$INF = \{ \langle M \rangle \mid L(M) \text{ is infinite} \}$

Last time: $A_{TM} \leq INF$

$\Updownarrow$ ⟸ Exercise: Prove this equivalence $\forall A, B$, $A \leq B \iff \overline{A} \leq \overline{B}$.

$\overline{A_{TM}} \leq \overline{INF}$

Hence, $\overline{INF}$ is not semi-decidable (since $\overline{A_{TM}}$ is not semi-decidable).

Now want to show INF is not semi-dec.
It suffices to show

$$\overline{A_{TM}} \leq INF.$$

Want a reduction $f : \Sigma^* \to \Sigma^*$
s.t. $\forall (M, w)$, $f(\langle M, w \rangle) = \langle M' \rangle$

(1) M not accept w $\Rightarrow L(M')$ is infinite

(2) M accepts w $\Rightarrow L(M')$ is finite

Attempt #1: M' = "On input x,
simulate M on w.
If M accepts w, then reject.
If M rejects w, then accept."

Attempt #2:

M' = "On input x,
simulate M on w for $|x|$ steps only.
If M accepted w within $|x|$ steps,
then reject. Otherwise, accept."

Correctness analysis:

(1) M does not accept w $\Rightarrow$ M' accepts <u>every</u> x

$\Rightarrow L(M') = \Sigma^*$ <u>inf.</u>

(2) M accepts w $\Rightarrow$
M accepts w within t steps

$\Rightarrow \forall x, \ |x| \geq t, \ M'(x)$ rejects

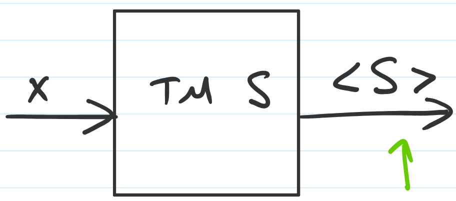
($\forall x, |x| < t$, $M'(x)$ accepts.
But, # accepted x's is $\leq 2^{t+1} - \text{const.}$)

$\Rightarrow L(M')$ finite. $\square$

Recursion Theorem

Informally, can construct TMs that are "aware" of their own source code.

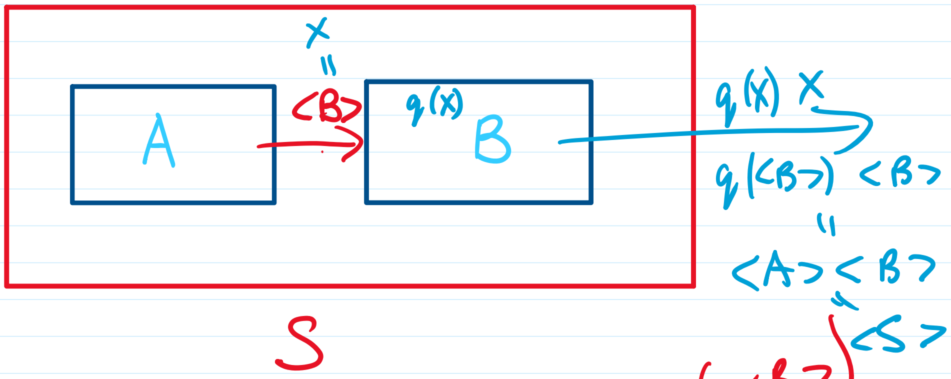
For example, can construct a TM S s.t.



TM S outputs

TM S outputs its own description $\langle S \rangle$.

Construction of a Self-Printing TM S



$A =$ " On input x, output $\langle B \rangle$. " $= q(\langle B \rangle)$

Define $q(w) =$ description of TM s.t. that TM prints w.

(e.g., $q(w) =$ " On input x, output w. "

$B =$ " On input x, compute $q(x)$. Output $q(x), x$. "

$A =$ " On input x, output " On input x, compute $q(x)$. Output $q(x), x$. " "

compute $q(x)$.
Output $q(x), x$. " "

Python:

1. a = "<line 2>"
2. take a, will produce line 1 & line 2.

Thm (Recursion Thm):

$\forall$ computable $t : \Sigma^* \times \Sigma^* \to \Sigma^*$

$\exists$ TM R