Last time: TQBF is PSPACE-complete.

$L = \text{class of logspace-computable languages}$

$NL = \text{class of nondeterministic logspace-computable languages}$

Complete problems:

1. undirected st-CONN: Given an undirected graph $G = (V, E)$, & nodes $s, t \in V$, decide if $t$ is reachable from $s$.

Theorem: Undirected st-CONN is
L-complete under FO-reductions.

(2) directed st-CONN: given a directed graph \( G=(V,E) \), & nodes \( s,t \in V \), decide if \( t \) is reachable from \( s \).

**Theorem:** Directed st-CONN is NL-complete under logspace-reductions.

**Open Question:** \( L = NL \) ?
Is there a logspace algorithm for directed graph reachability?

\[
NL = co\ NL
\]

\( co\ NL \) = the complement of \( NL \)

\( co\ NL \)-complete problem: directed st- UNREACH (given a digraph \( G=(V,E) \), \( s,t \in V \), decide if \( t \) is not reachable from \( s \)).
t is not reachable from s.

**Theorem [Immmerman–Szelepsényi]:**

Directed st-UNREACH ∈ NL.

Hence, coNL = NL.

(was open since 1964. Proved in 1987.)

**Proof Sketch:**

Imagine we know

\[ N = \# \text{ nodes reachable from } s \]

G has n nodes

\[ N \leq n \]

uses \( O(\log n) \) bits to write down.
Algo Unreach (G, s, t)
%%% given N = # nodes reachable from s

\[
\begin{array}{cccccc}
V_1 & V_2 & V_3 & \ldots & V_n \\
\text{Yes} & \text{Yes} & \text{No} & \ldots & \text{No}
\end{array}
\]

nondet. guesses : if the node is reachable from s

The number \( N = \# \text{nodes reachable from } s \) can be computed in \( NL \), iteratively.

\[
N_i = \# \text{nodes reachable from } s \text{ in } \leq i \text{ steps}
\]
in at most $c$ steps

\[ N_0 = 1, \quad N_n = N. \]

From $N_i$, can compute $N_{i+1}$, using the approach of Alg. Unreach above.

Randomization

\[ L \leq RL \leq NL \]

undirected

$\text{st.~CONN}$

Random walk for $O(n^3)$ steps from $S$ will likely see $t$, if $t$ is reachable from $S$. 
P \leq \text{ RP } \leq \text{ NP}