Lecture 13

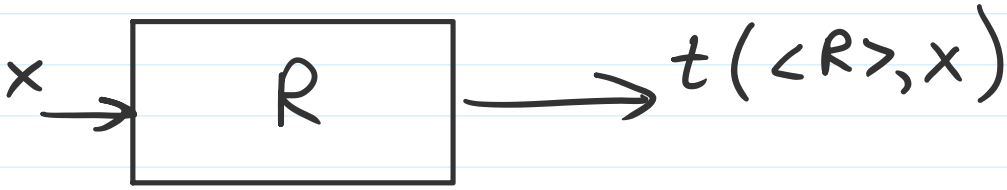
Sunday, June 9, 2019 9:44 PM

Last time:

Recursion Theorem:

$\forall$ computable $t: \Sigma^* \times \Sigma^* \to \Sigma^*$

$\exists$ TM R s.t.



applications:

We now can design TM as follows:

M = "On input x,

1. Get own description $\langle M \rangle$.
2. Do some computation on $\langle M \rangle$ & x."

Self-Print = $\{ \langle M \rangle \mid$ TM M outputs $\langle M \rangle \}$

<u>Claim</u>: Self-Print is undecidable.

Proof: Reduce from A_{TM}.

A_{TM}:

$(M, w) \mapsto M'$

Self-Print

Want: M acc $w \iff M'$ prints $\langle M' \rangle$

M' = "On input x,
1. Get own description $\langle M' \rangle$
2. Simulate M on w
3. If M acc w, then print $\langle M' \rangle$."

Fixed-Point Theorem

$\forall$ computable $t : \Sigma^* \to \Sigma^*$

$\exists$ TM F s.t.

$$L(F) = L(\underbrace{t(\langle F \rangle)}_{= \langle G \rangle})$$

Proof:

F = "On input x.

$F =$ " On input x,
1. Get own deser. $\langle F \rangle$
2. $\langle G \rangle = t(\langle F \rangle)$
3. Simulate G on x. "

$L(F) = L(G) = L(t(\langle F \rangle))$. ✓

Minimal TMs

Define $MIN_{TM} = \{ \langle M \rangle \mid M$ is a minimal TM $\}$

M is called minimal TM if

$\forall \; M'$, $|\langle M' \rangle| < |\langle M \rangle|$

$\Rightarrow L(M') \neq L(M)$.

Claim: MIN_{TM} is not semi-dec.

Proof:

Suppose MIN_{TM} is semi-decidable

Suppose MIN_{TM} is semi-decidable by some TM $\underline{M}$.

$C =$ " On input w,

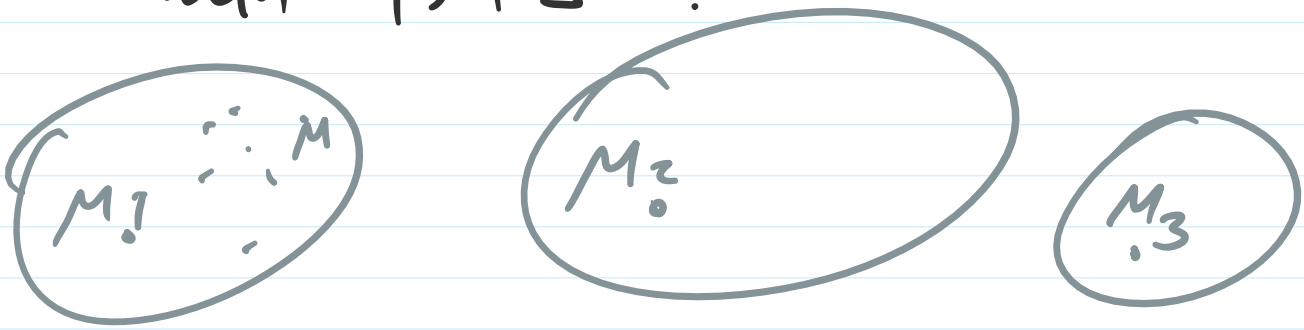
1. Get own code $\langle C \rangle$.
2. Let $M_1, M_2, M_3, \ldots$ be an enumeration of all TMs s.t. $|\langle M_i \rangle| > |\langle C \rangle|$
3. for $i = 1$ to ∞
   % want to find min TM M_i
   % for some i
   Simulate M on each $M_1, M_2, \ldots, M_i$ for $\leq i$ steps
   If M accepts some M_k $(1 \leq k \leq i)$ then go to step 4.
4. Simulate M_k on w. "

$L(C) = L(M_k)$

$|\langle C \rangle| < |\langle M_k \rangle|$

So M_k is not min. Contrad. $\square$

Claim: There are inf. many min TMs.



$L(M_1) = L(M)$
$\forall M \in$ Eq. Class

Summary of Basic Computability

1. TM = algorithm
2. $\exists$ undecidable problems
3. Diagonalization method
4. Reductions : (1) new algos
   (2) new lower bounds
5. Rice's Thm
6. Recursion Thm :
   (1) $\exists$ computer viruses
   (2) Virus Checking is undecidable!