1. Give the state diagrams for the DFA accepting the following languages. Assume the input alphabet $\Sigma = \{0, 1\}$.
   
   (a) $\{w \mid w$ begins with a 1 and ends with a 0$\}$.
   (b) $\{w \mid w$ doesn’t contain the substring 110$\}$.
   (c) $\{w \mid w$ contains an even number of 0s, or contains exactly two 1s$\}$.

2. Argue that the class of regular languages is closed under complementation. That is, show that if $L$ is a regular language over an alphabet $\Sigma$, then its complement $\bar{L} = \Sigma^* \setminus L$ is also regular.

3. Give an NFA accepting the language $(01 \cup 001 \cup 010)^*$. Next, convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.

4. For a string $w = a_1 \ldots a_n$, its reverse is $w^R = a_n \ldots a_1$. For a language $L$, its reverse is $L^R = \{w^R \mid w \in L\}$. Show that if $L$ is a regular language, then so is $L^R$.

5. Let $B_n = \{a^k \mid$ where $k$ is a multiple of $n\}$. Show that for each $n \geq 1$, the language $B_n$ is regular.

6. Let $\Sigma = \{0, 1\}$, and let
   
   $D = \{w \mid w$ contains an equal number of occurrences of the substrings 01 and 10$\}$.
   
   (So 101 $\in D$, but 1010 $\notin D$.)
   
   Show that $D$ is regular.

7. Let $B = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y$ contains at least $k$ 1s, for $k \geq 1\}$. Show that $B$ is regular.

8. Show that each of the following languages is not regular.
   
   (a) $\{1^k y \mid y \in \{0, 1\}^* \text{ and } y$ contains at most $k$ 1s, for $k \geq 1\}$,
   (b) $\{0^n 1^m 0^n \mid m, n \geq 0\}$,
   (c) $\{www \mid w \in \{0, 1\}^*\}$. 
   
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