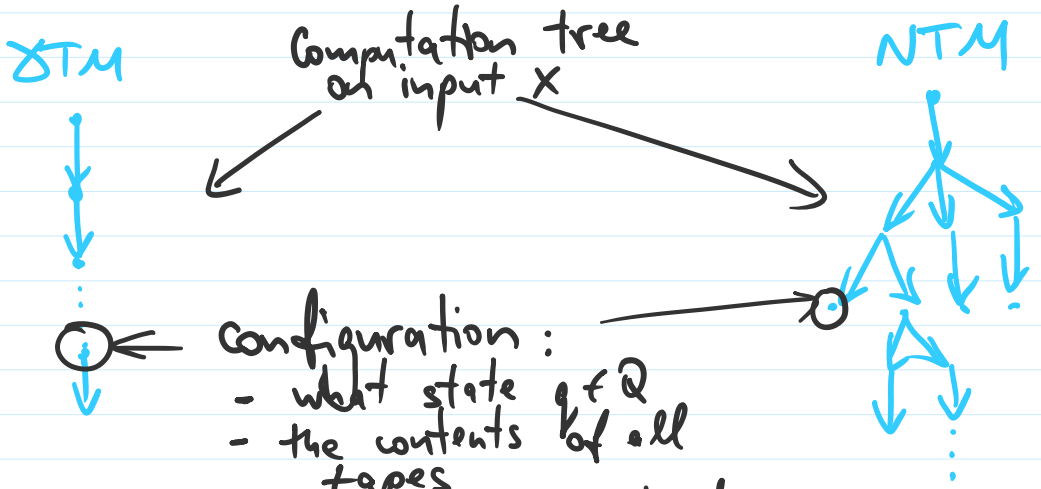
Lecture 10

Friday, May 31, 2019 12:17 PM

Decidability & Semi-Decidability for NTMs



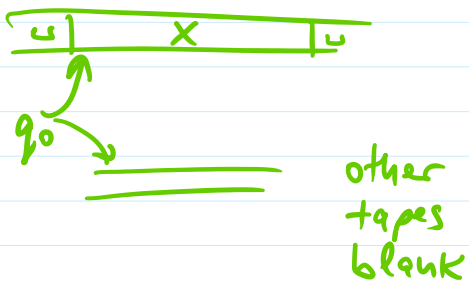
configuration:
- what state $q \in Q$
- the contents of all tapes
- positions of tape-head on all tapes

Computation of a TM on input x:
a sequence of configurations

$$c_1 \to c_2 \to c_3 \to \dots \to c_t \to \dots$$

where

c_1 = initial config.



c_{i+1} follows from c_i using δ of TM.

L is <u>decidable</u> by an NTM M if

$\forall x \in L \Rightarrow$ M on x accepts (on some branch)

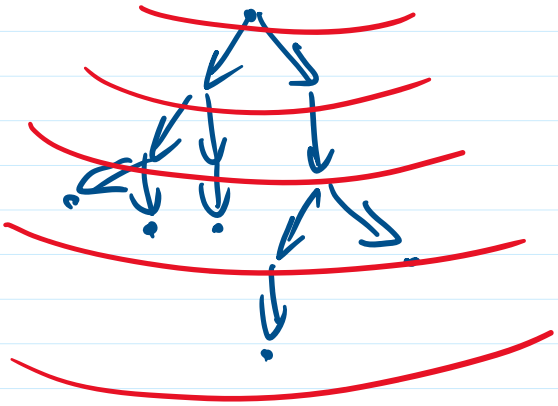
$\forall x \in L \Rightarrow M$ on x accepts (on some branch)

$\forall x \notin L \Rightarrow M$ on x rejects on all branches (hence halt)

Claim: L (semi-) decidable by an NTM

$\Longleftrightarrow$ L (semi-) decidable by a DTM.

x NTM M



BFS-style exploration & check if all leaves are rejects.

L is semi-decidable by an NTM M if

$\forall x \in L \Rightarrow M$ on x accepts (on some branch)

$\forall x \notin L \Rightarrow M$ on x does not accept (could be inf. branches)

NTM (semi) decidability $\equiv$

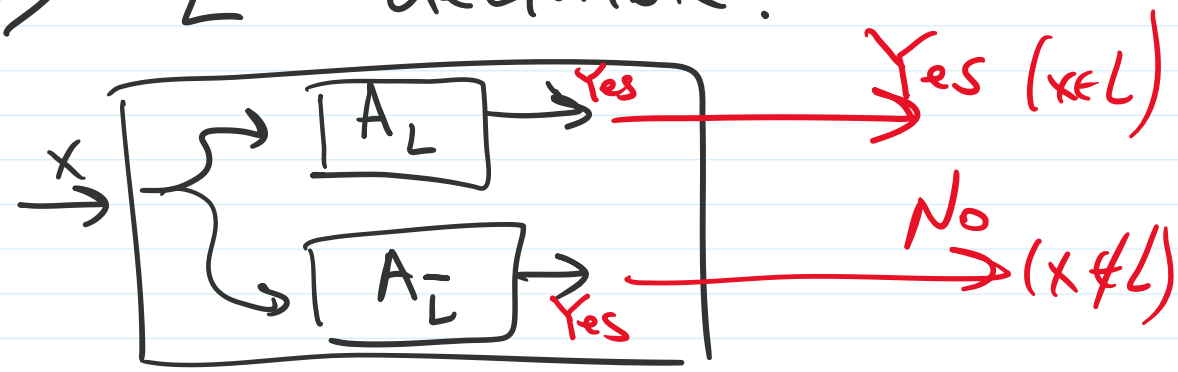
DTM (semi) decidability

TM (semi) decidability

Claim: $L \subseteq \{0,1\}^*$ semi-decidable

& $\overline{L} = \{0,1\}^* - L$ semi-decidable

$\Rightarrow$ L decidable.



Cor: $\overline{A_{TM}}$ is not semi-decidable.

Pf: A_{TM} is semi-decidable.

$= \{\langle M, w\rangle \mid \text{TM } M \text{ accepts } w\}$

On the other hand, we know

A_{TM} is not decidable.

Thm: (1) The class of decidable lang's is closed under complementation.

(2) The class of semi-decidable lang's

(2) The class of semi-decidable lang's is NOT closed under complement.

Pf. (1) $q_{acc} \leftrightarrow q_{rej}$. ✓

(2) A_{TM} . ✓ $\square$

Examples of undecidable problems

$$E_{TM} = \{ \langle M \rangle \mid L(M) = \emptyset \}$$

is undecidable.

Pf: Suppose E_{TM} is decidable.

Then can decide A_{TM}.

$\langle M, w \rangle \longrightarrow$

1. Construct M': "On input x, simulate M on w. If M acc w, then accept."
2. Check if $\langle M' \rangle \in E_{TM}$ $\xrightarrow{ALL_{TM}}$ No, Yes
   $\notin E_{TM}$ $\xrightarrow{ALL_{TM}}$ Yes, No

(1) M acc $w \Rightarrow L(M') = \Sigma^* \neq \emptyset$

(2) M not acc $w \Rightarrow L(M') = \emptyset$

$\{ \langle M \rangle \mid L(M) = \Sigma^* \}$?

$$ALL_{TM} = \{ \langle M \rangle \mid L(M) = \Sigma^* \}$$

is undecidable.

$$EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid L(M_1) = L(M_2) \}$$

is undecidable.

<u>Proof:</u> Suppose EQ_{TM} is decidable.

Then E_{TM} is decidable.