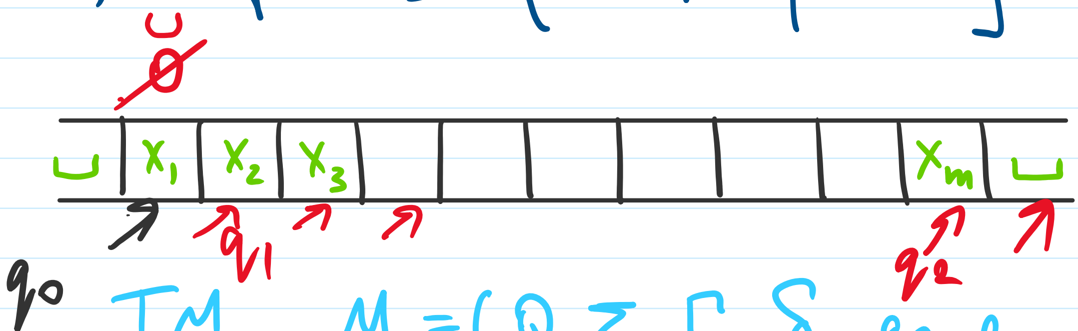
TM for $L = \{0^n 1^n \mid n \geq 0\}$



TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

- $\Sigma = \{0, 1\}$
- $\Gamma = \{0, 1, \sqcup\}$
- δ :

$$\delta(q_0, \sqcup) = (q_{acc}, \sqcup, R)$$
$$\delta(q_0, 1) = (q_{rej}, 1, R)$$
$$\delta(q_0, 0) = (q_1, \sqcup, R)$$

q_1 : will take us to the right-most input symbol

$$\delta(q_1, 0) = (q_1, 0, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, \sqcup) = (q_2, \sqcup, L)$$

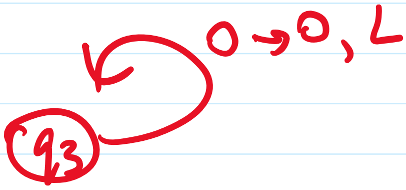
q_2 : check that we see 1 now!

$$\delta(q_2, 0) = (q_{rej}, 0, R)$$
$$\delta(q_2, \sqcup) = (q_{rej}, \sqcup, R)$$
$$\delta(q_2, 1) = (q_3, \sqcup, L)$$

$$\delta(q_2, 1) = (q_3, \sqcup, L)$$

q_3: takes us to the left most (remaining) input symbol

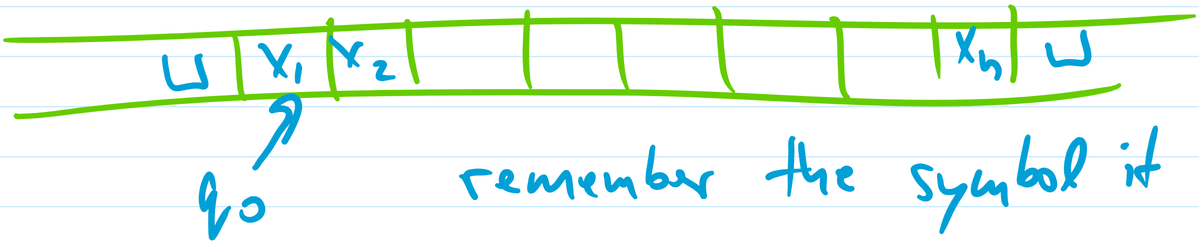
$$\delta(q_3, 0) = (q_3, 0, L)$$
$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, \sqcup) = (q_0, \sqcup, R)$$



$$PAL = \{ w \in \{0,1\}^* \mid w = w^R \}$$

010 , 11 , ε $\in$ PAL

01 $\notin$ PAL

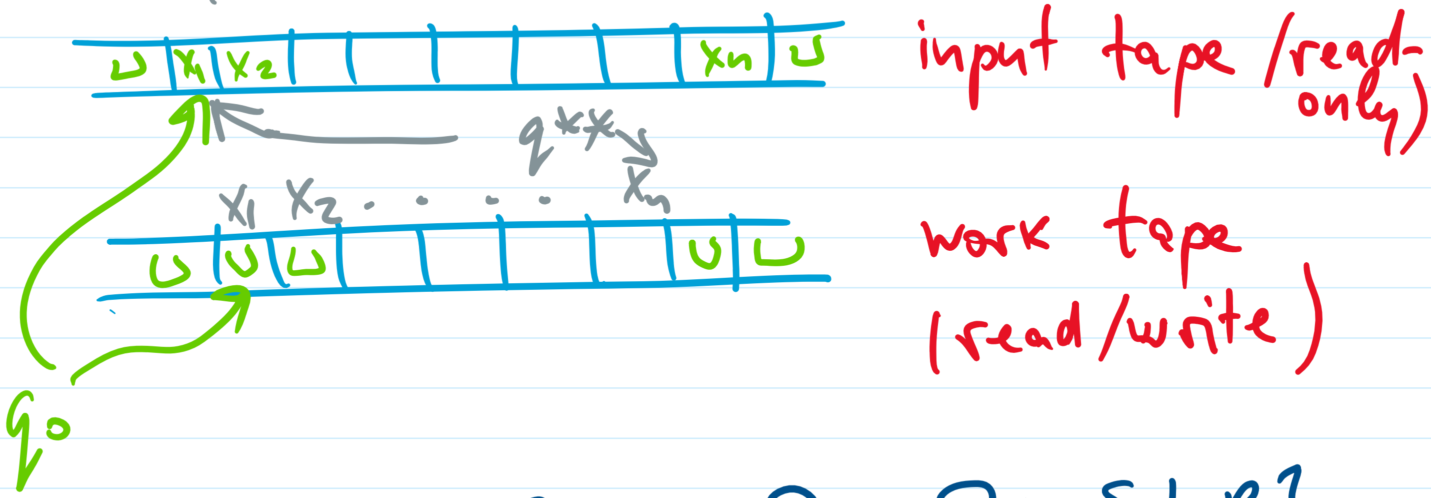


remember the symbol it sees

2-tape TM

2-tape TM

PAL $x_1 x_2 \ldots x_n \stackrel{?}{=} x_n x_{n-1} \ldots x_2 x_1$



1-tape $\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

2-tape $\delta: Q \times \Gamma^2 \to Q \times \Gamma^2 \times \{L, R\}^2$

Claim: PAL is in $O(n)$ time on a 2-tape TM.

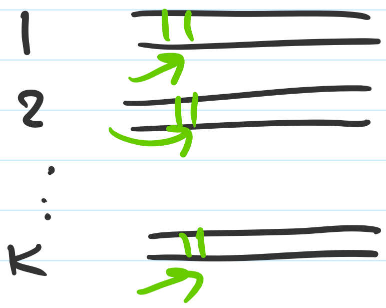
Remark: PAL requires $\Omega(n^2)$ time on a 1-tape TM.

Claim: $\forall L \in \mathrm{Time}(t_k)$ on a k-tape TM, $\exists$ a 1-tape TM for L with runtime $O(t_k^2)$.

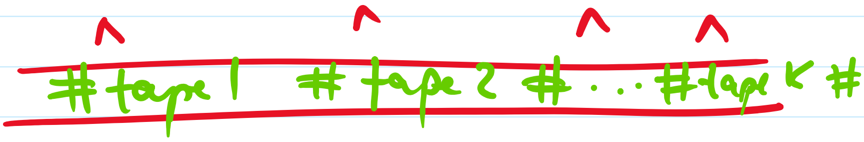
with runtime $O(t_k^2)$.

Idea:

tapes



1-tape TM



$\Gamma \rightarrow \Gamma \cup \hat{\Gamma}$

$\forall a \in \Gamma, \quad \hat{a} \in \hat{\Gamma}$

Other variants of TM

1-tape

2-D tape

k-tape

k-head

RAM

etc.

$\Downarrow$

1-tape TM, with only poly-blow-up in runtime.

UTM