Last time:

Theorem There exist unsolvable (uncomputable) problems.

Proof:

(1) Problem / function $f : \{0,1\}^* \to \{0,1\}$

truth table $\underbrace{f(\varepsilon)\ f(0)\ f(1)\ f(00)\ f(01) \dots}_{\text{infinite 0-1 string}}$

Have one-to-one correspondence

functions $\xleftrightarrow{1-1}$ infinite binary strings

(2) Computable function $g : \{0,1\}^* \to \{0,1\}$

that is computable by some Java program

$x_1 \dots x_n \in \{0,1\}^n$

$\downarrow$

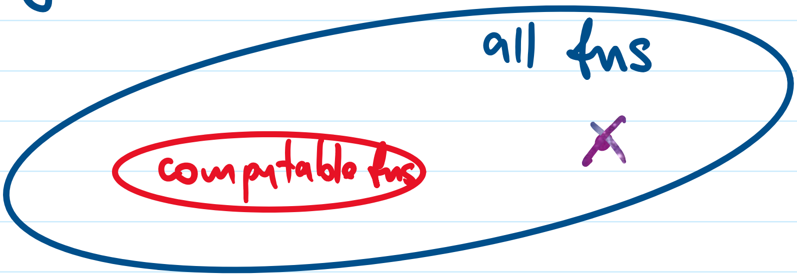
$\boxed{\text{Java program } P}$

$\downarrow$

$g(x_1, \dots, x_n)$

P computes g

all func



The set of computable fns is Countable / Enumerable. ✓
(Nat. numbers)

The set of infinite binary strings is NOT countable.

Cantor's Diagonalization: the set $\mathbb{R}$ is not countable.

By contradiction, assume 1-1 corresp. between $\mathbb{N}$ & set of inf. bin. strings

a_1	1	$\overline{0}$	0	1	1	0	0	...
a_2	2	1	$\overline{1}$	0	1	0	0	...
a_3	3	0	0	$\overline{0}$	0	0	0	...

q_3 3 | 0 0 0 0 0 0 . . .

⋮ . . . . . .

This inf. table contains **all** inf. bin. strings as rows.

New inf. bin. string: 1 0 1 . . . .

$$d_i = \neg(a_i)_i \qquad \forall i \in \mathbb{N}$$

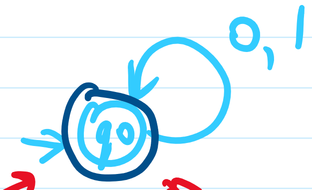
Define $d = d_1 d_2 d_3 \ldots$.

Claim: d is **not** a row in the table above.

Turing machine (TM) 1936

Finite Automata (FA) 1950's

A



init state

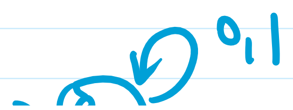
final state

0101110

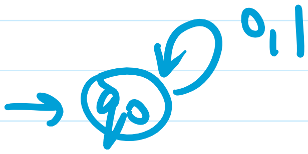
q_0

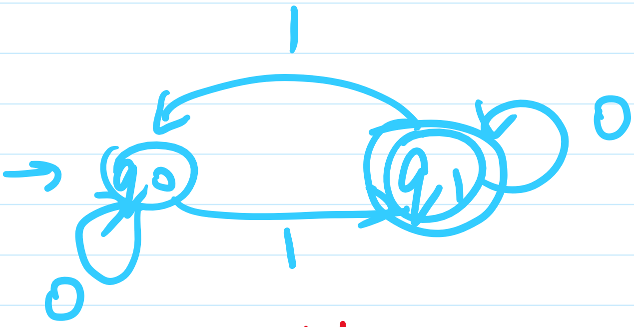
$L(A) = \{0,1\}^*$

$L(A)$ = **language** accepted by A



$L(B) = \emptyset$

B  $L(B)=\emptyset$

C 

q_0: 000 ← reject

001 ← accept

111

$L(C)=\{x\in\{0,1\}^* \mid x \text{ has odd \# of 1's}\}$

Ex: $\{0^n1^n \mid n\geq 0\}$ – not accepted by any FA

Nondeterministic FA (NFA)

Def FA

$\delta: Q\times\Sigma\to Q$

Q = set of states (finite)

Σ = input alphabet ($\Sigma=\{0,1\}$)