Mathematics For Industry

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PIMS Graduate Summit – Jasper

Aims of this talk

Aims: To explore the interface between mathematics and industry by answering the following questions:

What is industrial mathematics?

(from the viewpoint of an academic mathematician)

Where is industrial mathematics?

(interesting mathematical problems arise in the most unexpected places)

How to do industrial mathematics?

(how to go about initiating an industrial project)

Why would one want to do industrial mathematics?

(benefits in terms of research, teaching and mentorship)

I will attempt to answer all four questions with the help of case studies drawn from industrial projects.

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Caveat audiens

My emphasis is on industrial math in an academic research setting

Acknowledgements

Many students and postdocs were involved in these projects:

Bamdad Hosseini

Maryam Zarrinderakht

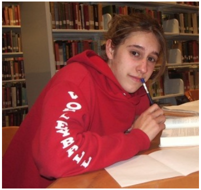
Isabell Graf

Sudeshna Ghosh

Jeff Wiens

Maurizio Ceseri

Reanne Bowlby



Enkeleida Lushi

Acknowledgements

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- AIS Technologies
- Alexander von Humboldt Foundation
- Ballard Power Systems
- Mitacs Network of Centres of Excellence
- North American Maple Syrup Council
- NSERC
- Teck Resources

Outline

1. Industrial mathematics
2. Case studies
   - Robotic pipe welding (consulting contract)
   - Land mine trip-wire detection (study group problem)
   - Hydrogen fuel cells (large multi-university project)
3. Interlude: Sometimes failure breeds success
   - Multi-class traffic flow modelling (a "failed" collaboration)
4. More case studies
   - Atmospheric pollutant dispersion (internships)
   - Maple sap exudation (private research grant)
5. Closing remarks

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Industrial mathematics = Industry + Mathematics ?

What is industrial mathematics?

First, we need definitions for **industry** and **mathematics**.

Industry

Definition (Dictionary.com):

1. the aggregate of **manufacturing** or technically productive enterprises in a particular field.

⋮

4. the ownership and management of companies, **factories**, etc.

I take a much broader view that encompasses all possible non-academic **end-users of mathematics**:

- public and private sector companies,
- federal, provincial or municipal government agencies or labs,
- hospitals and foundations,
- not-for-profit societies,
- ...

A **company** refers to any such end-user organization.

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OECD definition of “industry”

OECD Global Science Forum
Report on Mathematics in Industry, 2008:

“Industry was broadly interpreted as any activity of economic or social value including the service industry, regardless of whether it is in the public or private sector.”

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT
OECD

Mathematics

Take a correspondingly broad view of mathematics to include:

- Pure and applied mathematics: any mathematical techniques that can be employed to solve real problems.
- Statistics: "the science of learning from data, and of measuring, controlling, and communicating uncertainty" (American Statistical Association).
- Scientific computation: algorithms are simply mathematical equations expressed in the form of code.

The mathematician's objective is to develop new models and investigate new mathematics or new algorithms.

Contrast with "modelling and simulation" in engineering and applied science, which typically involves more routine applications of established mathematical techniques.

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Classifying industrial mathematics

Brian Wetton (2013 CAIMS-Mprime IM Prize lecture):

"Industrial mathematics is mathematics that industry is willing to pay for."

- The sum total of all mathematics that is employed in all industries covers a huge spectrum of activity.
- Industrial mathematics (IM) cannot be considered a (sub-)field of mathematics.

Classifying Industrial Mathematics (IM)

I propose dividing IM into 3 areas:

- Mathematics **IN** Industry (MII): performed by non-academic mathematicians who work as employees of a company.

  $\Longrightarrow$ Goal: advances in products or processes

- Mathematics **FOR** Industry (MFI): performed by academic mathematicians as part of a collaboration with a company.

  $\Longrightarrow$ Goal: advances in mathematics **and** products/processes

- Mathematics **INSPIRED BY** Industry (MIBI): refers to mathematical study of problems arising in industry, but within an academic setting, largely insulated from demands, pressures and constraints of industry.

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Spotlight on MFI

The focus of this talk is **Mathematics FOR Industry (MFI)**:

- Key aspects:
  - collaboration between company and academics.
  - aim to advance both mathematics and products/processes.
- Research is driven, but not dominated by, company interests.
- No requirement that company actually fund the research.
- Characterized by incommensurate time scales: academia (long-term) versus industry (short-term).
- MFI leads naturally to further problems in MIBI, lying outside the industrial collaboration.

Terminology problem

$$\textbf{IM} = \textbf{MII} \cup \textbf{MFI} \cup \textbf{MIBI}$$

This terminology conflicts with some usage in the IM community:

- In UK, “Maths-in-industry” refers to all of IM.
- Similarly for *Journal of Mathematics <u>in</u> Industry* and *European Consortium for Mathematics <u>in</u> Industry*.

BUT

- Tayler’s article (1990) is from the perspective of MFI.
- My MII is consistent with *SIAM Report on MII* (2012).

Other impressions

"Almost by definition, industrial research is **interdisciplinary***."*
(Ockendon, 2008)

"Industry needs mathematicians of an especially **broad** *type—men whose interests naturally extend beyond their special field, and who are* **flexible** *enough to cooperate with non-mathematicians."*
(Campbell, 1924)

"[Mathematics] rarely solves industrial problems, indeed most industrial problems are never solved and **compromise decisions** *have to be made."*
(Tayler, 1990)

"[Industrial mathematics] requires **barbarians***: people willing to fight, to conquer, to build, to understand, with no predetermined idea about which tool should be used."*
(Beauzamy, 2002)

Key words: interdisciplinarity, breadth, agility.

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Outline

Case studies

I'll present a wide range of problems that differ in ...

- **Mathematical technique:** algebra and trigonometry, partial differential equations, finite volume methods, integral transforms and Green's functions, Bayesian inversion, asymptotic analysis, homogenization theory.
- **Scientific discipline:** engineering, image processing, electrochemistry, fluid mechanics, atmospheric science, plant science.
- **Industry sector:** manufacturing, defense, automotive, alternate energy, mining, agriculture.
- **Company size:** small consulting firm, medium-sized R&D-intensive company, multi-national corporation, non-profit network of industry associations.
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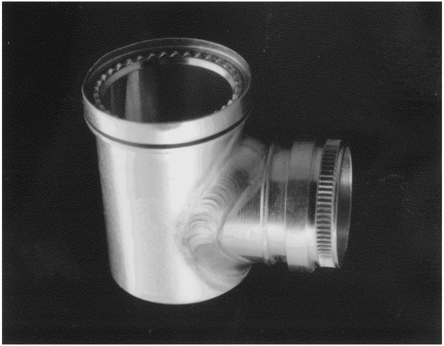
Case 1: Robotic pipe welding (consulting contract)

- A consulting contract with a Vancouver engineering firm.
- Initiated through personal contacts (a house party).

FANUC Robotics

Sample 90° pipe weld

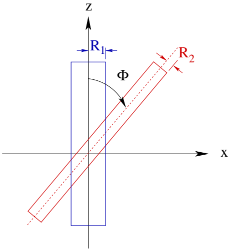


The problem: How to control the torch on a robotic pipe welding machine for two pipes of different cross-section?

The solution

- Cylindrical pipes of radius R_1, R_2
- Joint angle Φ

A straightforward exercise in algebra, trigonometry and parametric equations:



Pipe 1:
$$x = R_1 \cos\theta_1, \quad y = R_1 \sin\theta_1, \quad z = z_1$$

Pipe 2:
$$x = R_2 \cos\theta_2 \cos\Phi - z_2 \sin\Phi, \quad y = R_2 \sin\theta_2, \quad z = R_2 \cos\theta_2 \sin\Phi + z_2 \cos\Phi$$

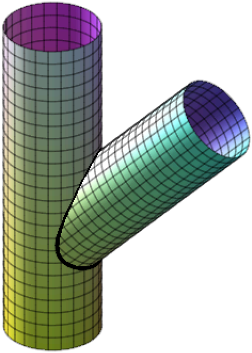
Equate x, y, z components to get parametric equations in θ_2:

$$x = \pm\sqrt{R_1^2 - R_2^2 + R_2^2 \cos^2\theta_2} \qquad y = R_2 \sin\theta_2$$

$$z = \frac{R_2 \cos\theta_2 \mp \cos\Phi \sqrt{R_1^2 - R_2^2 + R_2^2 \cos^2\theta_2}}{\sin\Phi}$$

Results

Pipe weld solution with
$\Phi = 45^\circ$ and $R_2 < R_1$



Some complications and extensions:

- Automatically select correct "±" solution branch.
- Choosing torch angle to avoid torch interference at small Φ.
- Weld length ("cost") yields an arclength integral that can only be approximated numerically (elliptic integral if $R_1 = R_2$).

Benefits and spin-offs

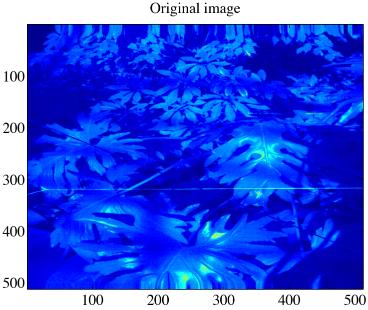
- Consulting fees covered one month's rent (basement suite).
- Published an educational article detailing the Maple code: *Maple Technical Journal*, 5, 1998
- Expanded code developed for MapleSoft Application Centre.
- I still get regular inquiries from pipe welding companies that stumble across my web site or Maple App.
- Great motivational example for (pre-)calculus students.

Case 2: Land mine trip-wire detection (study group problem)

- "Study group" problem from ITRES, a defense contractor specializing in thermal/hyperspectral imaging.
- PIMS Industrial Problem-Solving Workshop (Calgary, 1998).

Czech Talon mine disposal robot

Test image, with vegetation

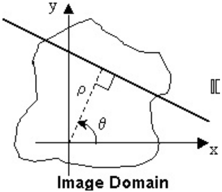


The problem: How to identify mine trip wires in images with noisy, cluttered backgrounds?

The solution

Given a 2D image with intensity $u(x, y)$, develop a simple image processing algorithm based on **Radon transform**:

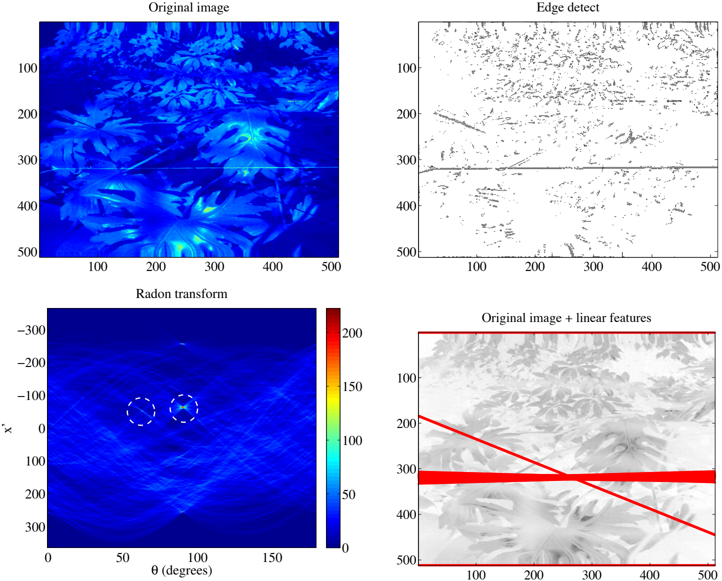
$$U(\rho, \theta) = \int u(x, y)\, \delta(x \cos\theta - y \sin\theta - \rho)\, dx\, dy$$



"Straight-line" detection algorithm

1. Apply a Laplacian filter to each point in the raw image u to accentuate regions of high curvature.
2. Perform edge detection on the filtered image.
3. Calculate the Radon transform U of the edge-detected image.
4. Find all points (ρ, θ) that exceed a threshold: $U(\rho, \theta) > T$.
5. Apply the inverse Radon transform to each point found in step 4.

Algorithm results



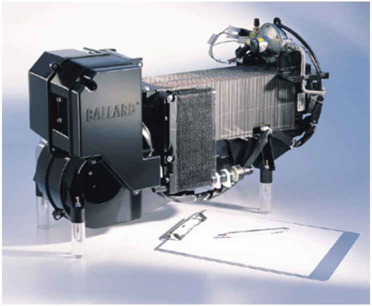
Benefits and spin-offs

- PIMS–IPSW proceedings paper.
- No further contact from the company after study group but . . .
- Vindication: Our work was actually implemented by ITRES according to a proceedings article of *International Society for Optical Engineering* (2000).
- Another fantastic illustration of the value of mathematics to industry, with lots of outreach potential:
  - *SIAM WhyDoMath*: `http://www.whydomath.org`
  - *Plus Magazine*: `http://plus.maths.org`
  - Numerous public talks (Chris Budd, ICIAM 2011 lecture)

Case 3: Hydrogen fuel cells (large multi-university project)

- Collaboration with Ballard Power Systems, a world leader in polymer electrolyte membrane (PEM) fuel cell technology.
- Multi-university project funded by Mitacs NCE, 1998–2008.
- **Aim:** To develop mathematical models of fuel cell components/processes to support Ballard's R&D efforts.

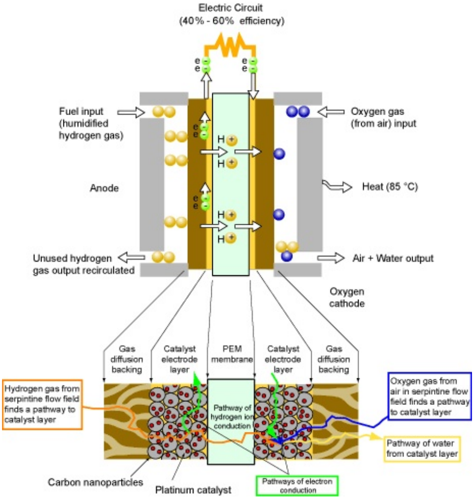
Van Hool bus powered by Ballard FCvelocity™-HD6 fuel cell module



Fuel cells

Hydrogen fuel cells are extremely complicated devices, involving:

- multi-phase flow (gas-liquid)
- multi-component gas mixtures (O_2, H_2, H_2O, ...)
- flow in porous media
- heat transport
- phase change
- chemical reactions
- ion transfer in polymers
- multi-scale catalyst structure
- etc.



A multitude of problems

Over 10 years, project team members investigated a multitude of problems including:

1. Heat and mass transport in a unit cell, with condensation.
2. Multi-scale models for the catalyst layer that account for nano/micro/macro-pore structure.
3. Electrical coupling in a stack of 100+ simple unit cells.
4. Reduced dimensional (1+1D) models of a stack of 100+ "complete" unit cells, exploiting multi-scale nature of materials (in space) and processes (in time).
5. Models for membrane transport and synthesis.

(my involvement was mainly in two projects)

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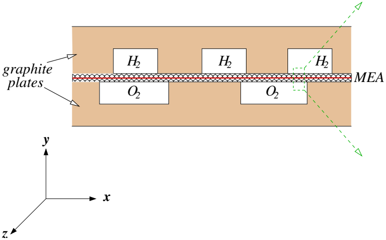
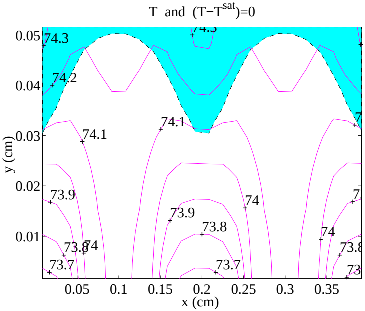
Problem #1: Condensation in porous electrodes

- Porous electrodes suffer from flooding at low temperature.
- Electrodes are treated with hydrophobic Teflon to improve their water transport characteristics – **Why?**

Unit cell geometry
(MEA = membrane electrode assembly)



Temperature contours and
liquid water region



Benefits and spin-offs

- **REAL DATA!!**
- Many journal papers:

  Promislow & JMS, *SIAM J. Appl. Math.*, 62, 2001

  JMS, Promislow & Wetton, *Int. J. Num. Meth. Fluids*, 41, 2003

  Kermani & JMS, *Int. J. CFD*, 18, 2004

  Promislow, JMS & Wetton, *Proc. Roy. Soc. A*, 462, 2006

  Dalasm et al., *Int. J. Hydrogen Energy*, 35, 2010

  Kermani & JMS, *Int. J. Hydrogen Energy*, 36, 2011

- Successes in postdoctoral training:

Mohammad Kermani
(Assoc Prof, Amir Kabir U)

Peter Berg
(Prof, U Alberta)

Akeel Shah
(Assoc Prof, U Warwick)

Outline

Case 4: Multi-class traffic flow modelling

- A past grad student introduced me to an engineer-consultant working with a Vancouver traffic consortium.
- A new highway (South Fraser Perimeter Road or SFPR) was being planned to service the Deltaport Terminal.

"Super-truck" study

- A novel feature of the traffic plan was to introduce "Super-trucks" or double container vehicles.
- Aim was to understand effects of multiple classes in hyperbolic PDE models for traffic flow, and then to optimize road/interchange design.

The failure

- Anticipated funding for the scientific study was never secured.
- As is often the case with industrial collaborations, this one never got off the ground.

The success

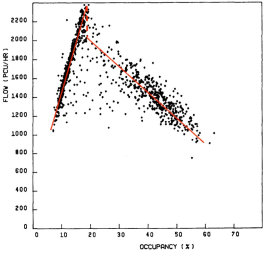
Nonetheless ...

- Two MSc students at SFU worked on related traffic flow projects, extending "standard" hyperbolic PDE models.
- Reanne Bowlby studied multi-class traffic:

Bowlby, "A Roe solver for a multi-class traffic flow model", MSc thesis, Dec. 2011

- Jeff Wiens derived analytical solutions and a novel algorithm for discontinuous fluxes seen in traffic experiments:

Wiens, "Kinematic wave and cellular automaton models for traffic flow", MSc thesis, Aug. 2011
Wiens, JMS and Williams, *J. Comput. Phys.*, 242, 2013



Greenshields (1935)

Outline

Case 4: Atmospheric pollutant dispersion (internships)

- Teck Resources operates one of the world's largest integrated lead-zinc smelters in Trail BC (600 km east of Vancouver).
- Airborne pollutant emissions must be reported annually to Environment Canada.
- Certain non-stack sources cannot be measured directly.

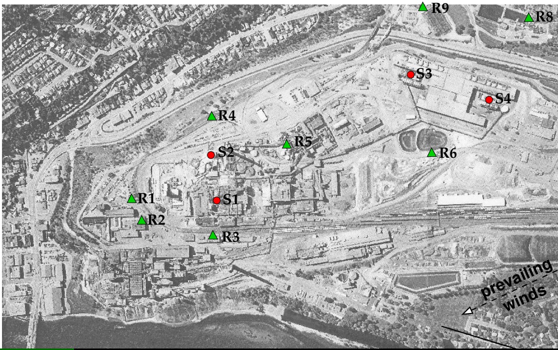
The source inversion problem

Given the following data:

- 4 sources: location Ss, emission rate Q_s.
- 12 dustfall receptors: location Rr, monthly accumulation D_r.
- Wind and other atmospheric parameters.

Dustfall jar

The problem: How to estimate non-stack source emission rates (Q_s) using dustfall data (D_r)?



The solution

- Pollutant concentration C from a point source in a constant wind is governed by steady advection-diffusion equation:

$$U \frac{\partial C}{\partial x} = K_y \frac{\partial^2 C}{\partial y^2} + K_z \frac{\partial^2 C}{\partial z^2} + Q\,\delta(x)\,\delta(y)\,\delta(z-H)$$

- Laplace transform yields the Gaussian plume solution:

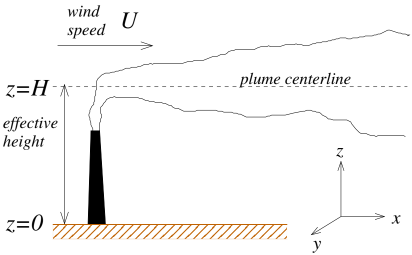
$$C(x,y,z) = \frac{Q}{2\pi U \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Q = emission rate

H = source height

U = wind speed

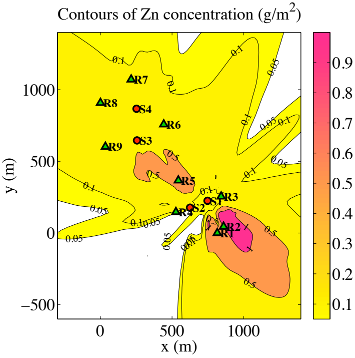
$\sigma^2_{y,z}(x) = 2xK_{y,z}/U$

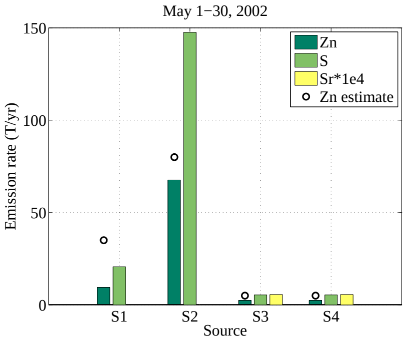


The solution

- Sum contributions from all sources to receptors $\vec{D} = (D_r)$.
- Concentration (and deposition) depend linearly on $\vec{Q} = (Q_s)$.
- Solving for $\vec{Q}$ is a (constrained) linear least squares problem:

$$\min_{\vec{Q}} \|\mathscr{L}\vec{Q} - \vec{D}\|^2$$





Most recent study of lead emissions

- Teck used other devices to collect time-dependent contaminant data $\Longrightarrow$ $\vec{D}(t)$, $\vec{Q}(t)$.
- Number of potential lead sources is much higher.
- Modify the optimization problem using regularization:

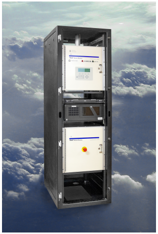
$$\min \left\{ \|\mathscr{L}\vec{Q} - \vec{D}\|^2 + \alpha\|\vec{Q}\|^2 + \beta\|\partial_{tt}\vec{Q}\|^2 \right\}$$

- Our results were consistent with a parallel Teck study, and will be used to deploy $10M's in site upgrades/improvements.

Dustfall jar (monthly)

$\Longrightarrow$

Xact (dense)



TSP/PM10 (sparse)

Benefits and spin-offs

- Graduate student training through Mitacs internships:
  - Enkeleida Lushi (initial zinc emissions study, 2006)
  - Sudeshna Ghosh (follow-up zinc study, 2011)
  - Bamdad Hosseini (time-dependent lead emissions, 2013)
- Journal publications:
  Lushi & JMS, *Atmos. Env.*, 44, 2010
  JMS, *SIAM Review*, 53, 2011 (SIAM Top 20)
  Hosseini, Nigam & JMS, *J. Comput. Phys.*, 305, 2016
  Hosseini & JMS, *Atmos. Env.*, 141, 2016
  Hosseini & JMS, *Comput. Fluids*, accepted
- Hosseini's thesis work:
  - MSc: Comparison of Gaussian plume and direct numerical simulations of the 3D advection-diffusion equation, 2013
  - PhD: Bayesian methods for source inversion (ongoing)
  - 3+ singly-authored papers on Bayesian inverse problems

New projects with Teck and other partners

- Study of emissions from an aluminum smelter at Kitimat, BC.
- Connection between pollutants and incidence of autism, with SFU Health Sciences and Metro Vancouver.
- Of course, it's not always smooth sailing! Work with Teck is on hold, and a new project with Teck's Carman de Andacallo mine in Chile was cancelled just before it was to start ...

Teck shares slide on coal demand woes despite earnings beat

SUSAN TAYLOR AND NICOLE MORDANT

TORONTO — Reuters

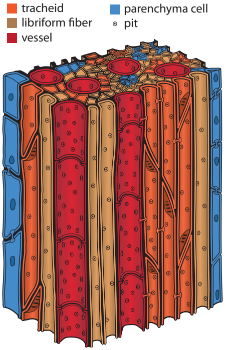
Published Wednesday, Feb. 15, 2017 5:07AM EST

Case 5: Maple sap exudation (private research grant)

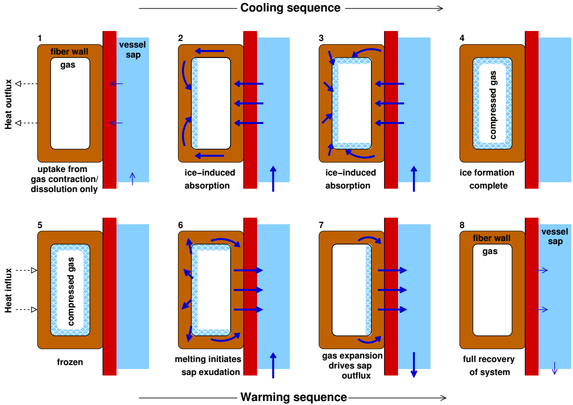
- Maple trees have a unique ability to **exude** sap in winter, with no leaves to drive photosynthesis or transpiration.
- A current controversy over (bio)physical causes of sap exudation sparked my interest: Cirelli et al., *Tree Physiol.* 28, 2008.
- Several years later, I stumbled across a call from the North American Maple Syrup Council (NAMSC) Research Fund . . .

Proposals were submitted to NAMSC and Mitacs PDF program to study the **Milburn-O'Malley hypothesis**:

Fibers (tracheids) and vessels



Milburn and O'Malley (1979) freeze/thaw process



The challenge

Melvin Tyree (1983):

"There is insufficient quantitative information to set up a system of physical equations to describe the model . . . There is no theoretical basis upon which to predict whether O'Malley's model will explain the effect of the rate of freezing on the time course of sap flow rate or pressure change."

Challenge: Can exudation be captured by a model incorporating:

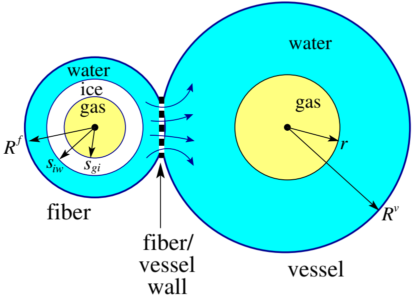
- sap flow in porous sapwood,
- freezing and thawing in fibers and vessels,
- gas dissolution and bubble formation,
- osmotic pressure across selectively-permeable membranes ?

. . . compelling similarities to PEM fuel cells!

The solution

- Developed a 1D cell-level model for freeze/thaw process in fibers and vessels.
- Novel features:
  - Gas bubbles in vessels
  - Freezing point depression (due to sugar content)
  - Root water uptake
- Governing equations are a coupled system of differential-algebraic equations for interface positions:

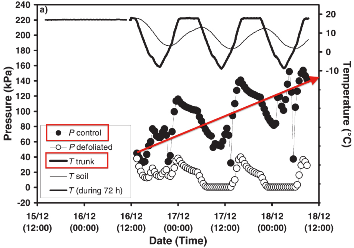
$$s_{gi}(t),\ s_{iw}(t),\ r(t)$$



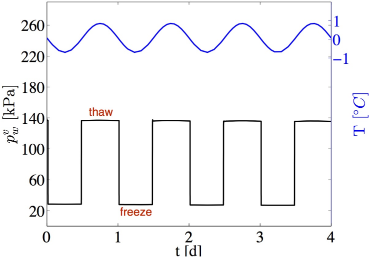
Breakthrough!

Our model results mimic the pressure build-up in a maple tree as air temperature oscillates above and below 0°C.

Experiments: Améglio et al. (2001)



Model results: Vessel water pressure



Cellular-level freeze-thaw model

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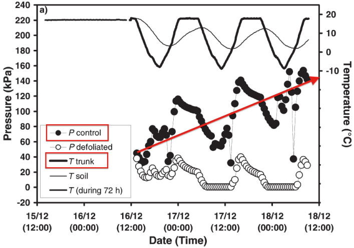

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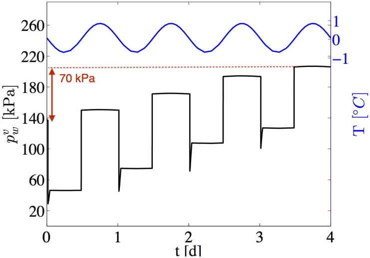

Cellular-level freeze-thaw model

... then add root water uptake

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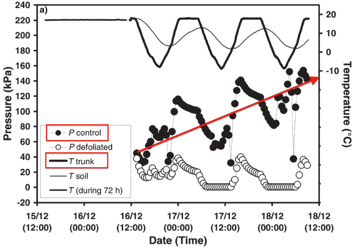
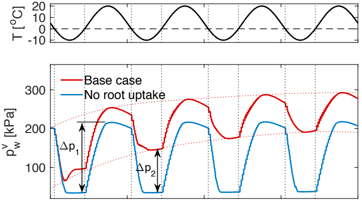


Model results: Vessel water pressure



Cellular-level freeze-thaw model

... then add root water uptake

... then apply periodic homogenization
("fancy" averaging over stem)

Benefits and spin-offs

- A modest NAMSC grant leveraged funding for 2 postdoctoral fellows and several grad students.
- Journal publications:
  Ceseri & JMS, *SIAM J. Appl. Math.*, 73, 2013
  Ceseri & JMS, *Euro. J. Appl. Math.*, 25, 2014
  Graf, Ceseri & JMS, *J. R. Soc. Interface*, 12, 2015
  Graf, Peter & JMS, *IMA J. Appl. Math.*, in press
  . . . with several more in preparation
- Solved a fundamental open problem in tree physiology!
- More to come:
  - comparison with maple stem pressure measurements
  - optimal sap harvesting strategies
  - growth of *X. fastidiosa* biofilms in sapwood

Outline

Underlying themes

- Industrial collaborations provide many opportunities for . . .
  - novel, difficult and exciting problems (multiphysics, multiscale)
  - student and postdoc training
  - funding for fundamental **and** applied research
  - personal professional development
- Access to real data can be of enormous benefit.
- Incommensurate time scales between academia and industry.
- The **"optimal"** mathematical solution is often not what industry partners need/want.
- Not every interesting problem leads to a viable collaboration.

  (Not every viable collaboration leads to an interesting problem)

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  (Not every viable collaboration leads to an interesting problem)

Advice and warnings

Interdisciplinarity. Breadth. Agility.

- IM requires a broad knowledge of mathematics.
- Network, network, network. Perfect your elevator speech. Maintain a web page profiling your industrial experience.
- Good communication skills are essential. Know your audience. Aim at an appropriate level.
- Prepare for disappointment. It's common for funding to evaporate or company interest to falter.
- If you want a competitive advantage in a **very tight** academic job market, proven expertise in IM may help.
- It's still important to **specialize** in something. Many in our discipline value specialists over generalists.

Thank you!

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