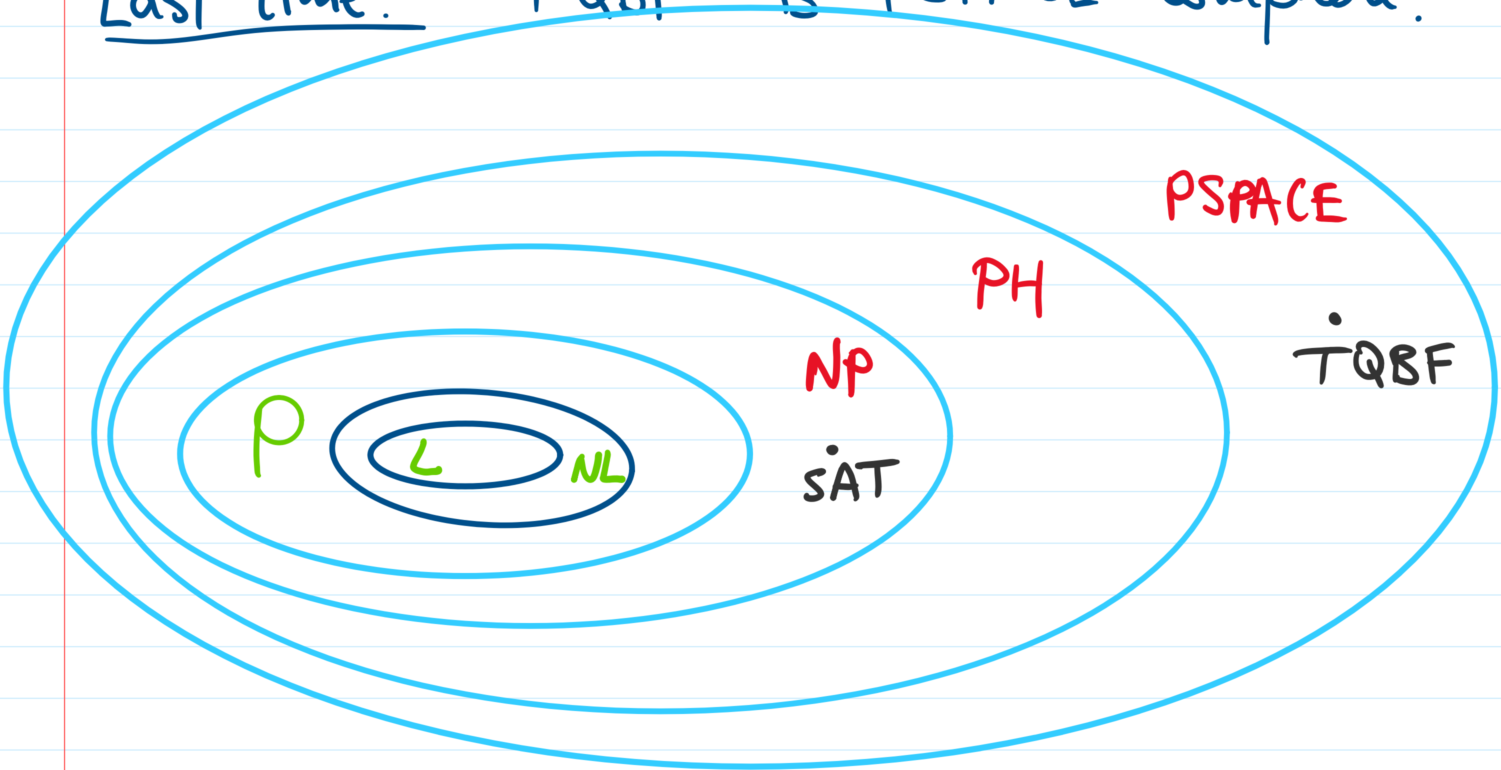
Lecture 26

Wednesday, July 24, 2019 11:44 AM

Last time: TQBF is PSPACE-complete.



L = class of <u>logspace</u>-computable languages

NL = class of <u>nondeterministic logspace</u>-computable languages

Complete problems:

(1) undirected st-CONN: Given an undirected graph $G=(V,E)$, & nodes $s,t \in V$, decide if t is reachable from s.

Theorem: Undirected st-CONN is

L - complete under FO-reductions.

(2) directed st-CONN: Given a directed graph $G=(V,E)$, & nodes $s,t \in V$, decide if t is reachable from s.

Theorem: Directed st-CONN is NL-complete under logspace-reductions.

Open Question: $L = NL$?

Is there a logspace algorithm for directed graph reachability ?

$NL = coNL$

$coNL$ = the complement of NL

$coNL$-complete problem: directed st-UNREACH (given a digraph $G=(V,E)$, $s,t \in V$, decide if t is not reachable from s).

t is not reachable from s).

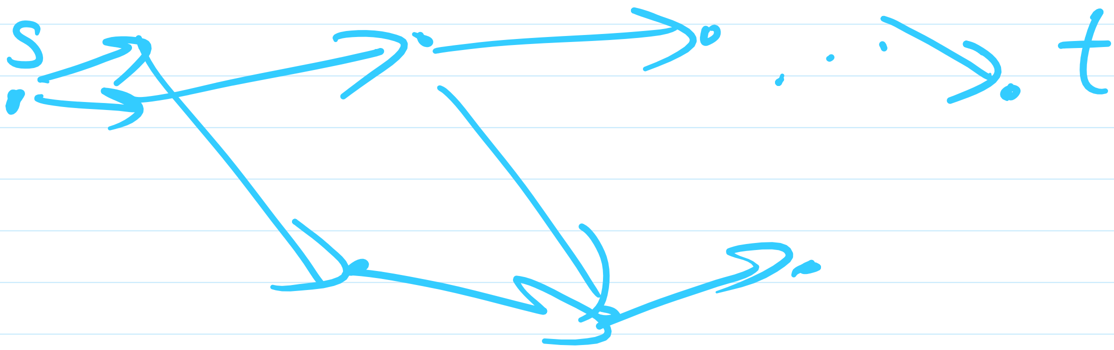
Theorem [Immerman - Szelepcsényi]:

Directed st-UNREACH $\in$ NL.

Hence, $coNL = NL$.

(was open since 1964. Proved in 1987.)

Proof Sketch:



Imagine we know

N = # nodes reachable from s

G has n nodes

$N \leq n$

uses $O(\log n)$ bits to write down

v_1 v_2 v_3 . . . v_n

Yes Yes No No

nondet. guesses : iff the node is reachable from s

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Algo Unreach ( G, s, t)
%%  given  N = # nodes reachable from  s

count = 0
for every node v
     "make a nondeterm. guess if v is reachable from s"
   if guess is YES then
      "nondterm. guess a path from s to v of length at most n"
       if "guessed path does not lead to v" then Reject
   endif
   if v=t then Reject
   else count = count+1
   endif
endfor
if count < N then Reject
else Accept
endif
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The number N = # nodes reachable from s can be computed in NL, iteratively.

N_i = # nodes reachable from s in at most i steps in at most i steps

$N_0 = 1$, $N_n = N$.

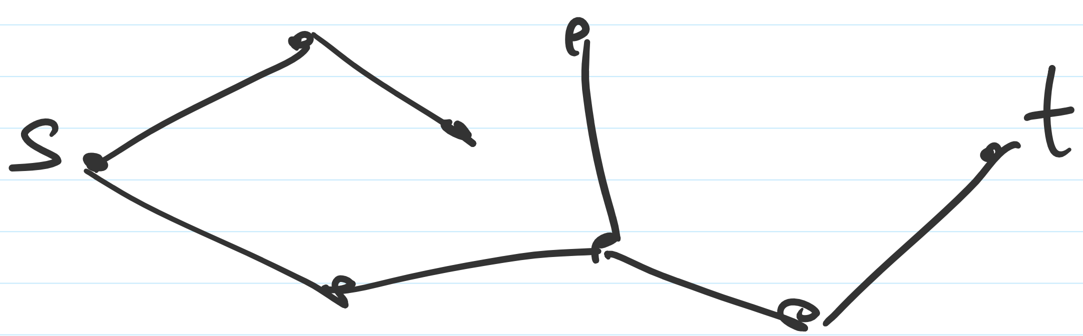
From N_i , can compute N_{i+1} , using the approach of Algo Unreach above.

$\square$

Randomization

$L \subseteq RL \subseteq NL$

undirected st-CONN



Random walk for $O(n^3)$ steps from s will likely see t, if t is reachable from s.

$P \subseteq \underline{\underline{RP}} \subseteq NP$