A NFA

\[ A = (Q, \Sigma, S, q_0, F) \]

- \( Q \) = finite set of states
- \( \Sigma \) = input alph.
- \( q_0 \in Q \) init state \( \alpha \)
- \( F \subseteq Q \)
- \( S : Q \times \Sigma \to Q \)

- state diagram
- table

\[ \begin{array}{c|c c c c}
S & a & \eta \\
\hline
q_0 & s & s
\end{array} \]

\[ L(A) = \{ x \in \Sigma^* \mid A \text{ on } x \xrightarrow{s} q_0 \xrightarrow{x} \Rightarrow q \in F \} \]

A language \( L \subseteq \Sigma^* \) is regular if it's accepted by some NFA.

NFA : Nondet. FA

\[ B = (Q, \Sigma, S, q_0, F) \]

- \( S : Q \times \Sigma \to S(Q) \)
\[ \Sigma_e = \Sigma \cup \{\varepsilon\} \]  
Set of all subsets of \( \Sigma \)

\[ \Sigma = \{0, 1\} \]

\[ \text{trap state} \]

We'll show: \( \text{NFA} \equiv \text{DFA} \).

Regular Operations (Expressions)

1. Union \( U \) : \( A \cup B = \{w \mid w \in A \lor w \in B\} \)
2. Concatenation \( \circ \) : \( A \circ B = \{xy \mid x \in A \text{ and } y \in B\} \)
3. Star \( \ast \) :
   \[ A^\ast = \bigcup_{k \geq 0} A^k \]
   \[ A^k = A \circ \ldots \circ A \quad (k \text{ times}) \]
   \[ A^\ast = \{x_1 \ldots x_k \mid k \geq 0, x_i \in A\} \]

\[ \text{So } S^\ast \]
Claim: The class of regular languages is closed under regular operations.

Proof:

\[ A, B : \text{regular languages} \]
\[ C = A \cup B \text{ is also regular} \]

\[ \text{FA for } A : \]
\[ \text{FA for } B : \]

\[ C = A \circ B \text{ is also regular} \]