APMA 930–4
Computational Fluid Dynamics (CFD)
Spring 2021

Instructor.

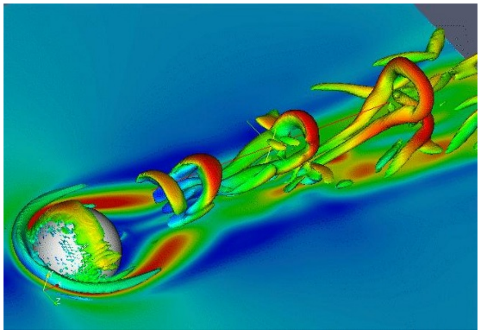
John Stockie
Department of Mathematics, SFU
jstockie@sfu.ca

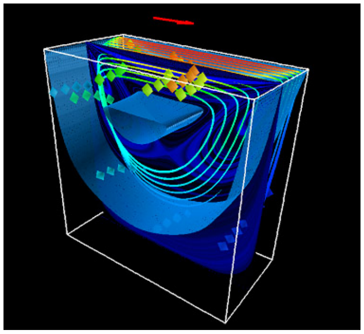
Classes.

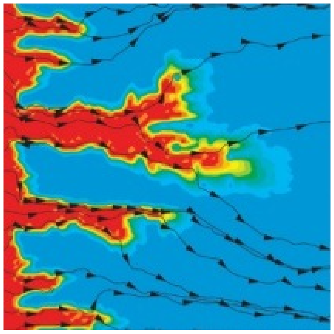
Time: Tuesdays & Thursdays 2:30-4:20
Location: On-line lectures (Zoom)

Description. This course will introduce students to a variety of computational approaches for solving the partial differential equations governing fluid dynamics, focusing on finite difference and finite volume techniques. Theoretical background material will be introduced as necessary, but the emphasis of the course will be on the numerical methods, their accuracy and stability, and applying them in practical calculations of real fluid flows. Students will gain experience writing their own codes, as well as employing existing open-source software packages. Applications will be drawn from a wide variety of problems arising in wave propagation, incompressible fluids, compressible gas dynamics, and porous media flow. In contrast with the common engineering approaches to teaching CFD, I will not emphasize the study of complex flows in sophisticated geometries using commercial codes, but will focus instead on the design of the underlying algorithms, and carefully assessing their correctness, accuracy, efficiency and robustness.

Prerequisites. Previous courses in ordinary and partial differential equations (such as MATH 310 or MATH 314) are required. A previous course in fluid dynamics (such as MATH 462) would be an advantage, but is not required. Some computing experience is highly recommended (with any programming language). The majority of algorithms presented in class will be implemented in MATLAB, although some compiled codes written in C or FORTRAN may also be used.


Flow over a cylinder.


Lid-driven cavity flow.


Fingering in a porous medium.

Grading. The grade for this course will be determined as follows:

- *Homework (40%):* homework questions will be assigned roughly every 2 weeks, consisting of both theoretical derivations and coding assignments
- *Mini-lectures (20%):* each student is responsible for delivering 2–3 “mini-lectures” (each 20–30 mins in duration) along with preparing accompanying course materials (slides, code, etc.)
- *Project (40%):* the final course project consists of a written report and an oral presentation

Textbook. There is no textbook for this course. Material will be drawn from a number of texts, most of which are available in the library in both hard-copy and electronic format:

- P. Colella and E.G. Puckett, *Modern Numerical Methods for Fluid Flow*, course notes, 1994. **[distributed as PDF]**
- C. Pozrikidis, *Introduction to Theoretical and Computational Fluid Dynamics*, 2nd edition (Oxford University Press), 2011. **[e-book]**
  Alternate: C. Pozrikidis, *Fluid Dynamics: Theory, Computation and Numerical Simulation* (Springer), 2009. **[e-book]**
- R.J. LeVeque, *Finite Volume Methods for Hyperbolic Problems* (Cambridge University Press), 2002. **[e-book]**
- K.W. Morton and D.F. Mayers, *Numerical Solution of Partial Differential Equations: An Introduction* 2nd edition (Cambridge University Press), 2005. **[e-book]**
- D.A. Nield and A. Bejan, *Convection in Porous Media*, 5th edition (Springer), 2017. **[e-book]**

Outline.

1. ***Background and Governing Equations*** *(2.5 weeks)*:
Navier-Stokes equations; boundary conditions; simplifications and extensions; analytical solutions.

2. ***Finite Difference and Finite Volume Methods for Linear Problems*** *(2.5 weeks)*:
Consistency, stability and convergence; CFL condition; Lax Equivalence Theorem; von Neuman stability analysis; common upwind and centered schemes; finite volume approach; time-stepping.
Applications: scalar advection; heat equation; wave equation.

3. ***Incompressible Fluid Flow*** *(2 weeks)*:
Stokes equations; pressure-Poisson equation; Navier-Stokes equations; projection methods.
Applications: creeping flow; potential flow; driven cavity flow.

4. ***Porous Media Flow*** *(2 weeks)*:
Darcy’s Law; capillarity; porous medium equation and nonlinear diffusion; IMPES method; Brinkman-Forchheimer extension.
Applications: oil reservoir simulation; groundwater transport; porous channels.

5. ***(tentative) Nonlinear Wave Propagation and Compressible Flow*** *(2 weeks)*:
Hyperbolic conservation laws; nonlinear systems; Riemann solvers; CLAWPACK code.
Applications: gas dynamics; shallow water waves; traffic and pedestrian flow; atmospheric transport.

Additional Topics:
Interspersed throughout the course, I will introduce additional material related to fluid mechanics, scientific computing, reproducible research, and validated numerics.