Stochastic Current Account Models

Assumptions

1.) Expected Utility (+ geometric discounting $\Rightarrow$ Dynamic Consistency)

2.) ~~Bonds~~ Rational Expectations (No Model Uncertainty)

3.) Quadratic Utility $\Rightarrow$ "Certainty Equivalence"

4.) Bonds Only (No default).

5.) Small country (r is exogenous)

6.) Domestic Mkts. Complete $\Rightarrow$ Rep. Agent

Objective

$$\max_{C_s, I_s} E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\}$$

s.t.

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s) = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s)$$

Note: Budget constraint holds w.p. 1, i.e., for "all" realizations

FOC

$$U'(C_t) = \beta(1+r)\, E_t\left[U'(C_{t+1})\right]$$

Assume : ① $U(\cdot) = C_t - \frac{a}{2} C_t^2$ $\quad$ } Ignore non-negativity constraint

② $\beta(1+r) = 1$

Then, $\quad C_t = E_t C_{t+1}$

Plug into budget constraint,

$$C_t = \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left(Y_s - G_s - I_s\right)\right] \Big\} \text{ Certainty Equivalence}$$

Let $Q_t = Y_t - G_t - I_t$ and $\tilde{Q}_t = \frac{r}{1+r} E_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Q_s$

Then,

$$\boxed{CA_t = Q_t - \tilde{Q}_t}$$

Suppose, $Q_t = \rho Q_{t-1} + \varepsilon_t$

Then, $\quad \tilde{Q}_t = \frac{r}{1+r-\rho} Q_t$, and

$$\boxed{CA_t = \rho\left[\frac{1-\rho}{1+r-\rho}\right] Q_{t-1} + \frac{1-\rho}{1+r-\rho}\, \varepsilon_t}$$

Note: Response of CA decreases with shock persistence, ρ.

Investment

$$Y_t = A_t F(K_t)$$

$$K_{t+1} = K_t + I_t$$

FOC for Ks

$$u'(c_t) = \beta E_t\left\{ u'(c_{t+1})(1 + A_{t+1} F'(K_{t+1}))\right\}$$

$$\Rightarrow 1 = E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot (1 + A_{t+1} F'(K_{t+1}))\right] \quad \Big\} \text{ Standard Asset Pricing Condition}$$

$$= E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\right] E_t\left[1 + A_{t+1} F'(K_{t+1})\right] + \operatorname{cov}_t\left(\frac{\beta u'(c_{t+1})}{u'(c_t)}, A_{t+1} F'(K_{t+1})\right)$$

Use consumption Euler Eq.,

$$E_t\left[A_{t+1} F'(K_{t+1})\right] = r - \operatorname{cov}_t\left[A_{t+1} F'(K_{t+1}), \frac{u'(c_{t+1})}{u'(c_t)}\right] \quad \Big\} \text{ Breakdown of Fisherian Separation}$$

$$A_{t+1}\uparrow \Rightarrow c_{t+1}\uparrow \Rightarrow u'(c_{t+1})\downarrow \Rightarrow \operatorname{cov} < 0$$

$$\Rightarrow E_t\left[A_{t+1} F'(K_{t+1})\right] = r + \text{risk premium}$$

$\Rightarrow K_{t+1} <$ "Certainty Equivalent" [uncertainty discourages investment]

Productivity and the Current Acct.

$$I_t = h(\overset{+}{\rho}) A_t$$

persistence of productivity shocks

$$CA_t = S_t - I_t$$

Consider 2 limiting cases:

a.) $\rho = 1 \implies I \uparrow$

$S \downarrow$ ($Y \uparrow$ more in future)

$\implies CA \downarrow$

b.) $\rho = 0 \implies \Delta I = 0$

$S \uparrow$ ($Y \uparrow$ only in current period)

$\implies CA \uparrow$

positive productivity shocks more likely to produce CA deficit, the more persistent they are. Empirically, productivity is very persistent

Empirical Tests (Campbell-Shiller Methodology)

$$CA_t = Q_t - \frac{r}{1+r} E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} Q_s$$

Key Issue: What information do individuals use when evaluating $E_t(\cdot)$?

Note: Model implies CA_t is a sufficient statistic for $E_t(\cdot)$. Can "soak up" extra info. by including CA_t when forecasting future values of Q_t.

Useful representation,

Let $\alpha = \frac{1}{1+r}$. Then,

$$CA_t = Q_t - (1-\alpha) E_t \sum_{j=0}^{\infty} \alpha^j Q_{t+j}$$

$$= Q_t - \frac{1-\alpha}{1-\alpha L^{-1}} Q_t$$

} can ignore E_t for now & put in later (certainty equivalence)

$L^{-1} Q_t = Q_{t+1}$

$$= \frac{1-\alpha L^{-1} - (1-\alpha)}{1-\alpha L^{-1}} Q_t$$

$$= -\alpha \frac{(L^{-1}-1)}{1-\alpha L^{-1}} Q_t$$

$$\Rightarrow \boxed{CA_t = - E_t \sum_{j=1}^{\infty} \alpha^j \Delta Q_{t+j}}$$

This is also useful from a statistical standpoint, since empirically, Q_t often appears to have a unit root.

future values of Q_t.

$$\begin{pmatrix} \Delta Q_t \\ CA_t \end{pmatrix} = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{pmatrix} \begin{pmatrix} \Delta Q_{t-1} \\ CA_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$E_t \Delta Q_s = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}^{s-t} \begin{pmatrix} \Delta Q_t \\ CA_t \end{pmatrix}$$

$$\hat{CA}_t = -\begin{pmatrix} 1 & 0 \end{pmatrix} \frac{1}{1+r}\Psi \left[I - \frac{1}{1+r}\Psi \right]^{-1} \begin{pmatrix} \Delta Q_t \\ CA_t \end{pmatrix}$$

$$= \begin{pmatrix} \Gamma_0 & \Gamma_1 \end{pmatrix} \begin{pmatrix} \Delta Q_t \\ CA_t \end{pmatrix}$$

$$\text{Theory} \Rightarrow \begin{pmatrix} \Gamma_0 & \Gamma_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Can write as a linear restriction,

$$(0 \quad 1) = -(1 \quad 0)\frac{1}{1+r}\Psi\left[I-\frac{1}{1+r}\Psi\right]^{-1}$$

$\Longrightarrow$

$$(0 \quad 1)\left[I-\frac{1}{1+r}\Psi\right] = -(1 \quad 0)\frac{1}{1+r}\Psi$$

That is,

Bottom row of $I-\frac{1}{1+r}\Psi$ = − Top row of $\frac{1}{1+r}\Psi$

This is equivalent to,

$$E_t\left[CA_{t+1} - \Delta Q_{t+1} - (1+r)CA_t\right] = 0 \quad \Big\} \text{ GMM orthogonality test}$$

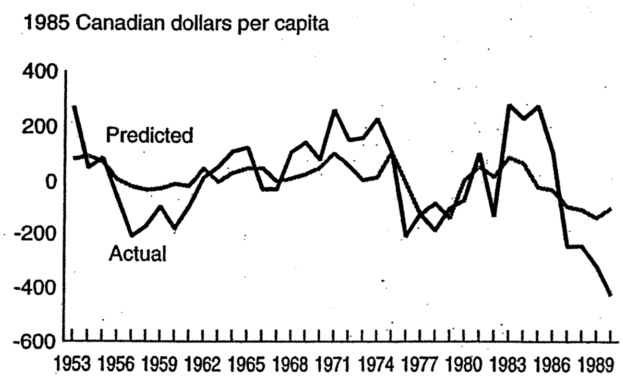


Figure 2.5
Canada: Actual and predicted current accounts

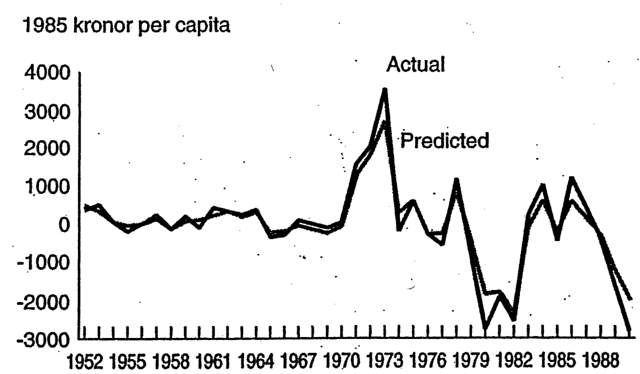


Figure 2.7
Sweden: Actual and predicted current accounts

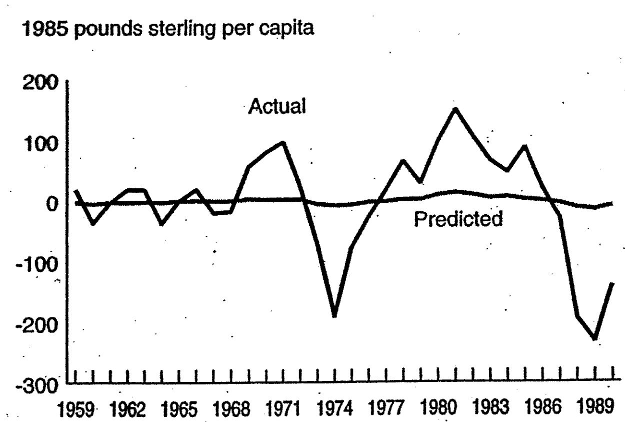


Figure 2.8
United Kingdom: Actual and predicted current accounts

Extensions

1.) Variable Interest Rates (+ other valuation effects)
- Gourinchas + Rey (JPE, 2007)

2.) Precautionary Saving

3.) Global vs. Country-Specific Shocks
- Glick + Rogoff (JME, 1995)

4.) Finite Horizons ("Twin Deficits")
- Kase (1994)

5.) Endogenous Labor

6.) Large Country

7.) Portfolio Diversification with Incomplete Mkts.

8.) Default + "Sudden Stops". Borrowing Constraints.

9.) Trade Costs

10.) Multiple Goods (Terms of Trade + Real Ex. Rate)

Variable Interest Rates

Define $$R_{t,s} = \frac{1}{\prod_{v=t+1}^{s} (1+r_v)}$$

Now get,

$$C_t = \frac{(1+r_t)B_t + \sum_{s=t}^{\infty} R_{t,s}(Y_s - I_s - G_s)}{\sum_{s=t}^{\infty} R_{t,s}\left[R_{t,s}^{-\sigma}\beta^{\sigma(s-t)}\right]}$$

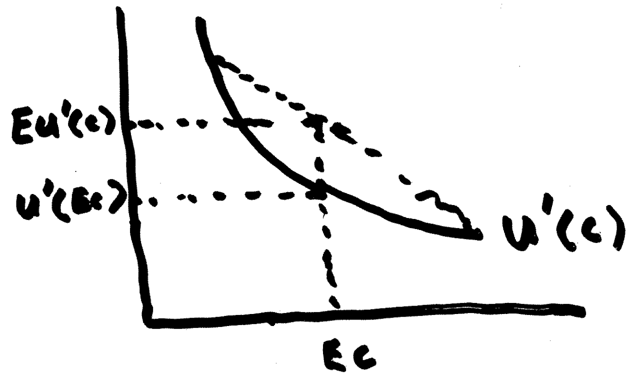
Now annuity values are,

$$\sum_{s=t}^{\infty} R_{t,s}\tilde{X}_t = \sum_{s=t}^{\infty} R_{t,s} X_s$$

$$\boxed{\begin{aligned} CA_t = {} & (r_t - \tilde{r}_t)B_t + (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) \\ & + \left(\frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t}\right)(\tilde{r}_t B_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t) \end{aligned}}$$

$$\tilde{\Gamma}_t = \frac{\sum_{s=t}^{\infty} R_{t,s}\left[R_{t,s}^{-\sigma}\beta^{\sigma(s-t)}\right]}{\sum_{s=t}^{\infty} R_{t,s}}$$ = discount rate weighted average of consumption growth

Precautionary Saving : $u''' > 0$



$$u'(c_t) = E_t u'(c_{t+1}) > u'(E_t c_{t+1})$$

$$\Rightarrow E_t c_{t+1} > c_t$$

Like consumption-tilting

Finite Horizons

Let γ = survival probability

$$CA_t = \gamma CA_{t-1} + \frac{\gamma}{R} E_{t-1}(\Delta Y_t - \Delta G_t) + \left(\frac{R-\gamma}{R}\right) E_{t-1} \sum_{j=1}^{\infty} \left(\frac{\gamma}{R}\right)^j \left[\Delta G_{t+j} - Y_{t+j}\right]$$
$$+ (1-\gamma)\left[\frac{R-\gamma}{R}\right] E_{t-1} \sum_{j=0}^{\infty} \left(\frac{\gamma}{R}\right)^j BS_{t+j} + u_t$$

Endogenous Labor

Suppose $U(C, \bar{L}-L) = \dfrac{1}{1-1/\sigma}\left[C^{\gamma}(\bar{L}-L)^{1-\gamma}\right]^{1-1/\sigma}$

Now get static efficiency condition $W = \dfrac{1-\gamma}{\gamma} \dfrac{C}{\bar{L}-L}$

Euler Eq. becomes,

$$C_{s+1} = \left(\frac{W_s}{W_{s+1}}\right)^{(1-\gamma)(\sigma-1)} (1+r)^{\sigma} \beta^{\sigma} C_s$$

Real Exchange Rates

Now assume more than one good.

Easy case first: Non-traded Goods

NT Goods $\Longrightarrow$ Prices don't need to be the same across countries.

Real Ex. Rate = Relative Cost of a common basket of goods in 2 countries.

If price indices use the same weights, just the ratio of national price levels (expressed in common currency).

Thus, $q_t = \dfrac{P_t}{E_t P_t^*}$ $\qquad q_t \uparrow \Longrightarrow$ Home real appreciation

PPP: Cost of living is the same everywhere

$\Longrightarrow q_t = 1 \;\; \forall t$ (Absolute PPP)

Problem: Price levels are index numbers, Expressed relative to a base year.

Weaker version: $q_t = \text{constant}$

$\Longrightarrow \hat{E} = \hat{P} - \hat{P}^*$

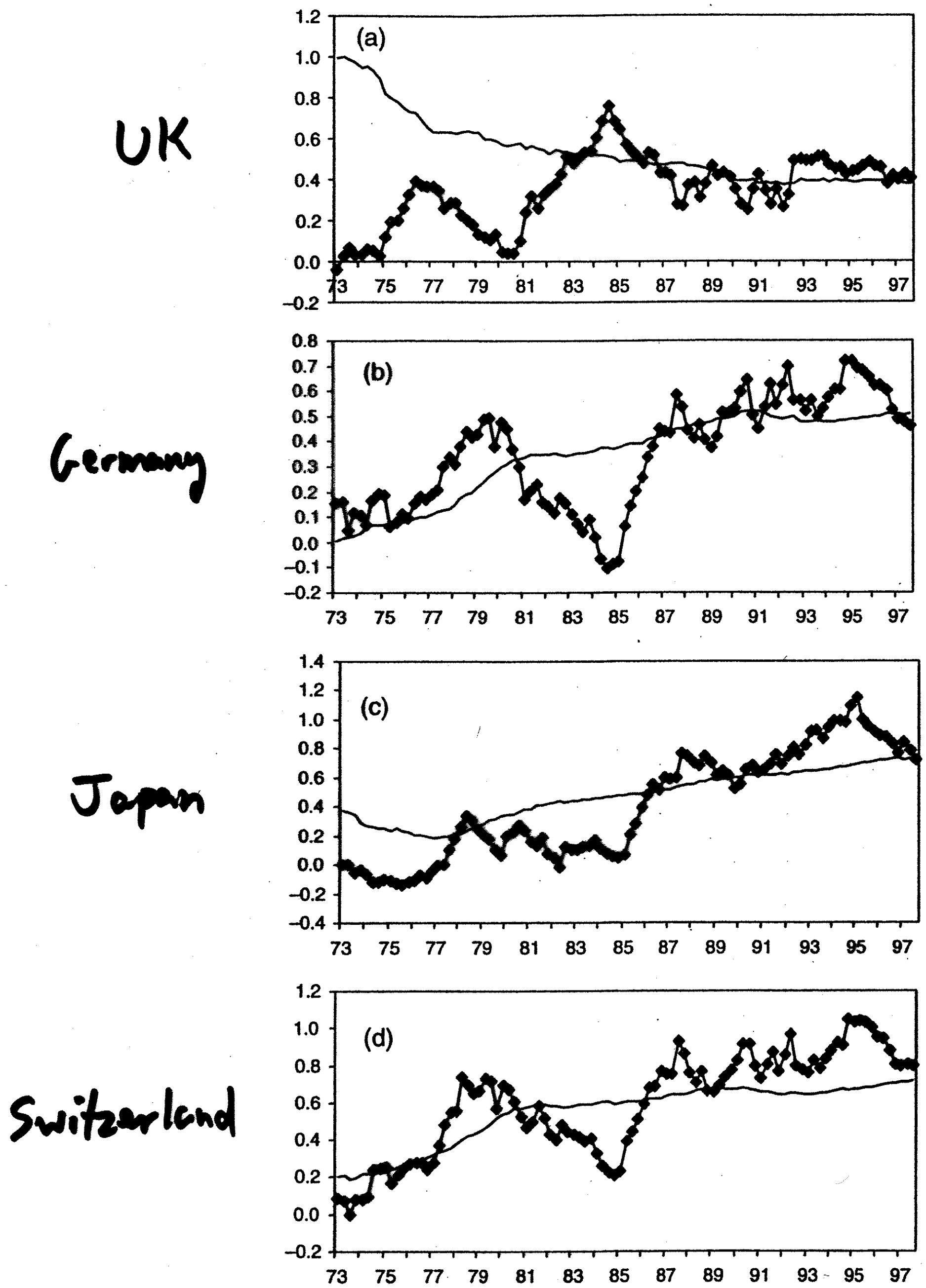


Figure 3.1 Log nominal exchange rates (boxes) and CPI-based PPPs (solid lines). (a) US–UK; (b) US–Germany; (c) US–Japan; (d) US–Switzerland.

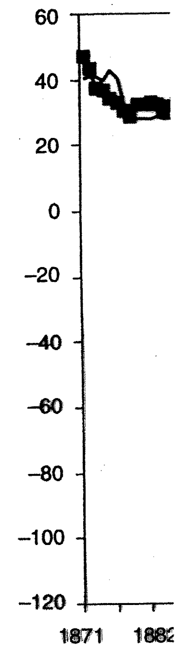


Figure 3.2

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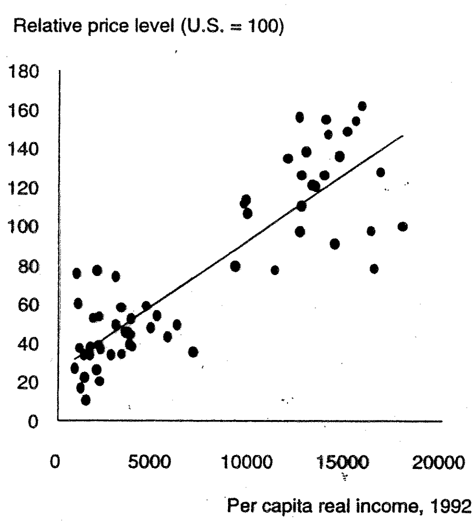

Figure 4.1
Real per capita incomes and price levels, 1992. (*Source:* Penn World Table)