Math and the Olympics

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$$\pi \; + \quad = \; ?$$

Outline

Outline

Why study sport?

- Sports and games have played a central role in human life for thousands of years – some researchers claim we're genetically programmed to compete in sports!
- Many sports are major cultural and social phenomena (e.g., soccer in Latin America).
- Everyone can participate in sport, at least recreationally.
- Everyone can be (likes to be) an "expert" on some sport.
- Today's media permits us to view sporting events from around the world in real time.
- Professional sport is a multi-billion dollar business!
- Sport is complex scientifically (i.e., interesting).
- It's fun!

Why study math in sport?

- **It's a challenge:** complexities in sport appear on many levels:

  individuals, pairs, teams, leagues, tools,
  and their interactions

  . . . and in many different aspects of sport:

  biomechanics, judging, tournament design, technology,
  rankings, gambling, crowd control, finance,
  media ratings, nutrition, doping, . . .

  . . . all of which can be described mathematically!
- **There's an opportunity:** advanced mathematical techniques have not penetrated sport science as much as other "hard" sciences (physics, chemistry, . . .)
- **Math is general:** methods developed for one sport can often be applied to others.
- **It's fun!**

Perspective: Benefits of sport

Sadovskiĭ and Sadovskiĭ, *Mathematics and Sports*:

"Sports have a beneficial effect on one's intellectual activities, state of mind, and will power . . . the extraordinary longevity of many of our outstanding mathematicians and physicists is due to their affinity for sports."

Examples: Niels Bohr (soccer, skiing), Einstein (yachting), Joe Keller (tennis, golf, . . .), and many others.

Perspective: Recognition of athletes vs. scientists

Society has an unreasonable fascination with athletes, and rewards them disproportionately . . .

Vitruvius (1st century BC):

"The wrestler, by training, merely hardens his own body for the conflict; a [writer], however, not only cultivates his own mind, but affords every one else the same opportunity, by laying down precepts or acquiring knowledge and exciting the talents of his reader . . . Since individuals as well as the public are so indebted to these [writers] for the benefits they enjoy, I think them not only entitled to the honour of palms and crowns, but even to be numbered among the gods."

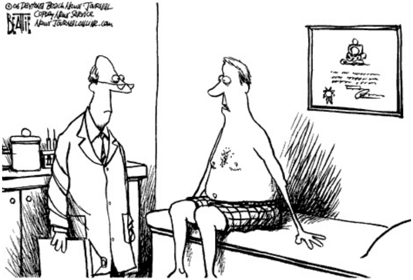
To the Greeks: "writer" = "scientist" or "mathematician"

Why study the Olympics?

- It involves a wide variety of sports.
- It's the largest sporting event in the world.
- The Winter Games descended on Vancouver in February/March 2010!

Why teach (take) a course on math in Olympic sport?

WHY NOT?!

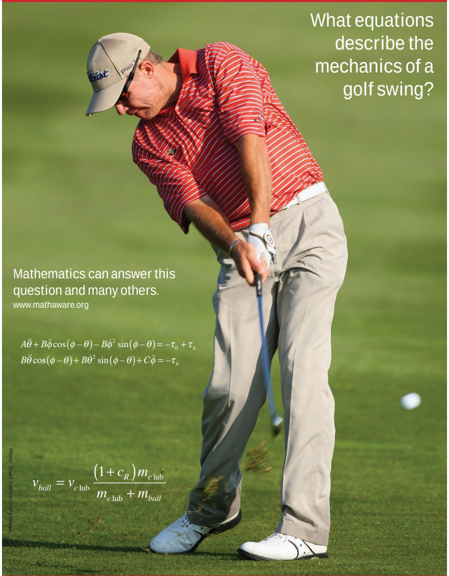


"Can you give me a prescription for something that'll get me excited about the Olympics?"

- Many resources available at `http://www.mathaware.org`.

Mathematics Awareness Month - April 2010

Mathematics and Sports



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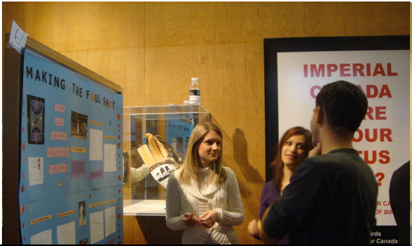
And yes, golf is an Olympic sport!!
(starting in London 2012)

Math 302: Topics

This talk is based on a course (MATH 302) taught in Fall 2009:

- Who really won the Olympics? (medals)
- What are the limits of human performance? (records)
- Who is the fastest person on the planet? (running)
- Is there an optimal technique for putting a shot? (projectiles)
- How miraculous was the "1968 Mexico City long jump miracle"?
- What are the chances of winning at tennis?
- Is there really a home ice advantage in the Stanley Cup playoffs? (tournaments)
- Is the judging system in figure skating a fair one?
- Does the Olympic triathlon penalize good swimmers?
- The economics of major sporting events.

Math 302: Projects



Mathematical content of this talk

In the next little while, we'll delve into some

- geometry
- calculus: derivatives, integrals and differential equations
- statistics ("descriptive statistics", nothing serious)
- . . . with some physics thrown in for good measure!

Outline

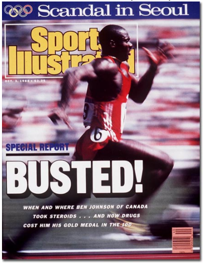
1. Introduction

2. The mathematics of running
   - Track geometry
   - Dynamic models for sprinting

3. Olympic medal rankings
   - Weighted medal counts
   - Biases in medal rankings

How fast can people run? And why?

Ben Johnson (CAN)



Oscar Pistorius (RSA)

Florence Griffith-Joyner (USA)

(l-r) Asafa Powell (JAM)
Tyson Gay (USA)
Usain Bolt (JAM)

Running is arguably the oldest sport, and also one of the simplest.

BUT there are still many aspects of running that are worthy of study:

- physics governing forces and propulsion,
- air resistance,
- oxygen intake and metabolism, and how they govern performance,
- race strategies,
- (for track events) design of the track itself,
- progression of records.

Origin of the word "stadium"

stadium (Greek) = *stadion* or *stade*:

An ancient Greek footrace. The word changed in meaning over time to mean a standard measure of length for the race, roughly 180–200 m. Later, it was also used to refer to the actual place where the race took place.

The original Greek *stadion* was long and narrow.

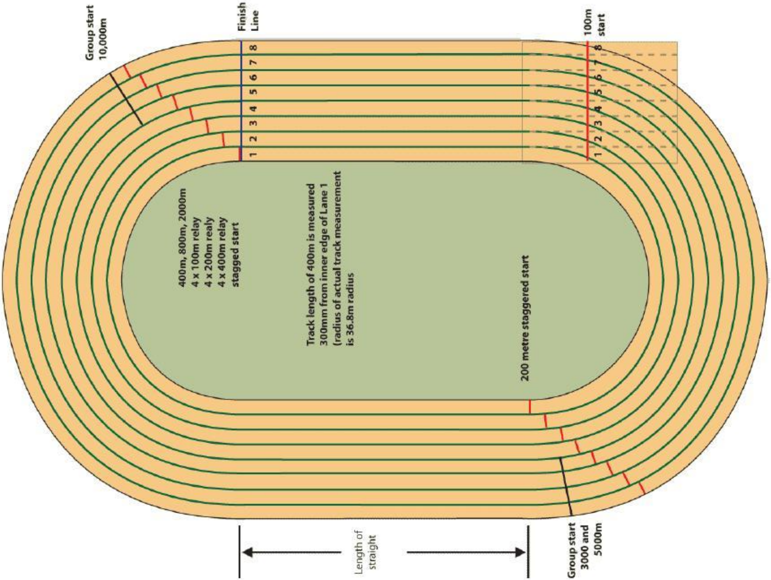
Constraints on outdoor track design

The typical outdoor track is designed as follows:

- Two straight sections joined by semi-circular ends.
- Semi-circular ends have radius 36.5 m.
- Complete circuit around lane 1 measures 400 m.
- (picky point) Running distance is measured along a line 0.3 m from the inner edge of lane 1, and 0.2 m from the inner edge of all other lanes.
- 8 lanes, each of width 1.25 m. Lanes are numbered starting from 1 on the inside.

Question 1:

Based on these constraints, what is the length of the straights?


Group start 10,000m
Finish Line
100m start
1 2 3 4 5 6 7 8
400m, 800m, 2000m
4 x 100m relay
4 x 200m realy
4 x 400m relay
stagged start
Track length of 400m is measured 300mm from inner edge of Lane 1 (radius of actual track measurement is 36.8m radius
200 metre staggered start
Length of straight
Group start 3000 and 5000m

Circular tracks?

Question 2:

Why don't they simply make running tracks circular?

There are a number of reasons:

- A straight stretch is required for events $\leqslant$ 100 m.
- The javelin event is held in the middle of the track. The longest throws are $\approx$ 100 m, which rules out circular and long/thin tracks for safety reasons.
- When running around curved sections:
  - Runners experience centrifugal force which slows their progress.
  - Circular tracks have a constant curvature, and so runners get no relief from this force.
  - Curvature is greatest in lane 1 which puts that runner at a disadvantage.

Why counter-clockwise?

Question 3:

Why are running events on a track always run counter-clockwise?

The answer to this question is not known as far as I can tell. Some possible answers (gleaned mostly from internet discussion groups):

- Asymmetry in human bodies means the right leg is stronger than the left leg – "leggedness" follows "handedness"??
- To please the crowd. When runners are nearest to spectators, they perceive the runners moving from left to right – the same direction our eyes move when we read.
- The Coriolis effect, due to the Earth's rotation. But then why are tracks the same in Australia?
- If runners ran clockwise, lane numbers would be upside down.
- The choice was arbitrary and has been passed down over time (think baseball).

Take your marks. Set. BANG!

- Running races are traditionally initiated by a starter who shoots a pistol from a location on the inside of the track, closest to lane 1.
- The starter is usually between 2 and 10 m away from lane 1's start position, located on the inside of the track.

- Since the speed of sound is 343 m/s, the retort of the pistol will be heard first by the runner in lane 1 and slightly later by the other runners.

Question 4:

Does the delay in hearing the starting pistol amount to a significant disadvantage for runners in lanes 2 through 8?

Consider the 100 m sprint (not staggered) and assume the starter is on the same line as the runners.

Fairness in starting II

Many (most?) international competitions now require an electronic pistol and speakers behind each starting block.

In the USATF Rules of Competition:

"In races where the competitors are not placed behind the same starting line, the Starter should use a microphone transmitting to speakers positioned at or near the starting line in each lane. Where such a device is not used, the Starter shall so be placed that the distance between the Starter and each of the competitors is approximately the same."

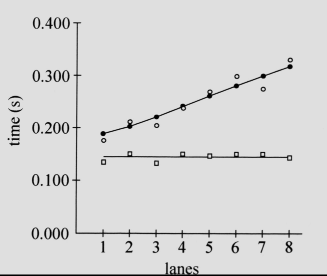
- World Championships (and other events) started using electronic guns in 1995.
- Olympics use a "loud gun," with the sound sent over a speaker behind the starting block.
- Loud gun could favour runners in lane 1, for several reasons . . .

[Science Update (www.sciencenetlinks.com): article — podcast]

The "loud gun" method should not be used

Julin and Dapena (2003) studied loud and silent guns:

- They measured starting/reaction times in all 8 lanes for events with "silent" and "loud guns."
- Reaction times for silent gun do not depend on lane.
- Reaction times for loud guns increase linearly, with slope closely matching the speed of sound.
- **Conclusion:** Runners don't react to the speaker sound even though it actually reaches them first!



○ = loud gun (1996 summer Olymipcs)
□ = silent gun (1995 world championships)
● = calculated using speed of sound

From *"Track Starter's Guide"* (US Department of Education, 1990):

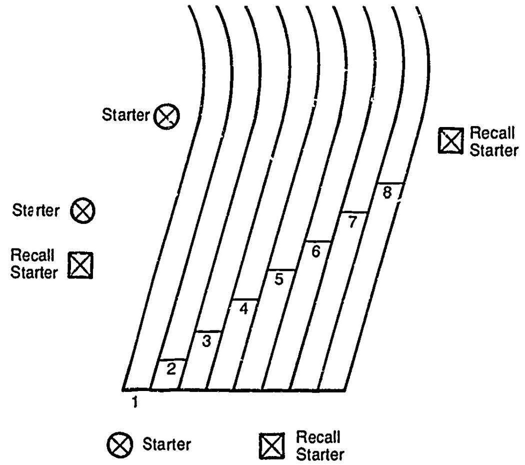


Figure 5. STAGGERED STARTS

Staggered starts

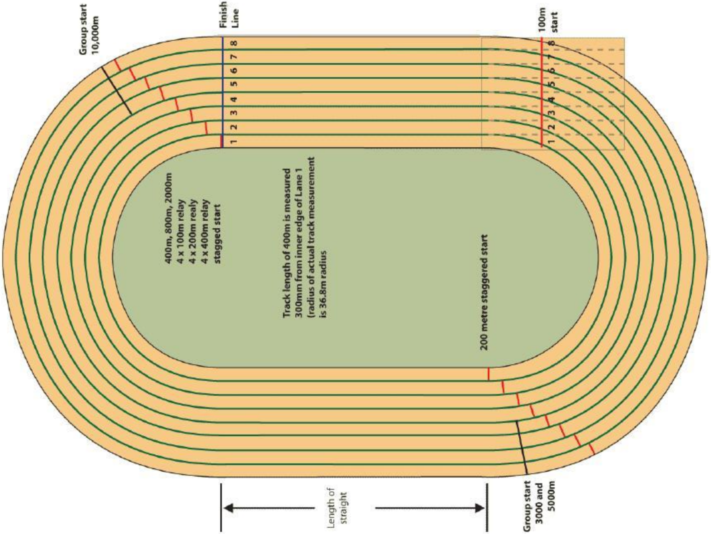
- In the 400 m run, assume that the starting line for lane 1 is at the start of a semi-circular section.
- Naturally, the finish line must be at the same location as lane 1's starting line (i.e., 1 lap $=$ 400 m).

Question 5:

By what distance must the runners in lanes 2–8 be staggered so that they also run 400 m when reaching the same finish line?

Note: Marking lanes, especially in international competitions, requires a surprising degree of care and precision!

Can you spot an error in this track diagram?



Other effects

- **Centripetal force:** reduces peak speed when rounding a curve, which is most disadvantageous to the runner in lane 1 – slows time by 0.12 s in the 200 m — HUGE!
- **Psychology:** runner in lane 8 starts ahead of others and so cannot see their competitors at the beginning of the race.

Question 6:

Putting all of these lane-dependent effects together (curvature, pistol sound delay, pistol loudness, psychology, . . .) which lane is really the best?

An excellent question! Lane 1 seems to be a bad selection, but no-one has a definitive "best choice" yet . . .

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1. Introduction
2. The mathematics of running
   - Track geometry
   - Dynamic models for sprinting
3. Olympic medal rankings
   - Weighted medal counts
   - Biases in medal rankings

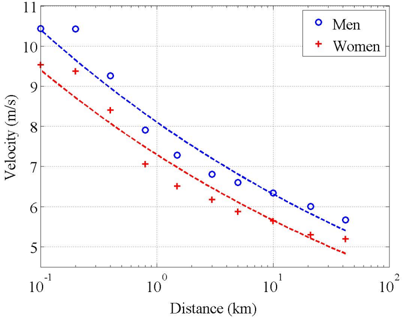
Motivation

Running record data indicates a clear trend in record time (or average velocity) versus distance run, suggesting that there is some predictable (bio-)physical process at work.

Question:

Can a mathematical model be found that captures this behaviour?

Average velocities from world record times



Basic physics

We'll develop a model using Newton's Laws of Motion:

1. A body remains in a state of rest or uniform motion unless acted upon by an external force.
2. The net external force acting on a body is equal to the product of its mass and its acceleration:

$$F = m \cdot a = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

$$\left(x = \text{distance}, \quad v = \tfrac{dx}{dt} = \text{velocity}, \quad a = \tfrac{d^2x}{dt^2} = \text{acceleration} \right)$$

3. For any force, there is an equal and opposite reaction force.

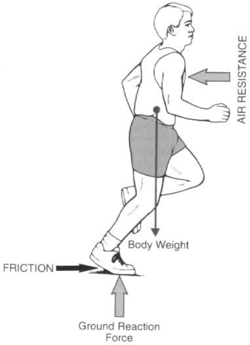
Here, we'll be mainly concerned with Newton's Second Law.

Forces on a runner

A number of forces act on a runner:

- propulsive force of the muscles,
- "internal friction," consisting of frictional and other losses within the muscles and converted into heat,
- friction between the feet and ground,
- air resistance ($\approx$ 3%).

The largest forces by far are due to muscle propulsion and internal friction.



Governing equations

Define:

$v(t) =$ speed of the runner (in m/s) at time t.

$F(t) =$ muscular propulsion force *per unit mass* (in m/s^2) – depends only on time.

$R(v) =$ internal resistance force *per unit mass* (also in m/s^2) – depends on running speed.

Newton's Second Law then becomes

$$m\frac{dv}{dt} = mF - mR \tag{1}$$

Governing equations II

- **Assumption #1 (linearity):** internal resistance is linear in velocity. So $R(v) = kv$ with k constant (units 1/s).
- **Assumption #2 (sprinting):** During a sprint, a runner exerts their maximum propulsive force for the entire race. So $F(t) = F_o$ constant.
- Using these two assumptions, Equation (1) becomes

$$\frac{dv}{dt} = F_o - kv$$

- This is a first order, linear, ordinary differential equation (ODE) with constant coefficients!

CAN YOU SOLVE IT?

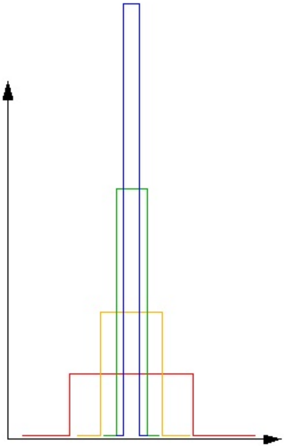
(Townend (1983) has a nice derivation)

Impulsive forces

- In real races, runners don't act so predictably. [video]
- Occasionally someone puts on a burst of energy, coming from behind and/or surging ahead.
- This burst corresponds to an impulsive force, where "impulsive" $\Longrightarrow$ large and short.
- Mimic using the function

$$f_h(t) = \begin{cases} \frac{1}{h}, & \text{if } -\frac{h}{2} \leqslant t \leqslant \frac{h}{2} \\ 0, & \text{otherwise} \end{cases}$$

where Area $= 1$ (constant).



Impulsive forces II

Mathematicians love to generalize!

- Consider the limit as $h \to 0$ and define $\delta(t) = \lim_{h \to 0} f_h(t)$.
- $\delta(t)$ is called the Dirac delta function and is an example of a "generalized function" or "distribution."
- **Example:** forces in collisions between pool balls are particularly large and act nearly instantaneously.
- Mathematical definition of the delta function:

$$\delta(t) = \begin{cases} +\infty, & \text{if } t = 0 \\ 0, & \text{if } t \neq 0 \end{cases} \qquad \text{and} \qquad \int_{-\infty}^{\infty} \delta(t) = 1$$

Running with an impulsive force

In our running model, we can add an extra term for an impulsive force (or "burst of energy") at time $t = t_1$:

$$\frac{dv}{dt} = F_o - kv + F_1 \delta(t - t_1)$$

The solution is then

$$v(t) = \begin{cases} \frac{F_o}{k}\left(1 - e^{-kt}\right), & \text{if } t < t_1 \\ F_1 + \frac{F_o}{k}\left(1 - e^{-kt}\right), & \text{if } t \geqslant t_1 \end{cases}$$

. . . which gives a discontinuous "kick" to the runner's velocity.

Extensions

Other effects that can be incorporated easily into the ODE model:

- Track curvature and centripetal forces,
- Wind resistance.

This ODE approach can also applied to other sports such as:

- cycling
- swimming
- wheelchair athletics
- and others . . .

More math of running . . . in the news!

- A new Constructal Law combines ideas of scaling and symmetry with Newton's laws of motion.
- It predicts that elite athletes will become taller and heavier.

Outline

Olympic medal rankings

Motivation:

There are many methods for determining Olympic medal standings. Consequently, different media reports can identify different "winners" for the same Games.

Which method is the "right one"? Which is the most fair?

Medal standings in the media (Beijing, Summer 2008)

Official IOC standings
(by gold medal count)

OLYMPIC GAMES
Beijing 2008 - Medal Table

Print Close

Nation	Gold	Silver	Bronze
1. People's Republic of China **CHN**	51	21	28
2. United States of America **USA**	36	38	36
3. Russian Federation **RUS**	23	21	28
4. Great Britain **GBR**	19	13	15
5. Germany **GER**	16	10	15
6. Australia **AUS**	14	15	17
7. Korea **KOR**	13	10	8
8. Japan **JPN**	9	6	10
9. Italy **ITA**	8	10	10
10. France **FRA**	7	16	17

www.olympic.org

NBC standings
(by total medals G+S+B)

2008 Olympics: Complete Medal Standings

As of 8/24, 5:14 AM ET

COUNTRY	MEDALISTS	GOLD	SILVER	BRONZE	TOTAL
USA United States	see names	36	38	36	**110**
CHN China	see names	51	21	28	**100**
RUS Russia	see names	23	21	28	**72**
GBR Great Britain	see names	19	13	15	**47**
AUS Australia	see names	14	15	17	**46**
GER Germany	see names	16	10	15	**41**
FRA France	see names	7	16	17	**40**
KOR South Korea	see names	13	10	8	**31**
ITA Italy	see names	8	10	10	**28**
UKR Ukraine	see names	7	5	15	**27**

www.nbc.com

Medal standings in the media (Vancouver, Winter 2010)

Official IOC standings (by gold medal count)

Rank	Nation	Gold	Silver	Bronze	Total
1	Canada (CAN)	14	7	5	26
2	Germany (GER)	10	13	7	30
3	United States (USA)	9	15	13	37
4	Norway (NOR)	9	8	6	23
5	South Korea (KOR)	6	6	2	14
6	Switzerland (SUI)	6	0	3	9
7	China (CHN)	5	2	4	11
7	Sweden (SWE)	5	2	4	11
9	Austria (AUT)	4	6	6	16
10	Netherlands (NED)	4	1	3	8

wikipedia.org

VANOC/USA standings (by total medals G+S+B)

Country	Gold	Silver	Bronze	Total
UNITED STATES	9	15	13	37
GERMANY	10	13	7	30
CANADA	14	7	5	26
NORWAY	9	8	6	23
AUSTRIA	4	6	6	16
RUSSIAN FEDERATION	3	5	7	15
KOREA	6	6	2	14
CHINA	5	2	4	11
SWEDEN	5	2	4	11
FRANCE	2	3	6	11

www.vancouver2010.com

Interesting observations:

- All media (except USA, CAN) report "official" IOC rankings, which use lexicographic ordering – nations are ordered by gold medal count, and silver/bronze only determine ties.
- USA/CAN ranking assumes that first, second and third placings have equal merit.
- USA consistently wins more silver/bronze than gold!
- Canada follows the USA even though the IOC ranking places Canada first in Winter 2010!!!
- Why? I'm guessing that the US media simply overwhelms us.

Question 1:

These two rankings are obviously different. Which is better? And are there other better ranking methods?

How much is a medal worth?

- Neither lexicographic ordering nor total medal count seem reasonable.
- Generalize: Both are examples of a weighted medal count

$$M = \alpha G + \beta S + \gamma B$$

[IOC: $\alpha = 1, \ \ \beta = \gamma = 0;$ USA: $\alpha = \beta = \gamma = 1$]

How much is a medal worth? II

$$M = \alpha G + \beta S + \gamma B$$

Certainly, we should restrict $0 < \gamma < \beta < \alpha$ so that:

- Each medal has a non-zero value.
- Strict inequality means that gold $\neq$ silver $\neq$ bronze.
- Hierarchy of medals is maintained.

Question 2:

Is there a "best" choice of weights α, β, γ?

Question 3:

What other factors might we be neglecting?

Choosing weights: One possible solution

- Try weights $\alpha = 10$, $\beta = 5$, $\gamma = 3$ (used by may others).
- Apply to the Beijing 2008 results, giving a different "top 10":

IOC	Change	Nation	G	S	B	Total	Weighted
1	0	China	51	21	28	100	699
2	0	USA	36	38	36	110	658
3	0	Russian Fed.	23	21	28	72	419
4	0	Great Britain	19	13	15	47	300
6	+1	Australia	14	15	17	46	266
5	-1	Germany	16	10	15	41	255
7	0	South Korea	13	10	8	31	204
10	+2	France	7	16	17	40	201
9	0	Italy	8	10	10	28	160
8	-2	Japan	9	6	10	25	150

- Top four finishers are unchanged (CHN, USA, RUS, GBR) and rest of the top 10 changes only slightly.

Choosing weights II

The next 6 in the rankings:

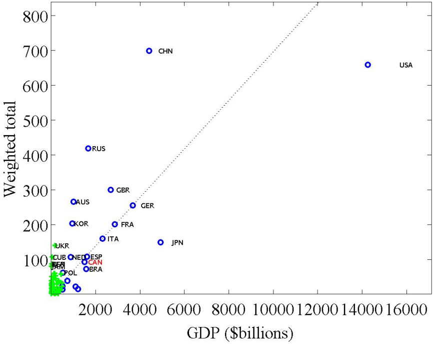
IOC	Change	Nation	G	S	B	Total	Weighted
11	0	Ukraine	7	5	15	27	140
14	+2	Spain	5	10	3	18	109
28	+15	Cuba	2	11	11	24	108
12	-2	Netherlands	7	5	4	16	107
19	+4	Canada	3	9	6	18	93
15	-1	Kenya	5	5	4	14	87

- Canada jumps 4 positions, from 19th to 15th.
- Cuba has the largest gain, from 28th to 13th.
- It's amazing that a nation as small/poor as Cuba (pop. 11M, GDP $55B) can surpass Canada (pop. 33M, GDP $1.5T).

Medal success and wealth

- Plot Gross Domestic Product (GDP) vs. weighted medal total for Athens (2004) Summer Olympics.
- Fit a straight line to the data using linear regression.

- Several countries fit the straight line quite well.
- The line separates countries that perform better (above) or worse (below) than "average".



Biases in medal rankings

1. The number of medals generally increases over time owing to the addition of new sports and events.

2. The number of countries is also increasing, which enhances competition (acts to counter bias #1).

3. Certain "special years" yield anomalous results:
    - Boycotts of Montréal 1976 (by 22 nations), Moscow 1980 (66 nations), and Los Angeles 1984 (13 nations).
    - Break-up of the Soviet Union between 1988–1996.
    - Exclusion of Germany after WWII (1948).

4. Historically, the host nation has a perceived advantage.

[NY Times Olympic Medal Map]

Biases in medal rankings II

5. All rankings ignore any placings below third.
6. Winter Games are heavily biased toward wealthy nations located in the upper latitudes. Summer Games are more inclusive.
7. Some countries are strategic in an "underhanded" way – they aim training programs at specific sports solely to maximize medal counts (e.g., China's "Project 119").
8. Some countries do exceedingly well in judged events (figure skating) relative to events that are measured/timed (swimming) or scored (hockey).

Inequities of simple weightings

Using a ranking based only on weighted medal counts biases the results in favour of countries that are . . .

- **wealthy:** and able to fund large, national training and recruitment programs in a wide range of sports
- **populous:** with a much larger pool of athletes to select from
- **healthy:** better health correlates with higher levels of fitness
- **well-educated:** ensures the population has the greatest possible access to opportunities

Note: These factors are not independent, e.g. health depends on wealth and education.

Example #1: Eliminate bias due to wealth

- **Idea:** Remove the advantage of wealth by defining a modified medal count that is scaled by GDP:

$$M = \frac{\alpha G + \beta S + \gamma B}{GDP}$$

- **Results:** Using a 10-5-3 weighting the top 10 are:

  Mongolia*, Jamaica, Zimbabwe*, Georgia, Kenya, Ethiopia, Cuba, Kyrgyzstan*, Tadjikistan*, Armenia*

  (* denotes GDP in the bottom 10)

- **Question:** Are these really the "winners of the Olympics"? (e.g., recent collapse of Zimbabwe's economy)

Example #2: Eliminate bias due to population

- Remove the effects of large populations, scaling M by population:

$$M = \frac{\alpha G + \beta S + \gamma B}{POP}$$

- **Results:** The top 10 are again totally different!

  Australia, Jamaica*, Bahamas*, Iceland*, Bahrain*, Slovenia*, Norway, New Zealand, Estonia*

  (* denotes population in the bottom 10)

- Neither approach identifies a clear winner, except Jamaica?
- **Note:** India sits at the bottom of the list of medals per population, and second-last in medals per GDP! ... chronic underperformer

Example #3: Host nation advantage

- The host nation typically sees a "spike" in medals won relative to other years.
- Some possible explanations:
  - The host nation typically has a large contingent of participants.
  - Athletes are more familiar with venues.
  - Less travel and jet-lag than other countries.
- Canada has "underperformed" as host, winning no gold medals in Montréal (summer 1976) or Calgary (winter 1988).

Idea: Compare the medal count $M(Y)$ in the host year Y to the average for the previous $M(Y-4)$ and next $M(Y+4)$ Olympics:

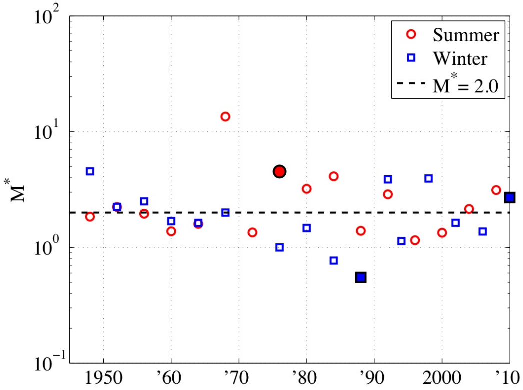
$$M^* = \frac{M(Y)}{\frac{1}{2}[M(Y-4) + M(Y+4)]}$$


artizans.com
CANADIAN ATHLETES UNDERFUNDED
THEY SHOULD DO IT FOR THE LOVE OF THE SPORT.
ELITE ATHLETES NEED MONEY TO STAY COMPETITIVE
WHY DON'T THEY GET JOBS AND SUPPORT THEMSELVES?
SHOULD CANADA'S BEST RECEIVE MORE FUNDS?
I'D PREFER TO SEE PUBLIC FUNDING PUT INTO ROADS.
BEIJING 2008 COVERAGE
WHY HAVEN'T WE WON MORE MEDALS?!

Aug. 15, 2008

Host nation advantage II

- Average host wins approximately 2 times as many medals as they win in other previous/succeeding Games:



- In only two years has the host won fewer medals ($M^* < 1$).
- Canada did well in the 2010 Winter Games ($M^* = 2.7$)!

Predicting medal performance

Many academics (and quacks) are in the game of predicting medal performance.

Luciano Barra's prediction for Canada in 2010:

"... winning 9 gold, 13 silver and 8 bronze medals, for a total of 30. That would be the second-highest count behind 42 for Germany."

[The Globe and Mail, 12 February 2010]

Actual results: Canada: $26 = 14G + 7S + 5B$
Germany: 30

Predicting medal performance II

In fact, I have an undergraduate student analysing medal data and looking at predictions for the 2012 Summer Olympics in London

. . .

Closing remarks

Conclusion: There is no perfect medal ranking **BUT** we can still do much better than the official IOC ranking.

Having said all of this, our stated aim in this section is actually counter to the spirit of the Olympic movement . . .

Rule 6 of the Olympic Charter:

The Olympic Games are competitions between athletes in individual or team events and not between countries.

But in reality, the success of a nation's athletes will always be an important source of national pride and prestige.

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