Supplementary Material S2:

DESCRIPTION OF MATLAB CODES AND DATA FILES FOR "A MATHEMATICAL MODEL FOR NORDIC SKIING"

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These notes provide an overview of the Matlab codes and data files used to perform the skier simulations in our paper submitted to the Education section of *SIAM Review*. Each Matlab file is described briefly below and a more detailed description, including a calling sequence for functions, can be obtained by typing `help func` where "`func`" is the name of the corresponding `.m`-file. All codes are commented generously, and we encourage the interested reader to consult these comments for details on the algorithm implementation.

The list of data files includes all CSV and GPX data from the examples and exercises.

S2.1 Matlab Codes

`skirun2d.m`: The main code that simulates the dynamics of a skier travelling along a 2D course corresponding to an elevation profile plot in the ξ, z plane. This code calls `setup2d`, `setup2dsz` or `setup3d` to initialize the course/skier data, solves the ODE system for the 2D skier dynamics, and produces a simple plot of the solution.

`skirun3d.m`: The 3D extension of `skirun2d` that simulates the dynamics of a skier travelling along a fully 3D course defined as a space curve $(x(\xi), y(\xi), z(\xi))$ parameterized by project arclength ξ. This code calls `setup3d` to initialize the course/skier data, solves the ODE system for the 3D skier dynamics, and produces a simple plot of the solution.

`setup2d.m`: Reads the course (ξ, z) coordinates from a CSV file, constructs a spline approximation for the course, and returns data structures containing both the original data and the splines. This code is suitable for data extracted from the usual course elevation profile plot.

`setup2dsz.m`: A modified version of `setup2d` that reads the (s, z) coordinates, with s being the arclength rather than projected arclength. This code is specialized to handle the data provided in the MSH paper [24, Fig. 1] and requires an additional step to first approximate the projected arclength ξ, before constructing the parametric splines.

`setup3d.m`: Reads the course coordinates (λ, ψ, z) from a GPS file, converts to Cartesian coordinates, constructs the spline interpolant, and returns data structures containing both the original data and the splines.

`cumsimpson.m`: Uses Simpson's quadrature rule to approximate the integral $\int y(x)\,dx$, where the function is sampled at a given array of y values, y_i for $i = 1, 2 \ldots, N$. The corresponding values of x_i are assumed to be equally-spaced with unit spacing. This code performs a "cumulative" Simpson's rule computation, analogous to the built-in function `cumtrapz`, and returns an array of integral approximations for $\displaystyle\int_{x_1}^{x_i} y(x)\,dx$.

`skimovie2d.m`: Generates an animated view of the skier moving along a 2D course which is generated by first running `skirun2d` or `skirun3d`.

`skimovie3d.m`: Generates an animated view of the skier moving along a 3D course which is generated by first running `skirun3d` only.

`starcourse.m`: Generates (x, y, z) data for the artificial "star-shaped" course needed in Exercise #19.

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S2.2 Data Files

`moxnes.csv`: CSV file containing data extracted manually from Figure 1 of Moxnes, Sandbakk and Hausken (MSH) [24] that is used in Example 1, Section 4.1.

`welde.csv`: CSV file containing data extracted manually from Figure 2 of Welde et al. [34] that is used in Example 2, Section 4.2.

`carlsson.csv`: CSV file containing data extracted manually from Figure 3 of Carlsson, Tinnsten and Ainegren [5] and is used in Exercise 15.

`Dolomiti4-2kmFISOle.gpx`: GPS data file for the FIS-certified "Ole" course used in examples throughout the paper in Sections 2, 4.3 and 6. This file is downloaded from the Dolomiti NordicSki web site [8].

`Dolomiti4-9kmAltpragsUphill.gpx`,
`Dolomiti5-5kmFISNathalie.gpx`,
`Dolomiti3-9kmFISStephanie.gpx`,
`Dolomiti5-3kmFISSaskia.gpx`,
`Dolomiti1-4kmStadiumTrackAlbertSprint.gpx`: These five GPS data files are used in Exercises 4, 5, 14, 18, and are also downloaded from the Dolomiti NordicSki site.