

# **AN OPEN-ACCESS CLICKER QUESTION BANK FOR NUMERICAL ANALYSIS**

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**Abstract:** We present a question bank consisting of over 250 multiple-choice and true-false questions covering a broad range of material typically taught in an introductory undergraduate course in numerical analysis or computational science. The questions are ideal for polling students during lectures by means of a student response system that uses *clicker* remotes or smartphones running a suitable app. We describe our experiences implementing these clicker questions in a recent class and provide evidence of their effectiveness in terms of testing students' prior knowledge, gauging understanding of new material, increasing participation, and especially improving student satisfaction. Our conclusions are supported by a mid-semester student survey as well as anecdotal observations. The question bank has been released as an open-access educational resource under a Creative Commons license (CC BY-NC-SA) for free use by the mathematics community.

**Keywords:** numerical analysis, computational science, clicker polling, student response systems, educational technology, MATLAB

## 1 INTRODUCTION

Student response systems, commonly referred to as *clickers*, have become commonplace in university classrooms as a way to interactively probe students' understanding of course material and to engage them more actively in lectures. These systems allow polling of students through a combination of specialized hardware and/or software that record, aggregate and display student responses in real-time. Clickers serve a variety of useful pedagogical purposes such as: testing prior knowledge, gauging student understanding of new concepts, providing instantaneous and visual feedback to students, increasing classroom participation, etc. A major advantage of being able to poll the entire class in this manner is that one can immediately identify situations where a significant number of students are confused or lacking understanding on an important point, thereby empowering the instructor to adjust their lecture on-the-spot to address the problem.

Although clickers have been utilized extensively and effectively in many university classes, it has been predominantly in lower division courses; and in the case of mathematics, attention has focused largely on development of clicker questions for introductory courses in calculus and linear algebra. There is a particularly conspicuous gap in openly-available clicker resources that cover material in the area of numerical analysis, which we discuss next. Here we use the term numerical analysis as a catch-all for courses that are offered in a number of academic departments under various titles including computational science, scientific computing, numerical methods and computational mathematics.

### 1.1 Review of Clicker Resources: The Numerical Analysis Gap

The use of short multiple-choice or true-false questions in a lecture setting was first proposed for physics courses by Eric Mazur in the form of *ConceptTests* [8, 15]. This approach was rapidly incorporated in other disciplines, and the polling of students during lectures was made easier and more interactive with the development of electronic student response

systems. These clicker devices, along with accompanying software, allow students' responses to be compiled immediately, presented as a summary table or bar chart, and then discussed further in class. An excellent literature review of clickers and their pedagogical uses in a wide range of disciplines is provided by Caldwell [5]. Clicker questions have become pervasive in mathematics instruction and there are numerous studies of clickers in entry-level undergraduate mathematics courses [3, 13, 16, 17, 20, 21]. These studies have established that use of clicker polling in lectures is "yet another source of a valuable learning tool for students" [16] and can generate a number of positive outcomes for both instructors and students. The most obvious and prosaic benefit for instructors is in terms of increased attendance, although they also clearly benefit from gaining an immediate measure of students' current understanding of course concepts. From the student's point of view, the advantages include immediate and regular feedback on their learning, an opportunity to measure their own knowledge against that of their peers, and improved satisfaction with their learning experience. Regular participation in clicker polls can also yield the interesting spin-off effect of stimulating other forms of in-class participation such as open class discussions followed by a re-polling, or personal consultations with individual peers (also known as "think-pair-share") – both of these techniques serve to engage students even more actively in lectures.

Devising interesting and insightful clicker questions can pose significant challenges. A concise and practical resource that provides useful tips on how best to formulate questions and implement them effectively in lectures is the *Clicker Resource Guide* by Wieman et al. [26]. In this guide, the authors identify ten question types that serve various purposes in a classroom setting, such as:

1. testing comprehension of assigned readings,
2. assessing recall of lecture points,
3. performing a step in a calculation,
4. surveying student opinions,
5. revealing pre-existing knowledge or thinking,
6. testing conceptual understanding,
7. applying ideas in a new context,
8. predicting results of a demo or simulation,
9. drawing on knowledge from everyday life,
10. relating different representations (mathematical, graphical, algorithmic, etc.).

Devising questions that satisfy such a wide variety of purposes for all topics in a course can be a daunting task, but thankfully there already exist extensive collections of openly available questions for certain subjects. The two most notable resources in mathematics are the GoodQuestions project for calculus [25], and the MathQUEST/MathVote [7, 27] project which covers a broader range of the first- and second-year undergraduate curriculum in calculus, linear algebra, differential equations and statistics. Several textbooks come bundled with a clicker resource [1, 10, 19] but these questions are subject to copyright and so are not freely available.

When it comes to upper-year undergraduate courses like numerical analysis, there are comparatively few open collections of clicker-type questions. Some multiple-choice and true-false questions can be found among the exercises in certain numerical analysis textbooks, the most noteworthy being Heath’s text [9]. The only freely-available question bank we are aware of is part of an open textbook written by Kaw et al. [11], although the number and diversity of questions are somewhat limited.

## 1.2 Aims and Outline

The main purpose of our paper is to remedy this “numerical analysis gap” and our aims in this respect are three-fold. First of all, we provide an account in Sections 2 and 3 of our efforts to construct a question bank that covers much of the standard curriculum in a third-year undergraduate course at Simon Fraser University called *Numerical Analysis I* (MACM 316 for short). Questions span the theory and application of algorithms for scientific computing and are posed as either true-false or multiple-choice questions (with 3 to 5 choices). Our intent was to assemble a sufficiently large collection of questions (at least 250 in number) that not only provides comprehensive coverage of the course material but also permits the instructor some flexibility in terms of both level of difficulty and question type. Because algorithms and MATLAB [14] programming are an integral part of MACM 316, we also include questions that test students’ understanding of basic algorithms in the form of MATLAB code, such asking them to identify bugs in erroneous code or to interpret tabulated or graphical output. This reflects the growing importance placed on integrating programming within undergraduate curricula [6, 22]; however, we also acknowledge that only a relatively small number of questions test programming concepts and that more effort could be expended in future to expand the question bank in this direction.

Secondly, we are releasing the clicker question bank as an open educational resource (OER) that should be of significant value to other teachers of numerical analysis. We describe in Section 3 how questions and solutions are conveniently written in  $\text{\LaTeX}$ , and macro files and slide templates are provided so that questions can be easily incorporated into other instructors’ lectures. Section 4 is a brief how-to guide that explains how we have integrated the clicker questions into our own lectures, along with some helpful in-class

tips. By releasing this resource under a Creative Commons Share-Alike (CC BY-NC-SA) license, we are strongly encouraging readers to contribute to this OER themselves by making corrections or improvements to existing questions, and more importantly by proposing new questions of their own that expand on the topics already covered. The following web site provides the complete resource in both `zip` and `tar.gz` formats for easy download:

<http://www.sfu.ca/~jstockie/NAclicker>

Our third and final aim is to share our experiences implementing the clicker question bank in a numerical analysis class during a recent semester (January to April 2020) and to illustrate how it has led to significant improvements in student satisfaction as well as the classroom learning environment. Support for these claims is provided in Section 5, which summarizes the results of a mid-semester student survey and also provides anecdotal observations comparing our experience teaching with clickers to several previous offerings of the same course without clickers.

If you have ever been tempted to experiment with clickers in your own numerical analysis course but were intimidated by the amount of work involved in writing good questions, then this just might be the OER for you!

## 2 COURSE OVERVIEW

We begin with a description of SFU's numerical analysis course MACM 316 to set the context for the development of the clicker question bank. MACM 316 is a course called *Numerical Analysis I* that covers the elementary topics from numerical analysis listed in the syllabus provided in Table 2. Admittedly, the name “numerical analysis” is considered by some to be old-fashioned and has been replaced in many universities by alternate titles such as computational science, scientific computing, computational mathematics, or numerical methods. The course can also be taught within a number of academic units, most commonly in mathematics but also computer science, engineering, physics, or other departments in science. The list in Table 2 is fairly standard for many introductory numerical analysis courses taught in universities around the world, and the topics map easily to the contents of most textbooks, one example being the seminal text by Burden, Faires and Burden [4] (which is required for MACM 316). The ordering of topics in the syllabus is slightly different from that in many books including [4] and is solely a personal choice of the instructor (JMS). A teaching semester at SFU consists of approximately 40 hours of instruction spread over 13 weeks and courses are parceled into 50-minute lectures taught three times per week.

### 2.1 Student Demographics

MACM 316 is a core course in the *Mathematics and Computing Science* (MACM) program at Simon Fraser University. Roughly two-thirds of students are enrolled in the School of

**Table 1.** Syllabus for MACM 316. The numbering of topics is the same as that used in the question bank, and the corresponding section number is listed from Burden, Faires and Burden [4].

#	Topic	BFB Section
1.	Introduction:	
	1a. Floating-point arithmetic and errors	1.2
	1b. Review of concepts from calculus	1.1
2.	Nonlinear equations:	
	2a. Bisection method	2.1
	2b. Fixed point method	2.2
	2c. Secant and Newton's methods	2.3
	2d. Convergence of iterative algorithms	1.3
3.	Systems of linear equations:	
	3a. Review of linear algebra	6.1, 7.1
	3b. Gaussian elimination, pivoting and matrix factorization	6.1, 6.2, 6.5
	3c. Plotting power laws (interlude)	
	3d. Iterative methods for sparse systems	7.3, 7.4
4.	Function approximation:	
	4a. Polynomial (smooth) interpolation	3.1, 3.3
	4b. Spline (piecewise) interpolation	3.5
	4c. Least squares fitting	8.1, 8.2
5.	Differentiation and integration:	
	5a. Numerical differentiation	4.1, 4.2
	5b. Numerical integration or quadrature	4.3, 4.4, 4.6
6.	Ordinary differential equations (ODEs):	
	6a. Background on ODE initial value problems	5.1
	6b. Euler's method	5.2
	6c. Taylor and Runge-Kutta methods	5.3, 5.4
	6d. Systems of first-order ODEs	5.9

**Table 2.** Grading scheme for MACM 316.

Quizzes (weekly, 10 mins long, in class)	18%
Computing assignments (bi-weekly)	24%
Clicker questions (participation only)	3%
Midterm test	25%
Final exam	40%

Computing Science, where the course is a graduation requirement. The remaining students are divided almost equally between programs in the Faculties of Science and Applied Science (Engineering). The prerequisites for MACM 316 are two first-year classes in differential and integral calculus, plus an introductory linear algebra class. Some prior computing experience is required because algorithms are central and students are obliged to submit MATLAB code as part of their assignment solutions. Major theorems and results on error estimates and convergence are all covered, but the emphasis is more on practical implementations of algorithms than on the theory behind them.

The usual enrollment in MACM 316 hovers around 200 students, with about 25% being in year 2 of their undergraduate programs and the remainder in years 3-4. The main purpose of this remark is to observe that the class is a relatively large one, and it is also an upper-year course so that most students are already more experienced. This distinguishes the student cohort in MACM 316 from that in most other clicker-based courses because clickers are most often implemented in large introductory classes [2, 3].

Furthermore, a significant number of students (especially those in Computing Science) tend to delay enrolling in this course until their final year, which means that there can be a 1–2 year gap since their last mathematics course. Consequently, many of these students have forgotten some of the important concepts or are out of practice with basic math skills that are a prerequisite for MACM 316. This demographic has guided the design of our clicker question bank, which has a noticeable emphasis on questions that either review prerequisite material in calculus and linear algebra, or test basic mathematical reasoning and calculations. For another class that consists of mostly math majors/minors, an instructor might choose instead to incorporate different questions that are more challenging or test deeper understanding of the material.

## 2.2 Grading Scheme

The serious computing component of this class is reflected in the grading scheme in Table 2 which puts significant weight on computing assignments. We also highlight that 3% of the final grade is assigned to participation in the in-class clicker polls, which is an all-or-nothing grade that students receive only if they respond to at least 75% of clicker questions posed during semester. Our aim is to encourage students to be present in class and to participate

in clicker polls, but to otherwise avoid imposing undue pressure on students to obtain the correct answer.

### 3 STRUCTURE OF THE CLICKER QUESTION BANK

The clicker question bank consists of a mix of multiple-choice and true-false questions based on the material from the syllabus in Table 2, which lists topics that are easily mapped onto chapters/sections from most numerical analysis textbooks. In authoring the questions, we aimed to employ a variety of the question types identified in the introduction (numbered 1–10) although we concede that our initial release of the question bank includes very few of types 1, 4 or 9.

Approximately 80% of questions were authored by ourselves, with the remaining 20% adapted mostly from two sources: Heath [9] and Kaw et al. [11]. A smaller number of questions testing background material from calculus and linear algebra were taken from the GoodQuestions [25] and MathQUEST/MathVote projects [27].

The clicker question bank (consisting of L<sup>A</sup>T<sub>E</sub>X source, image files and MATLAB code) can be obtained from the URL provided at the end of the introduction. Below are a few main features of the question bank that will help readers who are interested in implementing the questions in their courses and/or adding questions of their own:

- There are 267 questions in total, of which 242 are multiple-choice and the remainder true-false. The multiple-choice questions allow for 3, 4 or 5 choices (labeled A,B,C,D,E).
- Questions can be divided roughly according to the following classification:
  - 30% involve simple recall/application of definitions, theorems and formulas;
  - 15% each relate to review of prerequisite material, quick single-step calculations, visual interpretation of graphs or data, and “higher-order” reasoning (involving synthesis of several concepts or multi-step calculations);
  - the remaining 10% of questions relate either to algorithms (interpreting code or output) or concrete applications to real-world problems.

This clearly spans a wide variety of questions and levels of difficulty, although the last point does indicate an obvious opportunity for improving the question bank by incorporating additional questions that focus on algorithms and applications.

- Questions are encoded in L<sup>A</sup>T<sub>E</sub>X to ensure high-quality typesetting of the mathematics and to allow users to easily modify and cut-and-paste into other L<sup>A</sup>T<sub>E</sub>X documents. Several of the files deserve explanation:



- (a) `clickerQbank.{pdf,tex}`: A document containing all clicker questions that generates a compact 90-page listing of questions and solutions. Each question is defined as an item within a list environment called `clicklist`, and there is a macro named `\qitem...` defined for each question type. For example, a multiple-choice question with four choices (A,B,C,D) is encoded using the macro `\qitemMCfour` and is passed 8 arguments as follows:

---

```

\begin{clicklist}
...
\qitemMCfour{You have a system of three linear equations
with three unknowns. If you perform Gaussian
elimination and obtain the reduced row echelon form
\begin{gather*}
\left[ \begin{array}{rrr|r}
1 & -2 & 4 & 6 \\
0 & 1 & 0 & -3 \\
0 & 0 & 0 & 0
\end{array} \right]
\end{gather*}
then the system has \dots} % ARG1: question statement
{no solution} % ARG2: response (A)
{a unique solution} % ARG3: response (B)
{more than one solution} % ARG4: response (C)
{infinitely many solutions} % ARG5: response (D)
{4} % ARG6: correct answer (1=A, 2=B, 3=C,...)
{The last equation reads '$0=0$' so $x_3$ can be any
real number. Strictly (C) is also correct, but (D) is
the most accurate answer.} % ARG7: solution details
{MAH} % ARG8: author or source
...
\end{clicklist}

```

---

The formatted output for this question (numbered Q3b-16 in the question bank) is displayed in Figure 1. Additional macros are defined for the `clicklist` items `\qitemMCthree`, `\qitemMCfive` and `\qitemTF` corresponding to the other question types.

- (b) `clickerbeamer.{pdf,tex}`: A beamer [24] implementation of the `\qitem...` macros that instead generates a pair of slides – one containing the question by itself, followed by a second showing both question and answer. An example is pictured in Figure 2.
- (c) `clickerslides.{tex,pdf}`: A simple non-beamer slide implementation.

- (d) `clickerdefs.tex`: Defines several list counters and  $\LaTeX$  macros that are common to questions in all formats.
- Some questions include images, plots, tables, MATLAB code, or numerically-computed output. All images are supplied in both `.eps` and `.pdf` formats within the subdirectory `figures/`.
  - Plots are generated using MATLAB and the corresponding code can be found in the subdirectory `figures/matlab/`. This permits an instructor to easily modify a question by changing parameters, plot format, etc.

**Q3b-16.** You have a system of three linear equations with three unknowns. If you perform Gaussian elimination and obtain the reduced row echelon form

$$\left[ \begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

then the system has ...

(A) no solution  
 (B) a unique solution  
 (C) more than one solution  
 (D) infinitely many solutions

*Answer: (D). The last equation reads "0 = 0" so  $x_3$  can be any real number. Strictly (C) is also correct, but (D) is the most accurate answer.*

{ Source: MAH }

**Figure 1.** Formatted output (`clickerQbank.pdf`) for a sample multiple-choice question having 4 choices, typeset using the macro `\qitemMCFour`.

**Clicker Question #1**

You have a system of three linear equations with three unknowns. If you perform Gaussian elimination and obtain the reduced row echelon form

$$\left[ \begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

then the system has ...

(A) no solution  
 (B) a unique solution  
 (C) more than one solution  
 (D) infinitely many solutions

**Clicker Question #1**

You have a system of three linear equations with three unknowns. If you perform Gaussian elimination and obtain the reduced row echelon form

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then the system has ...

(A) no solution  
 (B) a unique solution  
 (C) more than one solution  
 (D) infinitely many solutions

**Answer: (D).** The last equation reads "0 = 0" so  $x_3$  can be any real number. Strictly (C) is also correct, but (D) is the most accurate answer.

(a) First slide with question only.

(b) Second slide showing question and answer.

**Figure 2.** Beamer slides (`clickerbeamer.pdf`) for the same multiple-choice question in Figure 1.

The question depicted in Figures 1 and 2 has the interesting feature that there are two valid responses (C,D). When this question was actually presented in class, students were

equally-split between responses (C) and (D), which then naturally initiated a follow-up discussion around why there were multiple correct answers and also why (D) was the best response. This is just one example of when an instructor must be prepared to respond to poll results by spontaneously allowing students to discuss and work through the solution for themselves rather than just uncovering the answer slide and moving on. Most questions in the question bank do have one unique answer, but there are several with multiple valid responses that can present opportunities for lively classroom debates of this sort. Indeed, Wieman et al. [26] remark that “such questions can generate the most educationally productive discussions.” Moreover, clicker polls and the animated peer discussions that accompany them provide a mechanism for what mathematics educators identify as “embodied actions to build up dynamic embodied concepts in mathematics” [23].

Another noteworthy feature of the question bank is that there are several groups of closely-related questions, consisting of minor variations on a common theme or differing levels of difficulty. On the one hand, this provides instructors with variety and flexibility to focus on specific aspects of the material. While it is not intended that all such related questions need to be posed together in a single lecture, mathematical cognition researchers such as Tall [23] do recognize that such repetitive questioning encourages “recognition of similarities, differences and patterns; repetition of actions to make them routine.”

Finally, we should recognize that several conspicuous gaps exist in the question bank corresponding to topics from the standard introductory numerical analysis curriculum. We note that the list in Table 2 fails to cover nonlinear systems of equations, Krylov-space methods, matrix eigenvalue problems, and numerical stability and implicit methods for ODEs. These are all areas where the question bank would benefit from additional questions.

#### 4 MECHANICS OF IN-CLASS CLICKER POLLING

[ Many small changes have been made in this section that leave the discussion largely unchanged ]

For the next two sections, we switch to using the personal pronoun “I” since only one of us (JMS) was the instructor for MACM 316. The aim here is to provide a how-to guide permitting interested instructors to incorporate clicker polling into their own lectures as easily as possible. Some of what follows relates specifically to the iClicker system (<http://www.iclicker.com>) which was chosen because it integrates so seamlessly with the Canvas learning management system (<http://www.infrastructure.com/canvas>) used at SFU. This requires that all students purchase or borrow an iClicker radio-frequency remote, two versions of which are pictured in Figure 3(left). Student responses are gathered via a base station that is connected to a computer running the *iClickerClassic* app. iClicker has an alternate web-based system in which students use smartphones or laptops running the *iClicker REEF* app instead, responses are transmitted through a Wi-Fi connection,



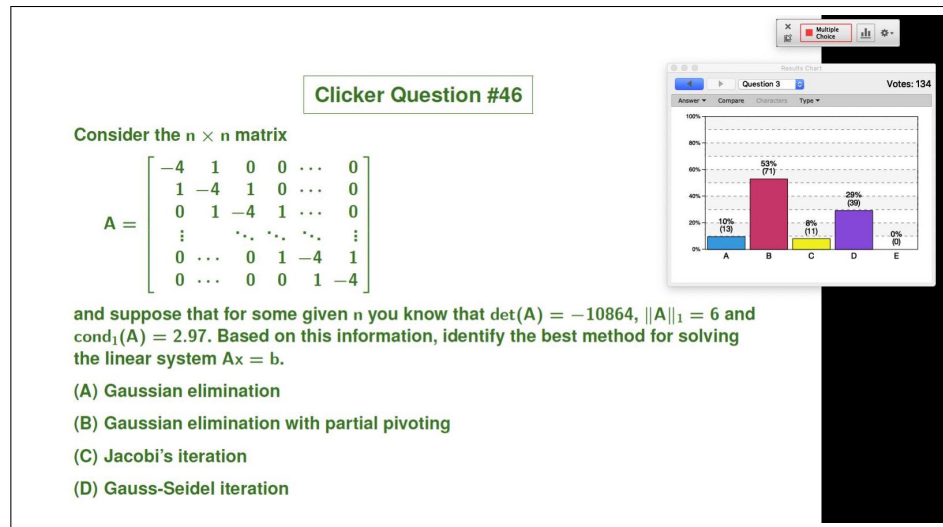
**Figure 3.** (Left) Two versions of the iClicker radio-frequency remote. (Middle) iClicker RF base station. (Right) Laptop and smartphone running the *iClicker REEF* app through a Wi-Fi connection. Source: <http://www.iclicker.com>.

and the base station is eliminated in favour of an internet-connected instructor app called *iClicker Cloud* (shown in Figure 3(right), which also integrates with Canvas). During this initial trial of the question bank, I decided to use the remote-only *iClickerClassic* option to minimize any difficulties handling input from multiple devices, as well as avoiding problems with intermittency in classroom Wi-Fi coverage. However, in future offerings of MACM 316 I envisage switching to *iClicker Cloud* to address the most common complaint on student surveys regarding the cost of the remotes (see Section 5 and Appendix A) and indicating a strong preference for using smartphones instead.

iClicker is only one company that markets student response systems, and several other systems are available. For example, Turning Technologies manufacture their own remote devices with a corresponding app that is also supported by Canvas. Other learning management systems (LMS) such as TopHat (<http://www.tophat.com>) and Acadly (<http://www.acadly.com>) have their own built-in polling systems that don't require specialized hardware but permit students to use their smartphones or laptops. Some free and open-access alternatives are the Moodle LMS (<http://moodle.org>) and the on-line testing systems provided by Kahoot! (<http://kahoot.com>), Nearpod (<http://nearpod.com>) and Itempool (<http://www.itempool.com>).

I found that it took a few lectures to familiarize myself with the process of clicker polling and develop a routine for incorporating clicker questions smoothly into lectures. The iClicker system makes this process relatively painless while also providing significant flexibility for customizing to suit individual instructors. Below is the procedure I followed when polling students, along with a few tips that may prove helpful to other instructors:

- Polling is especially easy to manage when lecture notes are displayed using an LCD projector connected to a laptop running the iClicker app. In MACM 316, I distribute skeleton lecture notes in advance of lectures that are typeset in  $\text{\LaTeX}$ , provide basic definitions, theorems, background material and examples, and leave students to fill in the blanks during class. Clicker questions are easy to cut and paste from the



**Figure 4.** Lecture slide displaying a clicker question, overlaid by the *iClicker Classic* widget (top right) and a bar plot summarizing student responses (middle right).

L<sup>A</sup>T<sub>E</sub>X question bank into the slides, although I chose not to include them in the skeleton notes for students. The clicker questions may also be used in a class taught on the whiteboard or blackboard, but then one must toggle back-and-forth from the LCD projector for each polling session.

- With the *iClickerClassic* app running in the background, a small widget is displayed on top of the slide window that contains a play/stop and plot button (see Figure 4). Upon uncovering a slide that contains a clicker question, I launch a new poll with the *iClicker* widget which then starts a timer and displays a live counter of responses. I provided students between 45–60 seconds to select a response, and gave a 10-second warning before closing each poll. At that time, clicking the plot button on the widget displays a histogram of student responses (see Figure 4).
- I typically pose between 3–5 clicker questions in each 50-minute lecture and I group questions in clusters of 2–3 to reduce the time students spend switching over to their remotes, and to minimize interruptions in the lecture. Working through each cluster required at least 5 minutes, which is a welcome break for students and also helps me to avoid my natural tendency to speed up and steam-roll through lectures.
- I impose no set pattern on timing of questions, variably using them as initial warm-ups, final wrap-ups, checks on student understanding after introducing a new concept, or simply to introduce a break. The key is to be flexible.
- I encourage students to discuss questions with their neighbours and (time permit-

ting) wander through the classroom to listen in on conversations and pinpoint any difficulties. Quieting students down after a particularly vigorous discussion can take a few extra moments, but the benefits of this peer instruction are worth the small disruption.

- When the histogram of responses is revealed, if at least 75% or so chose the correct answer then I advance directly to the solution slide, highlight an important point or two, and move on. Otherwise, this may be an ideal opportunity to discuss the problem more deeply and ensure that students who answered incorrectly understand the source of their error.
- If a question receives an unusually high proportion of incorrect responses then I make a separate note of it, recognizing that such questions are either: ill-posed or confusingly worded, in which case I modified them or removed them from the question bank; or especially difficult and require more time to solve, which I flag as potential test/homework problems or supplementary examples for lectures.

Clearly, any class time spent on clicker polling and discussions has a significant impact on the material that can be covered. Wieman et al. [26] observe that “most instructors do end up deciding to cover slightly ( $\sim 10\%$ ) less material in their courses after they have started using clickers effectively,” and my own experience is consistent with this. However, this is a small price to pay considering the substantial benefits in terms of student satisfaction and improved classroom learning environment. Latulippe [12] provides an excellent report of the use of clicker questions in a first-semester calculus course, including practical implementation tips and anecdotal comments from the perspective of both student and instructor.

#### 4.1 Unexpected Switch to On-line Instruction

The Jan-Apr 2020 semester on which this article is based was an exceedingly unusual one because it coincided with the outbreak of the COVID-19 pandemic. Classes at SFU ran as usual until mid-March (half-way through semester) at which time the university shifted all classes on-line to comply with provincial health guidelines on physical distancing. Because iClicker remotes function only in a classroom setting with an RF base station, I decided at that point to stop collecting clicker response data and to determine students' participation grades based only on their responses from the first half-semester of clicker questions.

Like many other universities, SFU has responded to the COVID-19 pandemic by moving almost exclusively to an on-line mode of teaching for the next academic year. This means that clicker remotes cannot be used since they operate by communicating RF signals to a base station. Therefore, at least in the near future, clicker polling in MACM 316 will have to be done via Wi-Fi connection using a system like *iClicker Cloud/REEF*.

Although the mid-semester interruption of this clicker experiment was disheartening, it did uncover an unforeseen by very convenient application of the clicker question bank. Upon the switch to on-line instruction, I simultaneously transformed my weekly in-person quizzes into an on-line multiple-choice format for which it was natural to re-purpose clicker questions. Based on this experience, I can confirm that the question bank is also an excellent source of multiple-choice and true-false questions for use on quizzes or tests.

## 5 IMPACTS ON TEACHING AND LEARNING

This section is a summary of several impacts on instructional practice and students' learning experience that we ascribed to the introduction of clicker questions in lectures.

### 5.1 Three Examples of Transforming Teaching Practices

Clicker polling provides many opportunities for an instructor to transform their teaching practices by engaging students more actively in the learning process and being more responsive to students' needs. Here I relate three examples of questions that either led to surprising results or were especially effective in terms of improving students' learning experience. These examples are also chosen to highlight certain subject-specific issues of particular relevance to material in numerical analysis and scientific computing.

**Example 1: Opportunities for agile/reactive interventions.** Every instructor has experienced the situation where they introduce a new concept that appears to go over smoothly during class, but then a test or homework question exposes a significant and unexpected shortcoming in students' understanding. Clicker polls provide an outstanding opportunity to address such problems in a timely manner by probing students' understanding of important concepts interactively. One of the fundamental goals of MACM 316 is to guide students in developing an intuitive sense for numerical error, and the concept of *relative error* in an approximation is absolutely fundamental. After introducing the definitions of absolute and relative error and working through several concrete examples, along with a discussion of the number of significant digits, the following question was posed:

*Below are four floating point approximations, each accompanied by its corresponding exact value. Which approximation is the most accurate?*

- (A) 315700, exact value 315690
- (B) 0.0005500, exact value 0.0005510
- (C)  $8.7362 \times 10^{-5}$ , exact value  $8.7743 \times 10^{-5}$
- (D)  $\varepsilon_M = 2.2204 \times 10^{-16}$  (machine epsilon), exact value 0

Surprisingly, just over 50% of students responded (D) for which the relative error isn't even defined. Only 17% of students identified the correct answer (A), which is easy to establish

by counting the number of correct significant digits among choices (A)–(C) after eliminating choice (D). Prompted by these results, I ran an impromptu mini-lecture on the importance of measuring accuracy using relative error (or number of significant digits) that provided students with a much better understanding of this important concept.

**Example 2: Developing computational intuition.** In numerical analysis, one of the great achievements that one strives to give students an appreciation for is that simple mathematical arguments can predict with surprising accuracy how practical algorithms behave in an order of magnitude sense. One example is measuring computational complexity by performing a rough count of floating-point operations which can then be used to appraise and compare the efficiency of different algorithms. After first reviewing the Gaussian elimination process with students (from their previous linear algebra course) and then reformulating it algorithmically in terms of an LU-decomposition, I posed the following question:

*You have a  $100 \times 100$  matrix  $A$  and your computer solves the system  $Ax = b$  in exactly one minute using the LU factorization. Roughly how much of that minute is spent performing the row-reduction operations to determine the factorization  $A = LU$ ?*

- (A) 30 secs      (B) 45 secs      (C) 53 secs      (D) 59 secs

This question was met largely with confused looks and an unusually slow response rate according to the widget counter. With 20 seconds remaining, I encouraged students to make a rough guess that reflects their intuition regarding the relative cost of various steps in the algorithm. That of course was the original intent, because accurately computing the correct response (D) requires students to have memorized (or look up from lecture notes) formulas for the total cost ( $2n^3 + 6n^2 - 2n$ ) and LU-decomposition cost ( $2n^3 + 3n^2 - 5n$ ), then estimate the ratio for  $n = 100$ , and then convert to a value in seconds. Expecting students to complete this multi-step calculation within the 60–90 seconds allowed for this poll is unreasonable. In the end only 25% of respondents answered correctly (no different from random choice) and student feedback suggested that most were genuinely surprised by just how much the matrix decomposition dominates the algorithm cost. This experience seemed to provide students with a much better appreciation for the importance of making order-of-magnitude estimates and they were well-prepared for a similar question that was posed on the final examination. This experience further highlights the side benefit of the question bank in that questions can be easily re-purposed for tests or exams.

**Example 3: Fostering in-class discussions.** It is far too easy to steam-roll through material in a lecture setting without providing students time to reflect or ask questions. Clicker polling provides an opportunity to change pace by giving students a welcome break and opening up space for classroom discussions. Although this applies to any clicker question, experience showed that the most lively and productive discussions often arose from



well-formulated true-false questions that weren't completely obvious or left some ambiguity. Such questions presented students with a simple choice between two options, which I then challenged them to justify before the correct answer was revealed.

Two questions are selected to illustrate this point with the first regarding iterative algorithms for solving nonlinear equations:

*TRUE or FALSE: Both Newton's method and Secant method are examples of fixed-point iterations.*

Responses came in quickly and settled on a majority of nearly 70% in favour of "TRUE", which is incorrect! This result follows simply from the definition of a fixed point iteration that I had clearly stated only a few minutes beforehand. When I provided students a few extra moments to compare and discuss their answers with their neighbour before re-voting, it was gratifying to see how quickly they convinced each other to re-orient towards the correct answer.

Another example was intended to test students' prerequisite knowledge of linear algebra:

*TRUE or FALSE: If a matrix  $A$  is nonsingular, then the number of solutions to the linear system  $Ax = b$  depends on the particular choice of right-hand side vector  $b$ .*

Again, a solid majority of 60% of students chose the incorrect response of "TRUE", which was initially surprising until the succeeding class discussions revealed a prevailing confusion over the meaning of "nonsingular". This led me to adjust my presentation of the material to more clearly emphasize that the terms nonsingular and invertible are interchangeable.

I remark in closing that only 10% of all questions are currently of true-false type, and so this is one potential area of future expansion for the question bank.

## 5.2 Student Satisfaction Survey

A student satisfaction survey was held at the mid-point of semester, which coincidentally was only two days before SFU switched all classes to an on-line mode of instruction in response to the COVID-19 pandemic. It was unfortunate that in-class clicker polling had to be terminated at that point owing to our reliance on iClicker remotes. However, because the survey was completed before the switch to on-line, we are confident that the results are unaffected by any of the ensuing pandemic-related disruptions.

A summary of survey questions and student responses is provided in Appendix A. The survey consisted of 7 multiple-choice questions (Q1–Q7), one set of 10 statements rated on a Likert scale (Q8, with 5 choices ranging from "strongly disagree" to "strongly agree"), and one free-response comment box (Q9). Several questions were modelled after similar surveys of clicker-based calculus classes [3, 13, 16], and the Likert-scale question Q8 was inspired by the human development class in [2]. A total of 137 responses were received out of 199 registered students. The corresponding response rate of 69% is relatively high for this

sort of survey and so we are confident that the results are representative of the entire class. The survey was administered as a Canvas Quiz, which allowed responses to be submitted anonymously and the data easily downloaded.

Rather than describing all survey responses in detail, we highlight what we see as the most important results and refer the interested reader to Appendix A for more details.

- Questions Q2 and Q3 suggest that more than 80% of students thought the clicker questions helped them to improve their understanding of the course material.
- Questions Q4 through Q7 indicate that the majority of students were satisfied with the level of difficulty of the problems, the method of classroom delivery, and the grading scheme based on participation only. Some instructors suggest assigning a clicker grade based on both participation and correctness [26] which is easy to implement in the iClicker system. However, the survey suggests that students derive sufficient benefit without pressuring them to submit correct responses.
- Ratings for the 10 statements listed in Question Q8 were an especially strong indicator of the high level of satisfaction and positive impacts on students' learning experience. The first three statements (A,B,C) received more than 80% positive responses (either "agree" or "strongly agree"), which reflects two aspects of clicker polling that are also recognized in other clicker studies:
  - Students have a strong preference for the anonymity of a clicker poll over answering questions individually, especially in a large class.
  - Clickers provide a positive break from the rest of lecture.

Of the remaining statements (D to J) all but one (H) received at least 50% positive responses and at most 20% negative, which reflects the beneficial contributions of clicker polling to the classroom learning environment.

- One result from question Q8 that deserves special mention concerns statement H: "I feel more connected to my classmates when we use iClickers". This attracted a much more mixed response, which exposes a distinct opportunity to involve students more actively and collaboratively with each other by incorporating a *peer instruction* or "think-pair-share" component more extensively into the clicker polling process. In this approach, students are provided extra time after an initial clicker poll (and before the solution is displayed) to discuss the results with each other, after which a second poll is taken to determine any changes in student opinion. Peer instruction is a well-established teaching practice [8, 13, 16, 17, 18, 26] that has proven effective in engaging students more actively in the learning process, improving their critical thinking and communication skills, and building a more collegial intellectual atmosphere.

The above results were strongly supported by individual free-form responses to Q9 (see Appendix A), which were assuredly the most encouraging indicator of student satisfaction with our clicker experiment.

### 5.3 Anecdotal Observations

We make no claim that clicker polling had any direct connection with improvements in students' grades or other quantifiable learning outcomes. Some previous studies have attempted to do so [8, 13, 17, 18, 20] and although the results suggest there are some positive correlations with student performance, this effect tends to be relatively small. Our focus in this paper is rather on the significant impact that devoting class time to clickers can have on the classroom atmosphere and students' learning experience. Outside of the above comments on teaching practices and student satisfaction, we also have a few observations from an instructor's perspective of a "before and after" nature (comparing the most recent semester with two other recent offerings of the same course) that provide anecdotal support for the positive effects of clickers in the classroom:

- *Increased attendance:* The percentage of enrolled students attending lectures (when there was no in-class quiz) was between 70–85% and normally lay closer to the high end of this range. This is in comparison with a 50% attendance estimated from other recent offerings of the course. It should be no surprise whatsoever that assigning a portion of the grade that requires attendance in class should increase attendance. Rather, we want to highlight the virtual absence of any negative feedback from the survey regarding the clicker participation grade, nor was there any suggestion that this was viewed as some form of coercion.
- *Student questions before/after lectures:* Compared with previous semesters, there was a marked increase in the number of students asking questions during class time (outside that devoted to clicker polling). But what was particularly striking was the long line-up of students waiting to ask questions after class, which often required an additional 20-30 minutes. It is unusual to see such enthusiasm from a large number of students, and we can only hypothesize that perhaps the clicker polling gave students a sense that their instructor is more approachable and cares about their learning.
- *On-line discussions:* A discussion board has always been provided as part of the Canvas container for this class, and there was more than a 100% increase in student postings *before* the move to on-line instruction at mid-semester.

When combined with the survey results, these observations suggest that engaging students more actively in the classroom through the use of clicker polling can have certain benefi-

cial side-effects in terms of not only improving the students' sense of engagement but also enhancing the atmosphere of collegiality both inside and outside the classroom.

## 6 CLOSING REMARKS

The use of clicker polling in university mathematics classes is commonplace and while many clicker question resources are available to support introductory courses, there is next to nothing developed for more advanced courses such as numerical analysis. We describe an open-access clicker question bank that we have created to fill this gap, and which covers a significant portion of the introductory numerical analysis curriculum. Based on the first semester in which the questions were implemented in class, results from a student survey suggest that clickers had a significant positive impact on the student learning experience.

This question bank is a work in progress, and we continue to improve on existing questions as well as adding new problems each semester that our numerical analysis course is taught. There are a few important topics that are completely missing at present (systems of nonlinear equations, eigenvalue problems, Krylov methods for linear systems) as well as specific areas that we believe would benefit from additional questions (ODEs, interpolation). There is also a great deal of room for incorporating new questions that deal with applications of numerical algorithms to problems in physics, biology, engineering, economics, health and other fields. Finally, our experience has shown that carefully-designed true-false questions can lead to excellent opportunities for in-class discussions and re-polling, and so this is another fruitful area for new question development.

We would like to close with a quote taken from Wieman et al. [26]:

*“We have found the effective use of clicker questions and discussion can have a transformative impact on both teachers and students, particularly in large classes. Students end up being far more actively engaged in the material and they learn more, and both students and instructors find the course much more rewarding as a result.”*

This has certainly been our experience and we strongly encourage readers to consider using clicker polling in their own classroom. Furthermore, with so many universities around the world recently transitioning to remote class delivery in response to the COVID-19 pandemic, clickers present a special opportunity for instructors to engage their students actively in an on-line lecture setting. It is our sincere hope that the open-access clicker question bank described here will help to reduce the cost of entry for instructors of numerical analysis who have an interest in experimenting with student response systems.

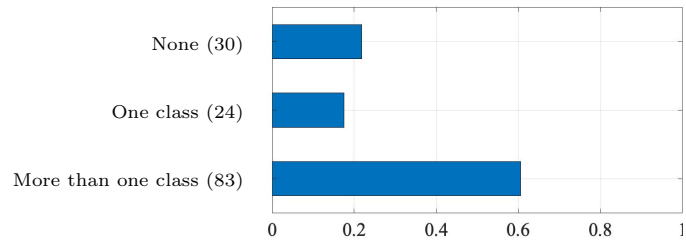
## APPENDIX A: STUDENT SURVEYS

### A.1 Responses to Survey Questions

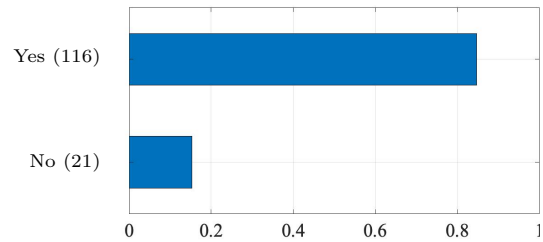
For the first 7 questions (Q1–Q7), responses are depicted as a bar chart showing the percentage of all 137 completed surveys, with the number of students who selected each response

also shown in parentheses.

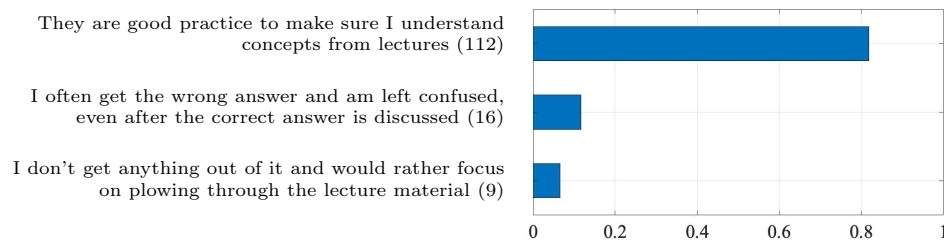
Q1. In how many other classes have you used iClickers before this semester?



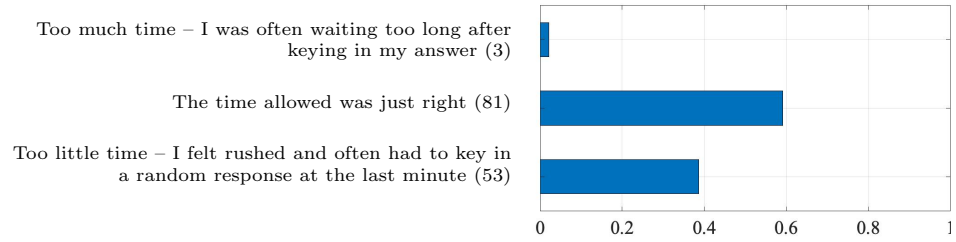
Q2. Overall, do you find that the iClicker questions have helped you to better understand the material presented in lectures?



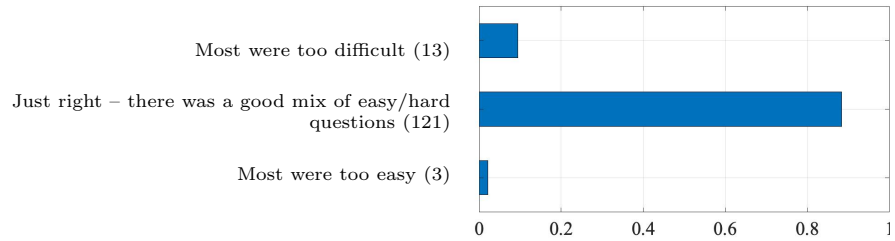
Q3. Pick the statement that best describes your experience with the iClicker questions.



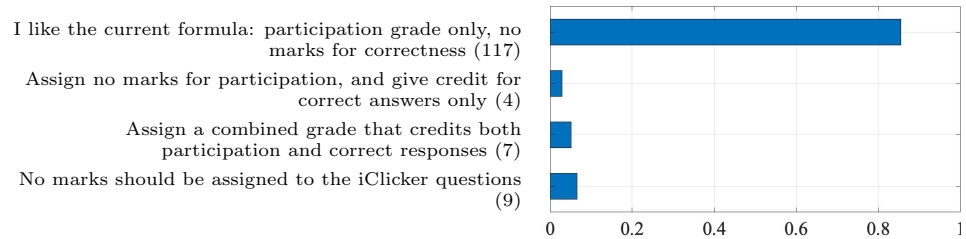
Q4. Can you comment on the amount of time permitted to answer each question?



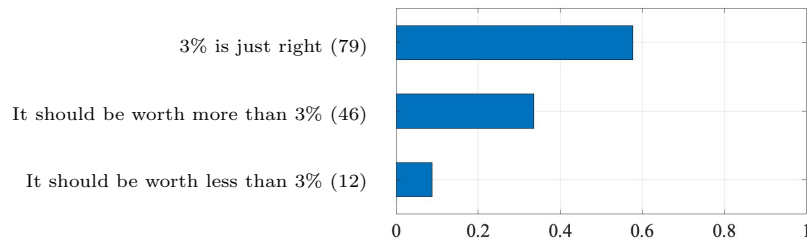
Q5. Can you comment on the average level of difficulty of the questions?



Q6. Do you have any preference regarding how the iClicker grade should be computed?



Q7. Assuming that some marks are assigned to completing the iClicker questions, what is your opinion of the portion 3% allocated to your final grade?

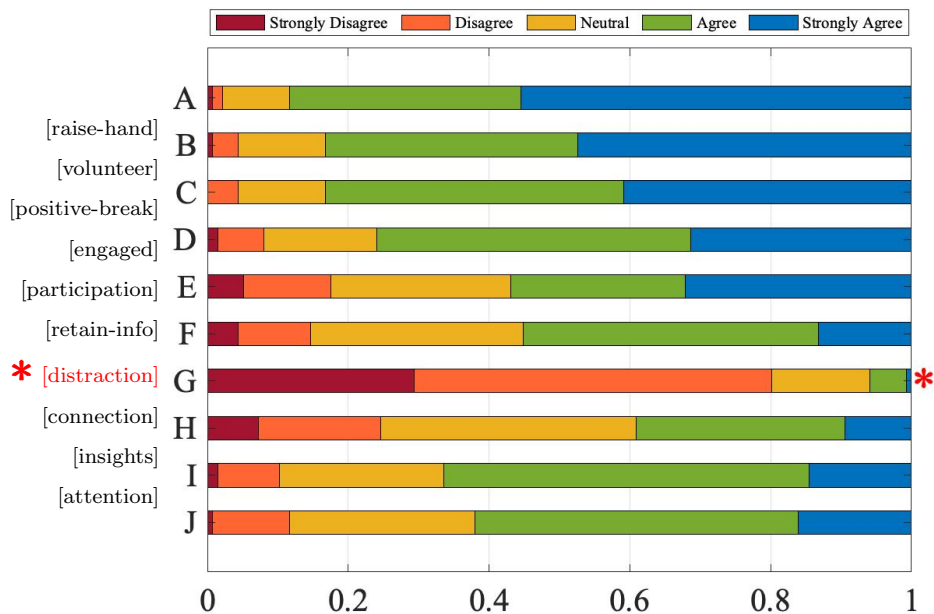


Q8. Rate your level of agreement with statements A-J on the following scale:

– Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree –

- A. [raise-hand] When a question is asked, I like the iClicker response option better than asking students to raise their hands to indicate their answer.
- B. [volunteer] When a question is asked, I like the iClicker response option better than having an open invitation for any student to volunteer an answer.
- C. [positive-break] I think iClickers provide a positive break from lectures.
- D. [engaged] I feel more involved and engaged in class when we use iClickers.
- E. [participation] I come to class more often because iClickers are used to track participation.
- F. [retain-info] I retain more information from lecture when iClickers are used.
- G. [distraction] I think iClickers distract from the learning process. \*
- H. [connection] I feel more connected to my classmates when we use iClickers.
- I. [insights] iClickers help me to discover more insights into the course material.
- J. [attention] It is easier to pay attention in lectures when iClickers are used.

Note that statement G (marked \*) is the only one where a response of “strongly agree” has a *negative* connotation, and so this bar should be interpreted in the opposite manner to the others. This was done to verify that students were indeed carefully reading and applying the Likert scale to each statement.



## A.2 Free-form Comments

[ This section has been transferred unchanged from the main text ]

The final survey question asked students to provide free-form comments in response to the prompt: “Do you have any other comments or suggestions related to the iClicker questions?”. We begin by summarizing the critical (but mostly constructive) comments which can be distilled down to four main points:

- *iClicker remotes are too expensive:* The cost of roughly \$50 for a remote is a significant expense, although not unreasonable when one considers that nearly 80% of the MACM 316 students needed them in at least one other class (Q1). It is not surprising that students would prefer to use their smartphones, but while the *iClicker REEF* app can be downloaded freely it does require an annual subscription fee of \$25. One possible concern about encouraging students to use their smartphones in class is that they may be more easily distracted by alerts from social media and other extraneous apps on their devices.
- *Can you post clicker questions and answers after class?:* In our opinion, the clicker component of this course is a major benefit of attending lectures in person. We have also found that certain clicker questions make excellent test or exam questions, which is why we preferred to not make the questions available to students. Wieman et al. [26] recognize that there are varying opinions about whether or not to provide clicker questions and solutions after class, and conclude that there is “no data that indicates one way or another.”



- *Polling time was insufficient, or the question text was too long:* This is a valid complaint and we have already made attempts to shorten the text in some of the wordier questions. We also plan in future to devise more short questions, in particular those of the true-false type.
- *Questions were spread unevenly between lectures:* Although our original plan was to evenly distribute 3–5 questions in every lecture, we failed to account sufficiently for the unpredictable delays involved with conducting polls and discussing the results. Based on our experience, we plan to modify the lecture notes and adjust the distribution of questions to address this concern.

Outside of these critical remarks, the vast majority of other comments were of a positive nature and highlighted students' high degree of satisfaction with their learning experience. Selected comments are provided below, which are far from an exhaustive list but help to convey a sense of the potential benefits that clicker polling can have on the learning environment:

- “I think iClickers serve as a useful addition to a class setting. It encourages active participation and encourages students to engage with the material. In this sense, it seems to work well, especially in a large class setting.”
- “It helps to keep my attention in class when we're switching back and forth (helps with short attention span).”
- “I do not feel the pressure of getting a wrong answer in front of a 150 person crowd. I also have more time to think about an answer rather than a sharp student immediately answering so I get more out of the question.”
- “At the beginning of the semester I was slightly annoyed with having to buy an iClicker. It was used quite effectively though, and I feel that it was worth the cost overall. It gave me a little extra push to make it to class, without the panic that I get from graded quizzes. I definitely prefer it to the alternatives like TopHat or Canvas quizzes, which are difficult to connect to in some classrooms.”
- “I usually enjoy classes with iClickers more because I usually dislike raising my hand so iClickers grant a anonymity that I enjoy.”
- “The depth and quality of questions asked made up for the cost and then some. It also helped knowing others in class via discussions, and the anonymity was helpful to know sometimes I am not the only one confused.”
- “The participation is a very positive way of doing the questions, rather than pressuring students to get them right. Learning is encouraged, rather than only focusing about getting the correct answer.”

- “I love your usage of iClickers, it acts like a little break in between class that helps us keep focused more before/after we do them. They also let us learn at our own pace, getting an answer wrong means I need to work on that certain material again.”
- “The questions gave me immediate feedback on how I was doing in the class and what I need more practice on.”
- “I particularly like tricky questions, edge case questions, or questions with an un-intuitive answer. I feel like these questions really help to solidify my understanding of the material.”

### BIOGRAPHICAL SKETCHES

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