Lecture 15: Risk Analysis

Compared with other engineering disciplines, engineering economics may seem frustratingly arbitrary and imprecise -- we perform an analysis, we get a result, but then if we don't like that result, we can always go back and consider a different study period, or consider an after-tax instead of a pre-tax analysis, or assume a different inflation rate. We are always basing our analyses on assumptions rather than facts, and the assumptions themselves don't seem to be based on anything. And yet we're supposed to be using our analyses as a guide to investing large sums of money.

These problems reflect the fact that economics deals with very complex systems -- systems in which human choice plays a large role. Nevertheless, there are ways of recovering some solid ground, and hence being able to put some confidence into our conclusions. We will discuss some of these issues in the mid-term *post mortem.* Here we take a more systematic look.

The basic problem is that we need hard numbers to go into our analyses, and most of the numbers we can actually lay our hands on are soft. At the best, we can assign [approximate] probabilities to the numbers falling into particular ranges; at worst, we are in a state of complete uncertainty. In the former case, we can employ *risk analysis*; in the latter case we can employ several techniques, including *break-even analysis*, *sensitivity analysis* and *decision-making under uncertainty*.

We will first examine

Break-even Analysis

Break-even analysis consists in determining the value of a parameter for which the conclusion of an analysis switches from `Do it' to `Don't do it.' It is commonly applied to determining the level of production of a particular product at which the operation starts to yield a profit.

We first need to establish some vocabulary. The profit made per unit of production is the difference between the selling price and the production costs. The costs are made up of two components: *fixed* and *variable* costs. For example, a company that offers photocopying services needs to have a photocopy machine to be in business at all; so the purchase or rental of that machine is a fixed cost. The number of sheets of paper purchased per month, however, depends on the volume of business, so that is a variable cost.

If the selling price of a product is **s**, the total fixed costs are **f**, the variable cost per item is **v**, and the number of items sold is **n**, then the total profit made will be

$$\mathbf{p = n\,s - nv - f}$$

So the volume of sales at which the profit is zero -- the break-even point -- is given by

$$\mathbf{n_{BEP} = f/(s-v)}$$

An organization is in good health if its sales are well above the break-even point. A statistic which measures this health is the *profit margin*,

$$\mathbf{(n - n_{BEP})/n_{BEP}}$$

To increase this margin, the company must lower fixed or variable costs, increase the selling price, or increase sales volume. Another alternative is to increase sales by *dumping*, that is, by increasing production and selling part of the product at a reduced price. Competition with the full-price version is avoided by selling under a different name or by selling to an overseas market.

Multi-Product Breakeven Analysis

By slightly modifying the format of a breakeven diagram, it is possible to show the contribution of a mix of products to profits. The *x*-axis represents total sales revenue, while the vertical axis represents total profit. For the first product in the mix, we begin on the vertical axis, below the zero, at the fixed cost for that product. Then we construct a line sloping diagonally upward, slope equal to the difference between variable costs and selling price for the item. At the highest point on this line, we drop vertically by an amount equal to the fixed cost for the second product in the mix, then climb diagonally again. This is repeated until all products are represented. The final point of the graph shows the total product realised by the mix.

Non-Linear Breakeven Analysis

The preceding analysis is slightly simplified, as the variable costs per item will rarely be completely independent of volume. Starting from an under-utilized facility, variable costs will initially drop as the slack in the organization is taken up; then, beyond a certain point, costs will rise again as overtime must be added and the facility becomes overcrowded. We may also expect the marginal selling price for producing one more item to fall as the market saturates and additional advertising or sales promotions become necessary. It is worth increasing production until the marginal cost of producing one more item is greater than the average revenue from selling one more item.

Summary of Break-even Analysis

Break-even analysis is a simple calculation which can be used to diagnose the overall health of a company and to prescribe measures for improving it. It helps us to deal with uncertainty by identifying the critical value of sales at which the operation becomes profitable; this can help to focus market research.

Sensitivity Analysis

Sensitivity analysis can be seen as a generalization of break-even analysis: we examine the effect of each parameter on the output of the model, so we can tell which unresolved uncertainties can significantly alter our results.

Given an economic analysis -- for example, calculating the present worth of an investment -- we repeat the analysis several times, each time varying one of the input parameters by a certain percentage. We can then plot the change in present worth versus percentage change in each parameter, and hence identify which parameters have a strong effect on the conclusion, and which are relatively unimportant.

The labour of these calculations can sometimes be reduced by the use of calculus; if the expression for present worth is a differentiable function of a particular parameter, the local slope of the sensitivity curve for that parameter is simply the value of the differential of the function with respect to that parameter.

Once the important parameters have been identified, the interaction of any two may be represented by an *isoquant* diagram. This is a graph, having the two parameters (*x* and *y*, say) on its two axes, with a curve corresponding to the solution of the equation

$$PW(x,y) = 0$$

This curve divides the plane into two regions, in one of which the proposal should be accepted, while in the other, it should be rejected.

Where we are interested in comparing two proposals, and inspection or sensitivity-diagram analysis shows that one particular parameter is critical in determining their present worths, it can be illuminating to plot present worths for both alternatives as a function of that parameter. based

The Goldilocks Method

Rather than start from an expression for present worth and calculating how large a change in each parameter would be needed to produce a 5% change in the expression, we could start from the other end and try to make a realistic assesment of how much change might be expected in each parameter. This can be done systematically by constructing three scenarios: an optimistic, realistic and pessimistic scenario. Then we calculate and compare the present worth of each.

In calculating the pessimistic scenario, it is not necessary to consider the worst possible case; there will always be possible events -- an outbreak of the Black Death among the workforce, a sudden nuclear holocaust -- which can make the present value of a proposal negative. Rather, the pessimistic scenario should be based on what might plausibly happen in the worst 10% of cases. Similarly for the optimistic scenario. This is particularly important when combining probabilities: if you select pessimistic values for each of ten independent parameters, where each parameter has a 10% chance of taking that value or something worse, the likelihood of all ten worst values occurring simultaneously is 0.1^{10}, or one in ten billion. In such a case, you should either be less pessimistic about each individual parameter, or use the Monte Carlo method (which will be discussed in the next-but-one lecture.)

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