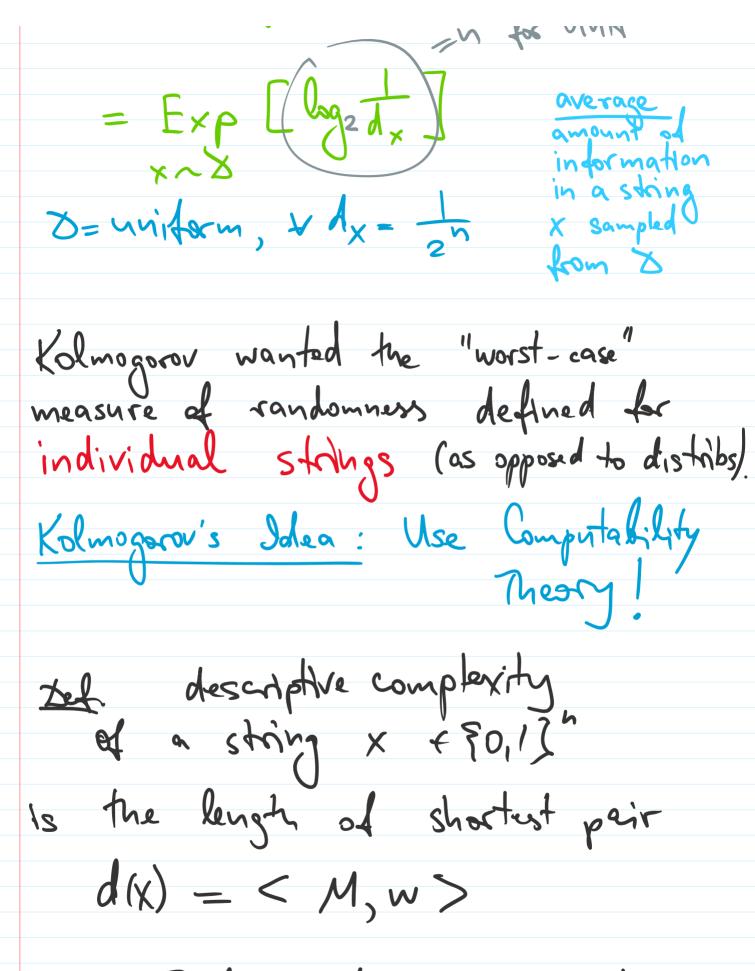
Lecture 14 Tuesday, June 11, 2019 10:25 PM
or how to measure randomness
Kolmogorov Complexity, or how to measure randomness d a given individual binary string
"Random" strings: 010101010101 < "repeat 01 6 times" 0011101010 : the string itself
0101010101 < : repeat 01 6 dimes
0011101010: the string itself
Randonness mensure de distributions et strings.
et strings.
$X \subseteq \{0,1\}^n$ $0 \le d_X \le 1$: probability $X = \{0,1\}^n$
$\sum d_{\times} = 1$
xeqoilin
Shannon's Entropy of 3:
XEROIZ" dx. log dx XEROIZ" Mithum



S.t. TM M on input W will output string X will output string X.

CM, W) = <m> : repetition code

Rep. Gode < 0100> +>>>> 00110000

D (D D

 $| \langle M \rangle w | = 2 \cdot | \langle M \rangle | + | w |$ Original (M)

(M) W = Rep. Code | M) pl, W

[Encoding length (M, W) |

= 2.1

= 2.1
/ M) + 2 + 1 W

|d(x)|= min over all cm, w) st Mon wouthput x

st. Mon wontport x et < M, w> Xet: A string x 1s called K.- random 1f 1d(x)1≥1X1. Observe: \(\forall \times \in \forall \cdot \in \forall \cdot \) $|d(x)| \leq n + O(1)$ Detine M = "On input w, "x can be described as <M, x7. why do K.-sandom strings exist? En 13° : 2° strings Suppose each of them is not K-rondom

1.e., e.,

X \rightarrow encoded using \le n-1

bits

X1 \rightarrow e2

All encodings

in the distinct!

X2n \rightarrow e2n 2+2+2+...+2=2-1 $\angle set: K(x) = |d(x)| Kolmogora fn$ Claim: (1) ∃c constant ∀x, K(XX) ≤ K(X)+C (2) \(\frac{1}{2}\), \(\text{Xy}) \(\frac{1}{2}\). \(\text{Xy}) \(\frac{1}{2}\). \(\text{Xy}) \(\frac{1}{2}\). \(\text{Xy}) \(\frac{1}{2}\). \(\text{Xy}) \(\frac{1}{2}\). Given computable en p, dp(x) = lex, shortest string S < + D(S)= X

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S.t. p(S) = XClaim: $\forall comp, p, \exists const. c$ s.t. $\forall x, K(x) \leq K_p(x) + C$

Thm: (1) K: 30,13 -> 30,13 to is not computable.

(2) { X { 30,13 to K(X) > 1X1}

Is not semi-decidable.

Computability Complexity

K-sandom X = \{0,19" \times that

x that has no small "circuit"