CMPT 308 - Computability and Complexity
Homework 1

1. Give the state diagrams for the DFA accepting the following languages. Assume the input alphabet $\Sigma = \{0, 1\}$.

   (a) $\{w \mid w$ begins with a 1 and ends with a 0$\}$.

   (b) $\{w \mid w$ doesn't contain the substring 110$\}$.

   (c) $\{w \mid w$ contains an even number of 0s, or contains exactly two 1s$\}$.

2. Argue that the class of regular languages is closed under complementation. That is, show that if L is a regular language over an alphabet Σ, then its complement $\bar{L} = \Sigma^* \setminus L$ is also regular.

3. Give an NFA accepting the language $(01 \cup 001 \cup 010)^*$. Next, convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.

4. For a string $w = a_1 \ldots a_n$, its reverse is $w^R = a_n \ldots a_1$. For a language L, its reverse is $L^R = \{w^R \mid w \in L\}$. Show that if L is a regular language, then so is L^R.

5. Let $B_n = \{a^k \mid$ where k is a multiple of n$\}$. Show that for each $n \geq 1$, the language B_n is regular.

6. Let $\Sigma = \{0, 1\}$, and let

   $$D = \{w \mid w \text{ contains an equal number of occurrences of the substrings 01 and 10}\}.$$

   (So $101 \in D$, but $1010 \notin D$.)

   Show that D is regular.

7. Let $B = \{1^k y \mid y \in \{0, 1\}^*$ and y contains at least k 1s, for $k \geq 1\}$. Show that B is regular.

8. Show that each of the following languages is *not* regular.

   (a) $\{1^k y \mid y \in \{0, 1\}^*$ and y contains at most k 1s, for $k \geq 1\}$,

   (b) $\{0^n 1^m 0^n \mid m, n \geq 0\}$,

   (c) $\{www \mid w \in \{0, 1\}^*\}$.