Logic Puzzles and How to Think Like a Mathematician

Crofton House School
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A Garden Variety Puzzle

Five friends have their gardens all in a row (#s 1-5), where they grow three kinds of crops each in 4 different varieties:

fruits (apple, pear, nut, cherry),
vegetables (carrot, parsley, gourd, onion), and
flowers (aster, rose, tulip, lily).

1. Each of the 12 varieties are grown in at least in one garden.
2. In every garden grows exactly 4 different varieties.

4. Only one variety is in 4 gardens.
5. Only in one garden are all 3 kinds of crops.
6. Only in one garden are all 4 varieties of one kind of crop.
7. Pear is only in the two end gardens.
8. Paul's garden is in the middle with no lily.
9. No garden with Aster has vegetables.
10. No garden with Rose has parsley.
11. Any garden with nuts also has gourd and parsley.
12. In the first garden are apples and cherries.
13. Only in two gardens are cherries.
14. Sam has onions and cherries.
15. Luke grows exactly two kinds of fruit.
16. Tulip is only in two gardens.
17. Apple is in a single garden.
18. Exactly one garden next to Zick's has parsley.
19. Sam's garden is not on either end of the row.
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pear 7		21		
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20. Hank grows neither vegetables nor asters.
~~21. Paul has exactly three kinds of vegetable.~~

		varieties			
crops	fruits	apple	cherry	nut	pear
	veggies	carrot	gourd	onion	parsley
	flowers	aster	lily	tulip	rose

People: Hank, Sam, Luke, Zick, Paul

gardens

varieties

row 1	row 2/4		row 3		row 5
Hank 20+	Sam 19	Zick 20+	Paul 8		Luke 15+
apple 12	onion 14		21		pear 7
cherry 12	cherry 14		21		nut 15+
pear 7			21		gourd 11
20+					parsley 11
no aster (20)		nut?	no lily (8) nut?		

Where is Hank?

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apple 12	onion 14	aster 16+	carrot 10+		pear 7
cherry 12	cherry 14	lily 16+	gourd 10+		nut 15+
pear 7	tulip	tulip 16+	onion 10+		gourd 11
rose 4+	rose 4+	rose 4+	rose 4+		parsley 11
no aster (20)		one crop (6) ~~veg~~ / flowers	no lily (8) ~~nut?~~		

varieties

1. Each of the 12 varieties are grown in at least in one garden.
2. In every garden grows exactly 4 different varieties.

~~4. Only one variety is in 4 gardens.~~
5. Only in one garden are all 3 kinds of crops.
~~6. Only in one garden are all 4 varieties of one kind of crop.~~
~~7. Pear is only in the two end gardens.~~
~~8. Paul's garden is in the middle with no lily.~~
9. No garden with Aster has vegetables.
~~10. No garden with Rose has parsley.~~
~~11. Any garden with nuts also has gourd and parsley.~~
~~12. In the first garden are apples and cherries.~~
~~13. Only in two gardens are cherries.~~
~~14. Sam has onions and cherries.~~
~~15. Luke grows exactly two kinds of fruit.~~
~~16. Tulip is only in two gardens.~~
~~17. Apple is in a single garden.~~
18. Exactly one garden next to Zick's has parsley.
~~19. Sam's garden is not on either end of the row.~~
~~20. Hank grows neither vegetables nor asters.~~
~~21. Paul has exactly three kinds of vegetable.~~

		varieties			
crops	fruits	apple	cherry	nut	pear
	veggies	carrot	gourd	onion	parsley
	flowers	aster	lily	tulip	rose

People: Hank, Sam, Luke, Zick, Paul

gardens

varieties

row 1	row 2/4		row 3		row 5
Hank 20+	Sam 19	Zick 20+	Paul 8		Luke 15+
apple 12	onion 14	aster 16+	carrot 10+		pear 7
cherry 12	cherry 14	lily 16+	gourd 10+		nut 15+
pear 7	tulip	tulip 16+	onion 10+		gourd 11
rose 4+	rose 4+	rose 4+	rose 4+		parsley 11
no aster (20)		one crop (6) ~~veg~~ / flowers	no lily (8) ~~nut?~~		

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		varieties			
crops	fruits	apple	cherry	nut	pear
	veggies	carrot	gourd	onion	parsley
	flowers	aster	lily	tulip	rose

People: Hank, Sam, Luke, Zick, Paul

gardens

varieties

row 1	row 2		row 3	row 4	row 5
Hank 20+	Sam 19		Paul 8	Zick 20+	Luke 15+
apple 12	onion 14	aster	carrot 10+	aster 16+	pear 7
cherry 12	cherry 14	lily	gourd 10+	lily 16+	nut 15+
pear 7	tulip	tulip	onion 10+	tulip 16+	gourd 11
rose 4+	rose 4+	rose	rose 4+	rose 4+	parsley 11
no aster (20)			no lily (8) ~~nut?~~	one crop (6) ~~veg~~ / flowers	

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		varieties			
crops	fruits	apple	cherry	nut	pear
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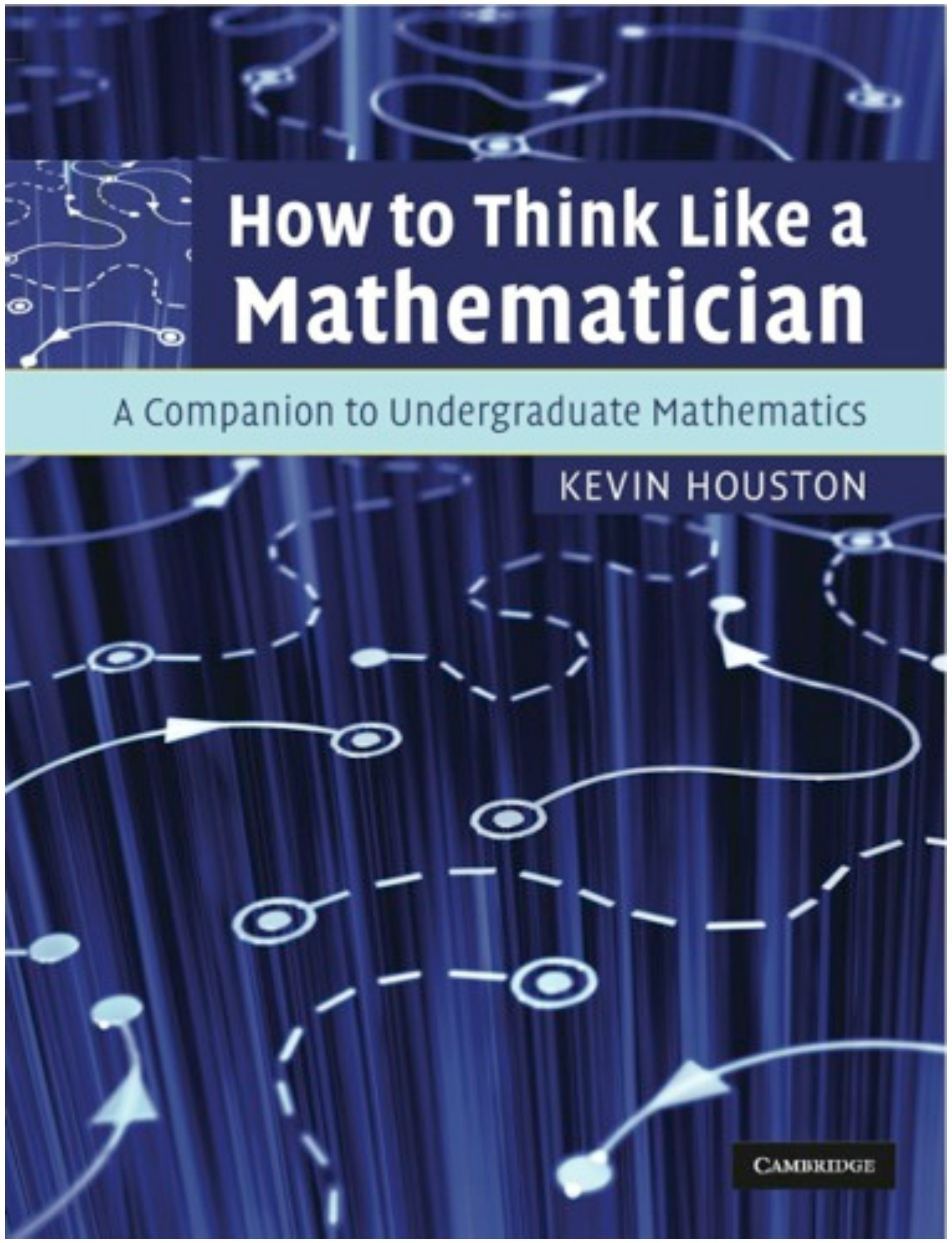
	flowers	aster	lily	tulip	rose

People: Hank, Sam, Luke, Zick, Paul


How to Think Like a
Mathematician
A Companion to Undergraduate Mathematics
KEVIN HOUSTON
CAMBRIDGE

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Definitions, theorems and proofs

The highest form of pure thought is in mathematics.
Plato

We now come to the heart of how mathematicians organize and present their work. Look through any high-level mathematics book (or see Chapter 1) and you will find that mathematics is not presented as one continuous piece of prose like a novel. Instead the text is divided up into small nuggets of information such as a Theorem, Proposition, Lemma, Corollary, Proof, Definition and Conjecture. All of these have special meanings and we will see how to approach them in the following chapters.

Meanings

We shall now briefly describe the meanings of the above words.

- Definition: an explanation of the mathematical meaning of a word.
- Theorem: a very important true statement.
- Proposition: a less important but nonetheless interesting true statement.
- Lemma: a true statement used in proving other true statements.
- Corollary: a true statement that is a simple deduction from a theorem or proposition.
- Proof: the explanation of why a statement is true.
- Conjecture: a statement believed to be true, but for which we have no proof.
- Axiom: a basic assumption about a mathematical situation.

Definitions

Much more will be said about definitions in the next chapter. For the moment let us say that in higher mathematics close attention must be paid to definitions – much more than at lower levels. Definitions allow us to separate one class of objects from another or single out some interesting property.

The last two have been made up to use as examples so do not expect to see them in other books.

The purpose of a definition

The main purpose of a definition is so that everyone knows what we are talking about. For example, we saw in Chapter 1 that some mathematicians define the set of naturals to include 0 and some do not. This is confusing, but even greater confusion would result if the definition being used by an author was different to that of a reader. By making the definition explicit the author avoids this.

In the case of natural numbers two different concepts are being given the same name. We can also have the situation where two different definitions are given but they are in fact the same concept.

The main mathematical reason for giving a definition is to identify some interesting objects worthy of study (although sometimes we give definition to exclude certain 'bad' objects). Also, psychologically, it is easier to deal with a concept once it has been given a name.

Definitions can be used as a solution to a problem. For example, if we define i to be the square root of -1, then we can begin to define complex numbers. Defining complex numbers leads to a good theory of quadratic equations (solutions always exist) and surprisingly helps to solve problems such as ordinary differential equations.

In studying mathematics it is vitally important that you can recall definitions precisely. You have to have all the right conditions or else you are defining something else. A common problem I find amongst students is that they cannot advance in some problem because they do not know the definition of a word. This is an example where students are viewing mathematics as an 'apply-a-process' subject rather than 'understand-the-concepts'.

The 'if and only if' nature of mathematical definitions

Giving definitions is one area where mathematicians are imprecise. In a definition with an 'if', what is intended is an 'if and only if'. The 'only if' part of a definition is considered to be such an obvious part that it is omitted.

For example, the definition above 'A positive integer n is a square number if $n = x^2$ for some integer x' should be read as an 'if and only if' statement:

> 'A positive integer n is a square number if *and only if* $n = x^2$ for some integer x.'

In other words the number is called square if the condition is true but, more than that, only if the condition is true. There are no square numbers that do not satisfy the condition.

This is an important point to bear in mind when reading mathematical definitions. It is the only time that an 'if' can be read as an 'if and only if'. Do not do this when reading theorems for example.

Making a statement

When dealing with people, let us remember we are not dealing with creatures of logic. We are dealing with creatures of emotion, creatures bristling with prejudices and motivated by pride and vanity.
Dale Carnegie, *How to Win Friends and Influence People*, 1936

When I tell people that I am a mathematician it is often not very long before they bring up the subject of how logical mathematics is. Sometimes they consider this a downside to the subject as it makes it so much harder. After all, with logic you have to be right whereas in other areas of life opinions matter more. And it's easy to have an opinion – even if you can't back it up with evidence.

Logic is in essence quite simple despite its reputation for difficulty. A small number of simple rules exist. We need only to apply these and reduce complicated statements with them to achieve success.

In the next few chapters we shall concentrate on the logic used by mathematicians in their day-to-day work rather than on deep conundrums and paradoxes or on technical material such as predicates, compound statements and so on. I will, however, explain truth tables, which are rarely used by mathematicians in their everyday work, as they do provide a lot of clarity for beginners.

Mathematics is the business of proving mathematical statements to be true or false. So, first let's look at statements.

Statements

Defining precisely what is meant by a mathematical statement is surprisingly difficult – we could get into some very deep philosophical work at this point. However, I wish to be very practical – I want to give you the tools that a mathematician uses on a day-to-day basis – and hence we use the following definition.

Definition 6.1

*A **statement** is a sentence that is either true or false – but not both.*

Implications

Mathematics consists of propositions of the form: P implies Q,
but you never ask whether P is true.
Bertrand Russell

Russell's quote above is extremely incisive. Modern mathematics is indeed made up of statements of the form statement P implies statement Q. That is, we have 'If statement P is true, then statement Q is true also.' Usually, however, this structure is hidden, mainly to make mathematics more comprehensible – it would be hard to read if we always wrote it that way.

The second part of Russell's quote is also true but a lot more subtle. One could argue that the statement 'The Moon is made of cheese implies the Moon is a tasty snack' is true because if the Moon was cheese, then it would be tasty. The point is that the statement makes sense and is true yet it has nothing to say on whether the Moon really is made of cheese or whether it really is a tasty snack. All it says is that *if* it is cheesy, then it is tasty. It is worth bearing this example in mind as we proceed.

Instead of Russell's P and Q we will, in general, use A and B to denote our statements.

'If ..., then ...' statements

Most mathematical statements are of the form

'If statement A is true, then statement B is true.'

They may be heavily disguised but when you break them down, that is what you will find. This type of statement is called an **implication**. We say A implies B and sometimes write $A \Longrightarrow B$. Please refer to page 37 concerning the correct use of this symbol.

Examples 7.1

(i) If I am Winston Churchill, then I am English.
(ii) If I am English, then I am Winston Churchill.
(iii) If I am President George Washington, then I am the first President of the United States of America.
(iv) If $a < b$, then $a^2 < b^2$.

Definitions, theorems and proofs

The highest form of pure thought is in mathematics.
Plato

We now come to the heart of how mathematicians organize and present their work. Look through any high-level mathematics book (or see Chapter 1) and you will find that mathematics is not presented as one continuous piece of prose like a novel. Instead the text is divided up into small nuggets of information such as a Theorem, Proposition, Lemma, Corollary, Proof, Definition and Conjecture. All of these have special meanings and we will see how to approach them in the following chapters.

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Definitions

Much more will be said about definitions in the next chapter. For the moment let us say that in higher mathematics close attention must be paid to definitions – much more than at lower levels. Definitions allow us to separate one class of objects from another or single out some interesting property.

True statements

The words theorem, proposition, lemma and corollary denote statements that are true.

Theorems and propositions

The most important mathematical statements are called **theorems**. Any result of importance will be called a theorem. We use **proposition** for statements that we think are of less importance but which are of some intrinsic interest. It is very difficult to give examples of the distinction between the concepts of theorem and proposition as different authors will put the same statement in different categories. I once saw two distinguished mathematicians argue like children about the difference, but this is rare; most can't be bothered to draw a precise distinction. In fact, in this book we will use theorem to mean any type of true statement.

Examples 14.1

(i) The following is true: Napoleon was a Corsican.
(ii) Every natural number can be written as a product of primes. (This will be shown in Theorem 25.5.)

Lemmas

A **lemma** is a statement that is a step on the road to proving another statement. Lemmas are considered to be less important than propositions and again the distinction between categories is rather blurred.

An interesting point to note is that often they eventually turn out to be more useful than the statement they are used to prove.

Corollaries

A **corollary** is a statement of interest that is deduced from a theorem or proposition.

Example 14.2

Napoleon was French. (This is true because Corsica was part of France and as we have seen in Example 14.1(i) Napoleon was a Corsican.)

The other terms

Proofs

Mathematicians solve problems – proof is the guarantee that our solutions are correct.

A **proof** is an explanation of why a statement is true. Much, much more will be said about proofs since the defining feature of high-level mathematics is the emphasis on proof. In some sense this book is about proof.

Analysing theorems

Find the assumptions and conclusions

As you know from Chapter 7, Implications, theorems can usually be written in the form

'a collection of assumptions imply some conclusion'.

That is, they are in the form, 'If …, then …' Identifying precisely what these assumptions and conclusions are is our first goal in dealing with a theorem. There is frequently room for debate but you should be precise in *your* mind what the assumptions and conclusions are.

For the first theorem we have

'm and n are natural and odd' as assumptions

and

'mn is an odd integer' as the conclusion.

We rephrase the second theorem as

'If X is the set of prime numbers, then X is infinite.'

This is an ugly, ugly statement of the theorem – it introduces an unnecessary X for a start – but we can clearly see the assumptions and conclusions.

The third theorem can be rewritten as

'If $x = 2$, then $\sqrt{x}$ is irrational'

or

'If $x = \sqrt{2}$, then x is irrational.'

Is the theorem telling us something about the number 2 or $\sqrt{2}$? The answer is open to debate. Either way, the assumption concerns a specific number and the conclusion is about irrationality.

Rate the strength of the assumptions and conclusions

We want to know how strong the assumptions and conclusions are. The best theorems have weak assumptions and strong conclusions. What constitutes weak and strong is subjective; again there is room for debate.

A **strong assumption** refers to a small set of objects. A **strong conclusion** says something very definite and precise about those objects. In both cases the opposite of strong is **weak**.

Mathematicians want weak assumptions and strong conclusions, that is, we take a very wide collection of objects (weak assumption) and say something very definite about them (strong conclusion).

Proof

Do not confuse reasons which sound good with good, sound reasons.
Anon.

Mathematics is fantastic. It is a subject where we do not have to take anyone's word or opinion. The truth is not determined by a higher authority who says 'because I say so', or because they saw it in a dream, the pixies at the bottom of their garden told them, or it came from some ancient mystical tradition. The truth is determined and justified with a mathematical proof.

What is a proof?

A proof is an explanation of why a statement is true. More properly it is a *convincing* explanation of why the statement is true. By convincing I mean that it is convincing to a mathematician. (What that means is an important philosophical point which I am not going to get into; my interest is more in practical matters.)

Statements are usually proved by starting with some obvious statements, and proceeding by using small logical steps and applying definitions, axioms and previously established statements until the required statement results.

The mathematician's concept of proof is different to everyday usage. In everyday usage or in court for instance, proof is evidence that something is likely to be true. Mathematicians require more than this. We like to be 100% confident that a statement has been proved. We do not like to be 'almost certain'.

Having said that, how confident can we be that a theorem has been proved? Millions have seen a proof of Pythagoras' Theorem; we can be certain it is true. Proofs of newer results, however, may contain mistakes. I know from my own experience that some proofs given in books and research journals are in fact wrong.

Why prove statements?

In other subjects most statements are open to debate and whether you believe a particular one may be down to personal tastes or prejudices. The existence of proofs means this is

So what about the converse? That is, 'if mn is odd, then m and n are odd'. Certainly, this is not done in a direct method: we do not begin by assuming that mn is odd and proceed with a series of implications to show that m and n are odd. Instead, it is assumed that one of m and n is even. This is the negation of 'm and n is odd'. Thus the contrapositive statement is being proved (recall that 'not $B \implies$ not A' is the same as '$A \implies B$'; see Chapter 8). Notice we are not explicitly told this; it is for us to realize, there is no big announcement 'We will prove the contrapositive statement . . .'.

In more general situations it may be that another theorem is used to prove the new one, or it may the use of a definition, etc. Notice which is used.

Find where the assumptions are used

Identify where the assumptions are used. They will be used once (maybe more) or else they will have been unnecessary. (More will be said about unnecessary assumptions in Chapter 33.) This includes finding where previously proved theorems are used. These will also have assumptions; make sure they are satisfied – be active! An additional point to remember here is that if a theorem gets used again and again in different proofs, it must be important and has the potential to be used in your proofs, so learn it well.

In our proof the assumption that the natural numbers m and n are odd (in the 'if' part of the statement) is used a number of times. In the first paragraph, it is used in setting $m = 2k+1$, etc., since odd numbers are of this form. The identification of the assumptions for the 'only if' is a bit harder since we assume the negation of a statement so we can apply the contrapositive method – it is a bit disguised but it is used nevertheless.

Apply the proof to an example

A very effective method to understand a proof is to apply each step to a particular special or concrete case that satisfies the assumptions. This is an important nugget of information to take away from this book.

In the above example this is largely trivial, yet we can have a go. Suppose that $m = 3$ and $n = 7$ and rewrite the proof using those figures. Of course for the proof of the 'only if' part you need to assume that m is even, so let's say it is 6. No assumption is made on n being odd or even, try both. Again, this a matter of being active!

This method can work particularly well for problematic proofs, but unfortunately doesn't shed light in every case.

Draw a picture

Reading a proof is like solving a problem so apply the techniques from Chapter 5, How to solve problems, such as drawing a picture.

In the above proof the picture would look like the following.

Definitions:

We must be of like mind in our interpretation of language.

Five friends have their gardens all in a row (#s 1-5), where they grow
three kinds of crops each in 4 different varieties:

fruits (apple, pear, nut, cherry),
vegetables (carrot, parsley, gourd, onion), and
flowers (aster, rose, tulip, lily).

1. Each of the 12 varieties are grown in at least in one garden.
2. In every garden grows exactly 4 different varieties.

Statements:

The whole truth & only truth!

4. Only one variety is in 4 gardens.

6. Only in one garden are all 4 varieties of one kind of crop.
7. Pear is only in the two end gardens.
8. Paul's garden is in the middle with no lily.

10. No garden with Rose has parsley.
11. Any garden with nuts also has gourd and parsley.
12. In the first garden are apples and cherries.

Simple Facts: 7, 8, 12, 21

gardens

varieties

row 1	row 2	row 3	row 4	row 5
		Paul		
		no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
						✓	✓				✓									✓

Simple Facts: 7, 8, 12, 21

gardens

varieties

row 1	row 2	row 3	row 4	row 5
		Paul		
apple				pear 7
cherry				
pear				
		no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
						✓	✓				✓									✓

Simple Facts: 7, 8, 12, 21

Lemma (fruit): accepting Simple Facts, 13, 14, 15, 17, 19
- we know where apple/pear/cherry are.

gardens

varieties	row 1	row 2	row 3	row 4	row 5
		Sam?	Paul	Sam?	
	apple				pear 7
	cherry				
	pear				
			no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
						✓	✓				✓	✓	✓	✓		✓		✓		✓

Simple Facts: 7, 8, 12, 21

Lemma (fruit): accepting Simple Facts, 13, 14, 15, 17, 19
- we know where apple/pear/cherry are.

Theorem: accepting previous results, and 11
- 5th garden is Luke's and we know what he grows.

gardens

varieties	row 1	row 2	row 3	row 4	row 5
		Sam?	Paul	Sam?	Luke
	apple				pear 7
	cherry				nut
	pear				gourd
					parsley
			no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
						✓	✓			✓	✓	✓	✓	✓		✓		✓		✓

Simple Facts: 7, 8, 12, 21

Lemma (fruit): accepting Simple Facts, 13, 14, 15, 17, 19
- we know where apple/pear/cherry are.

Theorem: accepting previous results, and 11
- 5th garden is Luke’s and we know what he grows.

Theorem: accepting previous results, and 20
- first garden is Hank.

gardens

varieties

row 1	row 2	row 3	row 4	row 5
Hank	Sam?	Paul	Sam?	Luke
apple				pear
cherry				nut
pear				gourd
				parsley
		no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
						✓	✓			✓	✓	✓	✓	✓		✓		✓	✓	✓

Simple Facts: 7, 8, 12, 21

Lemma (fruit): accepting Simple Facts, 13, 14, 15, 17, 19
- we know where apple/pear/cherry are.

Theorem: accepting previous results, and 11
- 5th garden is Luke's and we know what he grows.

Theorem: accepting previous results, and 20
- first garden is Hank.

Lemma (flowers): accepting previous results, and 6
- Zick grows one crop: flowers.

gardens

varieties

row 1	row 2/4		row 3		row 5
Hank	Sam	Zick	Paul		Luke
apple	onion	aster			pear
cherry	cherry	lily			nut
pear		tulip			gourd
		rose			parsley
			no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
					✓	✓	✓			✓	✓	✓	✓	✓		✓		✓	✓	✓

Simple Facts: 7, 8, 12, 21

Lemma (fruit): accepting Simple Facts, 13, 14, 15, 17, 19
- we know where apple/pear/cherry are.

Theorem: accepting previous results, and 11
- 5th garden is Luke’s and we know what he grows.

Theorem: accepting previous results, and 20
- first garden is Hank.

Lemma (flowers): accepting previous results, and 6
- Zick grows one crop: flowers.
- with 4, 16 the popular variety is rose in row 1-4.

gardens

varieties

row 1	row 2/4		row 3		row 5
Hank	Sam	Zick	Paul		Luke
apple	onion	aster			pear
cherry	cherry	lily			nut
pear	tulip	tulip			gourd
rose	rose	rose	rose		parsley
			no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
			✓		✓	✓	✓			✓	✓	✓	✓	✓	✓	✓		✓	✓	✓

Simple Facts: 7, 8, 12, 21

Lemma (fruit): accepting Simple Facts, 13, 14, 15, 17, 19
- we know where apple/pear/cherry are.

Theorem: accepting previous results, and 11
- 5th garden is Luke's and we know what he grows.

Theorem: accepting previous results, and 20
- first garden is Hank.

Lemma (flowers): accepting previous results, and 6
- Zick grows one crop: flowers.
- with 4, 16 the popular variety is rose in row 1-4.

Theorem: accepting previous results, and 10, 18
- second garden is Sam.
- fourth garden is Zick

gardens

varieties	row 1	row 2	row 3	row 4	row 5
	Hank	Sam	Paul	Zick	Luke
	apple	onion	carrot	aster	pear
	cherry	cherry	gourd	lily	nut
	pear	tulip	onion	tulip	gourd
	rose	rose	rose	rose	parsley
	no lily (8)				

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
			✓		✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Simple Facts: 7, 8, 12, 21

Lemma (fruit): accepting Simple Facts, 13, 14, 15, 17, 19
- we know where apple/pear/cherry are.

Theorem: accepting previous results, and 11
- 5th garden is Luke’s and we know what he grows.

Theorem: accepting previous results, and 20
- first garden is Hank.

Lemma (flowers): accepting previous results, and 6
- Zick grows one crop: flowers.
- with 4, 16 the popular variety is rose in row 1-4.

Theorem: accepting previous results, and 10, 18
- second garden is Sam.
- fourth garden is Zick

Corollary: accepting previous results
- only Sam and Zick can grab a carrot from Paul.

gardens

varieties	row 1	row 2	row 3	row 4	row 5
	Hank	Sam	Paul	Zick	Luke
	apple	onion	carrot	aster	pear
	cherry	cherry	gourd	lily	nut
	pear	tulip	onion	tulip	gourd
	rose	rose	rose	rose	parsley
			no lily (8)		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
			✓		✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

What have we proved?

- statements **deduced** to be truths
- only Paul grows carrots - both Sam and Zick can steal some
- the garden arrangement is **unique**
- statements 5 & 9 are redundant (but consistent)