Stochastic Current Account Models

Assumptions

1.) Expected Utility (+ geometric discounting $\Rightarrow$ Dynamic Consistency)
2.) Rational Expectations (No Model Uncertainty)
3.) Quadratic Utility $\Rightarrow$ "Certainty Equivalence"
4.) Bonds Only (No default).
5.) Small country ($r$ is exogenous)
6.) Domestic MKts. Complete $\Rightarrow$ Rep. Agent

Objective

$$\max_{C_s, I_s} E_t \left\{ \sum_{s=t}^{\infty} \beta^s U(C_s) \right\}$$

s.t.

$$\sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^s (C_s + I_s) = (1+r)B_t + \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^s (Y_s - C_s)$$

Note: Budget constraint holds w.p. 1, i.e., for ‘all’ realizations
\[ U'(C_t) = \beta (1+r) E_t \left[ U'(C_{t+1}) \right] \]

Assume:
1. \( U(\cdot) = C_t - \frac{1}{2} \alpha C_t^2 \) \( \text{Ignore non-negativity constraint} \)
2. \( \beta (1+r) = 1 \)

Then, \( C_t = E_t C_{t+1} \)

Plug into budget constraint,
\[ C_t = \frac{r}{1+r} \left[ (1+r) B_t + \sum_{s=0}^{\infty} (1+r)^s E_t (Y_s - C_s - I_s) \right] \]

Let \( Q_t = Y_t - C_t - I_t \) and \( \tilde{Q}_t = \frac{r}{1+r} E_t \sum_{s=0}^{\infty} (1+r)^s Q_s \)

Then,
\[ CA_t = Q_t - \tilde{Q}_t \]

Suppose, \( Q_t = \rho Q_{t-1} + \varepsilon_t \)

Then, \( \tilde{Q}_t = \frac{r}{1+r-\rho} Q_t \), and
\[ CA_t = \rho \left[ \frac{1-\rho}{1+r-\rho} \right] Q_{t-1} + \frac{1-\rho}{1+r-\rho} \varepsilon_t \]

Note: Response of \( CA \) decreases with shock persistence, \( \rho \).
Investment

\[ Y_t = A_t F(K_t) \]

\[ K_{t+1} = K_t + I_t \]

**FOC for \( K_t \)**

\[ u'(C_t) = \beta E_t \{ u'(C_{t+1})(1 + A_{t+1}F'(K_{t+1})) \} \]

\[ \Rightarrow 1 = E_t \left[ \frac{\beta u'(C_{t+1})}{u'(C_t)} \cdot (1 + A_{t+1}F'(K_t)) \right] \] \quad \text{Standard Asset Pricing Condition}

\[ = E_t \left[ \frac{\beta u'(C_{t+1})}{u'(C_t)} \right] E_t \left[ 1 + A_{t+1}F'(K_t) \right] + \text{cov} \left( \frac{\beta u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_t) \right) \]

Use consumption Euler Eq.,

\[ E_t \left[ A_{t+1}F'(K_{t+1}) \right] = r - \text{cov} \left( A_{t+1}F'(K_{t}), \frac{u'(C_{t+1})}{u'(C_t)} \right) \] \quad \text{Breakdown of Fisherian Separation}

\[ A_{t+1} \uparrow \Rightarrow C_{t+1} \uparrow \Rightarrow u'(C_{t+1}) \downarrow \Rightarrow \text{cov} < 0 \]

\[ \Rightarrow E_t \left[ A_{t+1}F'(K_{t+1}) \right] = r + \text{risk premium} \]

\[ \Rightarrow K_{t+1} < \text{"Certainty Equivalent" \ [uncertainty discourages investment]} \]
Productivity and the Current Account

\[ I_t = h(p)A_t \]
\[ \uparrow \text{persistence of productivity shocks} \]

\[ CA_t = S_t - I_t \]

Consider 2 limiting cases:

a.) \( p = 1 \) \( \implies I \uparrow \)
\[ S \downarrow \] (\( Y \uparrow \) more in future)
\[ \implies CA \downarrow \]

b.) \( p = 0 \) \( \implies \Delta I = 0 \)
\[ S \uparrow \] (\( Y \uparrow \) only in current period)
\[ \implies CA \uparrow \]

Positive productivity shocks more likely to produce CA deficit, the more persistent they are. Empirically, productivity is very persistent.
Empirical Tests (Campbell-Shiller Methodology)

\[ CA_t = Q_t - \frac{r}{1+r} E_t \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^s Q_s \]

**Key Issue**: What information do individuals use when evaluating \( E_t (\cdot) \)?

**Note**: Model implies \( CA_t \) is a sufficient statistic for \( E_t (\cdot) \). Can "soak up" extra info. by including \( CA_t \) when forecasting future values of \( Q_t \).
Useful representation,

Let $\alpha = \frac{1}{1+r}$. Then,

$$CA_t = Q_t - (1-\alpha)E_t \sum_{j=0}^{\infty} \alpha^j Q_{t+j}$$

$$= Q_t - \frac{1-\alpha}{1-\alpha^2} Q_t$$

$$= \frac{1-\alpha \bar{L}^{-1} - (1-\alpha)}{1-\alpha^2} Q_t$$

$$= -\alpha \frac{(\bar{L}^{-1} - 1)}{1-\alpha \bar{L}^{-1}} Q_t$$

$$\Rightarrow CA_t = -E_t \sum_{j=1}^{\infty} \alpha^j \Delta Q_{t+j}$$

This is also useful from a statistical standpoint, since empirically, $Q_t$ often appears to have a unit root.
future values of $\Delta t$.

$$
\begin{align*}
(\Delta Q^t) &= (q_{11}, q_{12}) (\Delta Q^{t-1}) + (e_{1t}) \\
(CA^t) &= (q_{21}, q_{22}) (CA^{t-1}) + (e_{2t})
\end{align*}
$$

$$
E_{\Delta Q^t} = (1, 0) \left[ \begin{array}{cc}
q_{11} & q_{12} \\
q_{21} & q_{22}
\end{array} \right] (\Delta Q^t) ^{s-f}
$$

$$
\hat{CA}^t = -(1, 0) \left[ I - \frac{i}{\tau_{\Delta t}} \Psi \right] ^{-1} (\Delta Q^t)
$$

$$
= (\Gamma_0, \Gamma_1) (\Delta Q^t)
$$

Theory $\Rightarrow$ $(\Gamma_0, \Gamma_1) = (0, 1)$
Can write as a linear restriction,

\((0, 1) = -(1, 0) Y [I - \# Y]^T\)

\[\Rightarrow (0, 1) [I - \# Y] = -(1, 0) Y \]

That is,

Bottom row of \( I - \# Y \) \(-\) Top row of \( \# Y \)

This is equivalent to,

\[E_+ \left[ CA_{t+1} - \Delta Q_{t+1} - (1 + r) CA_t \right] = 0 \] \(\text{GMM orthogonality test}\)
Figure 2.5
Canada: Actual and predicted current accounts

Figure 2.7
Sweden: Actual and predicted current accounts

Figure 2.8
United Kingdom: Actual and predicted current accounts
Extensions

1.) Variable Interest Rates (and other valuation effects)
   - Couriñchak & Rey (JPE, 2007)

2.) Precautionary Saving

3.) Global vs. Country-Specific Shocks
   - Glick & Rogoff (JME, 1995)

4.) Finite Horizons ("Twin Deficits")
   - Kase (1994)

5.) Endogenous Labor

6.) Large Country

7.) Portfolio Diversification with Incomplete Markets

8.) Default + "Sudden Stops", Borrowing Constraints

9.) Trade Costs

10.) Multiple Goods (Terms of Trade & Real Exchange Rate)
**Variable Interest Rates**

Define
\[ R_{t,s} = \frac{1}{\prod_{v=t+1}^{s} (1+r_v)} \]

Now get,
\[ C_t = (1+r_t)B_t + \sum_{s=t}^{\infty} R_{t,s} (Y_s - I_s - G_s) \]
\[ \frac{\sum_{s=t}^{\infty} R_{t,s} \left[ R_{t,s}^{-\alpha} \beta^{\alpha(s-t)} \right]}{\sum_{s=t}^{\infty} R_{t,s} } \]

Now annuity values are,
\[ \sum_{s=t}^{\infty} R_{t,s} \tilde{X}_s = \sum_{s=t}^{\infty} R_{t,s} X_s \]

CA\(_t\) = \[(r_t - \tilde{r}_t)B_t + (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t)\]
\[ + \left( \frac{\tilde{r}_t - 1}{\tilde{r}_t} \right) (\tilde{r}_t B_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t) \]

\[ \tilde{r}_t = \frac{\sum_{s=t}^{\infty} R_{t,s} \left[ R_{t,s}^{-\alpha} \beta^{\alpha(s-t)} \right]}{\sum_{s=t}^{\infty} R_{t,s} } \],  
*discount rate weighted average of consumption growth*
Precautionary Saving: \( U'''' > 0 \)

\[ U'(c_t) = E_t U'(c_{t+1}) > U'(E_t c_{t+1}) \]
\[ \Rightarrow E_t c_{t+1} > c_t \]

finite horizons

Let \( \tau = \text{Survival probability} \)

\[ CA_t = \tau CA_{t-1} + \frac{1}{\tau} E_{t-1} \left( \Delta y_t - \Delta c_t \right) + \frac{\left( \frac{\tau - \lambda}{\tau} \right) E_{t-1} \sum_{s=1}^{\infty} \left( \frac{\tau}{\tau - 1} \right)^s \lambda^s (1+\gamma)^s \right] \]
\[ + (1-\tau) \left[ \frac{\tau - \lambda}{\tau} \right] E_{t-1} \sum_{s=1}^{\infty} \left( \frac{\tau}{\tau - 1} \right)^s \beta^s BS_{t+s} + \psi_t \]

Endogenous Labor

Suppose \( U(c, L) = \frac{1}{1 - \lambda} \left[ C^\lambda (c - L)^{1-\lambda} \right] \]

Now get static efficiency condition \( W = \frac{1-\tau}{\tau} \frac{c}{L} \)

Euler Eq. becomes,

\[ C_{t+1} = \left( \frac{W_t}{W_{t+1}} \right)^{(1-\gamma)(\sigma-1)} (1+\gamma)^\sigma \beta^\sigma C_t \]
Real Exchange Rates

Now assume more than one good.

Easy case first: Non-traded Goods

NT Goods $\Rightarrow$ Prices don't need to be the same across countries.

Real Ex. Rate = Relative Cost of a common basket of goods in 2 countries.

If price indices use the same weights, just the ratio of national price levels (expressed in common currency).

Thus, \( q_t = \frac{P_t}{P_t^*} \)  \( q_t \uparrow \Rightarrow \) Home real appreciation

PPP: Cost of living is the same everywhere

\( \Rightarrow q_t = 1 \forall t \) (Absolute PPP)

Problem: Price levels are index numbers, expressed relative to a base year.

Weaker version: \( q_t = \text{constant} \)

\( \Rightarrow \hat{E} = \hat{\rho} - \hat{\rho}^* \)
Figure 3.1 Log nominal exchange rates (boxes) and CPI-based PPPs (solid lines).
(a) US-UK; (b) US-Germany; (c) US-Japan; (d) US-Switzerland.

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Relative price level (U.S. = 100)

Per capita real income, 1992

Figure 4.1
Real per capita incomes and price levels, 1992. (Source: Penn World Table)