Decision

The object of calculating a rate of return is to enable us to make a decision. In the case of a single project, the decision is simple: we calculate the IRR for the project, and compare it with the minimum rate of return we consider acceptable (the MARR). What value we set this at is a matter of informed choice; it will certainly be bounded below by the interest rate available from external agencies (k). If we have a small company, chronically short of capital, we may want to set the MARR very high before we risk some of our capital on a new project.

In the case where we have several possible investment opportunities, more care is needed. Here we run into the second pitfall, as illustrated by the following example:

Option A involves an initial investment of $500, yielding a return of $1,000 two years in the future. Option

B requires an initial investment of $1,000, and yields $1,900 two years in the future. A simple calculation shows that the rate of return on A is 41%, while the rate of return on B is 37%. However, *Option B is the better option*.

The reason for this is that we want money, not a high rate of return. If we have $1,000 now -- and we must have at least that much, otherwise we couldn't consider B -- then our two options for the $1,000 are really: invest $500 in A, invest the remaining $500 in the bank; or, invest it all in B. So, supposing we've calculated the IRR on A and noted that it looks worthwhile, we now have to consider whether it's better to cancel A and invest in B, or to go ahead with A and put the spare money in the bank. To compare these two strategies, we need to calculate the *incremental* rate of return on replacing A with B. To do this, we construct a cash-flow diagram for B-A. This gives us an initial cost of $500, and an income of $900 in year 2. This corresponds to a rate of return of 34% on the $500, which is better than we could do by putting it in the bank. So we should go with the strategy of cancelling A and doing B.

Reasonable Solutions

Consider the following case: a logging company owns a thousand-acre woodlot. Trees take approximately 50 years to grow to maturity. So if the company logs 2% of the woodlot every year, they will be able to sustain the same annual harvest forever. If the value of the timber now on the lot is $P, this represents a 2% annual return on the value of the investment.

If, on the other hand, they clearcut the lot, they can obtain $P at once and earn at least 5% interest by putting it in the bank. Thus, sustainable forestry doesn't make economic sense.

Is there anything wrong with this analysis?

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