Balassa - Samuelson

Price Indices

$$P = P_T^{\gamma} P_{NT}^{1-\gamma} \qquad P^* = P_T^{*\gamma} P_{NT}^{*1-\gamma}$$

Assume traded goods are numeraire.

$$\frac{P}{P^*} = \left(\frac{P_T}{P_T^*}\right)^{\gamma} \left(\frac{P_{NT}}{P_{NT}^*}\right)^{1-\gamma}$$

By assumption, $P_T = P_T^*$

So,

$$\frac{P}{P^*} = \left(\frac{P_{NT}}{P_{NT}^*}\right)^{1-\gamma}$$

The Question is: Why might wealthy countries have higher relative prices of NT goods?

Balassa-Samuelson explains this via differences in sectoral productivities

Assumptions

1.) Each country produces 2 composite goods, an identical traded good and a NT good.

2.) Goods produced in competitive industries via CRS production functions.

3.) Capital is mobile internationally

4.) Labor is immobile internationally, but mobile across sectors within a country.

$$Y_T = A_T F(K_T, L_T) \qquad Y_N = A_N G(K_N, L_N)$$

$$L_T + L_N = \bar{L}$$

Assume tradeables are numeraire $\quad p =$ price of NT in terms of tradeables

Profit-Maximizing FOCs

$k_T = K_T / L_T$

a.) $A_T f'(k_T) = r$

b.) $A_T [f(k_T) - k_T f'(k_T)] = w$

c.) $p A_N g'(k_N) = r$

d.) $p A_N [g(k_N) - g'(k_N) \cdot k_N] = w$

Productivity & the Real Ex. Rate

Zero Profit Conditions

a) $A_T f(k_T) = r k_T + w$

b) $p A_N g(k_N) = r k_N + w$

Log differentiate both sides,

a) $\hat{A}_T + \frac{f' \cdot k_T}{f} \hat{k}_T = \frac{r k_T}{A_T f} \hat{k}_T + \frac{w}{A_T f} \hat{w}$

$\Longrightarrow \hat{A}_T = \frac{w \cdot L_T}{A_T F(\cdot)} \hat{w} = \mu_{LT} \hat{w}$

(b) $\hat{p} + \hat{A}_N = \mu_{LN} \hat{w}$

Sub-in from (a)

$$\boxed{\hat{p} = \frac{\mu_{LN}}{\mu_{LT}} \hat{A}_T - \hat{A}_N}$$

$\frac{\mu_{LN}}{\mu_{LT}} > 1 \Longrightarrow \hat{p}$ rises when $\hat{A}_T > \hat{A}_N$

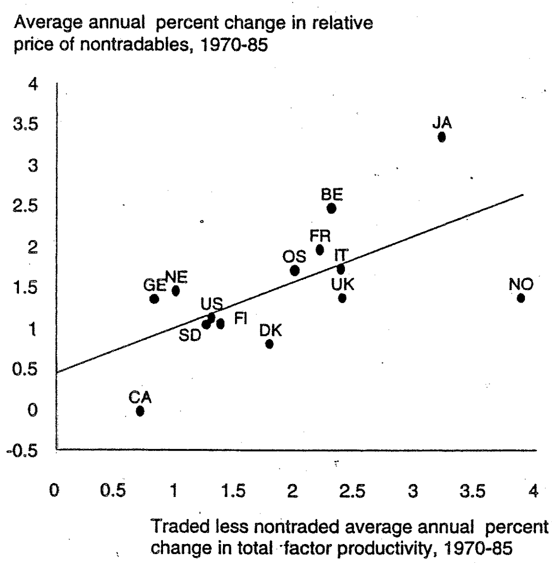


Figure 4.4
Differential productivity growth and the price of nontradables. (*Source:* De Gregorio, Giovannini, and Wolf, 1994)

Table 4.1
Average Annual Labor Productivity Growth in Manufacturing, 1979–93

Country	Productivity Growth (percent per year)
Belgium	4.3
Canada	1.7
Denmark	1.5
France	2.8
Germany	1.9
Italy	4.1
Japan	3.8
Netherlands	2.6
Norway	2.3
Sweden	3.2
United Kingdom	4.1
United States	2.5

Source: Dean and Sherwood (1994). Data for Italy cover 1979–92 only.

Intuition

1.) When $A_T \uparrow$ wages in T-sector rise
(P_T + r fixed)

2.) Intersectoral labor mobility $\Rightarrow$ wages in NT rise

3.) Rise in NT wages forces p up. (zero profits)

Implications for Real Ex. Rate,

$$q = \left(\frac{P_N}{P_N^*}\right)^{1-\gamma}$$

$$\boxed{\hat{q} = (1-\gamma)\left[\hat{P}_N - \hat{P}_N^*\right] = (1-\gamma)\left[\frac{\mu_{LN}}{\mu_{LT}}(\hat{A}_T - \hat{A}_T^*) - (\hat{A}_N - \hat{A}_N^*)\right]}$$

Caveat

Assumption of identical T-goods could be important. Without this, there could be offsetting TOT effects. [Fitzgerald (2003)].

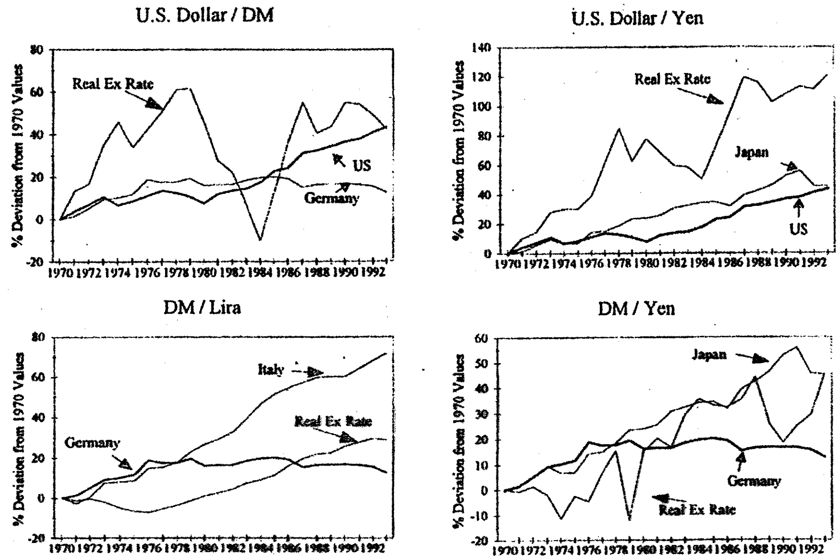


Fig. 1. Real exchange rates and relative labor productivities.

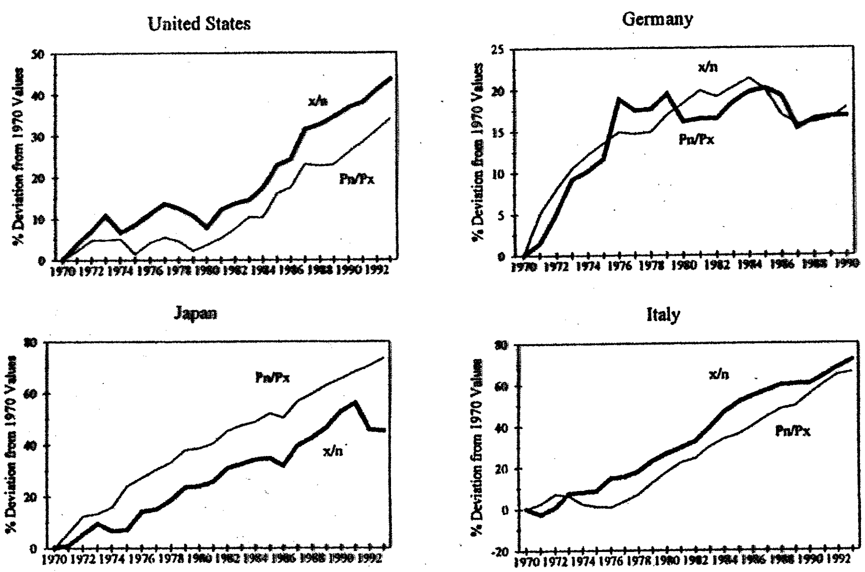


Fig. 2. Relative productivities and relative prices.

Engel (JPE, 1999)

Suppose $P = P_T^{\alpha} P_N^{1-\alpha}$ } Domestic Price Index

$P^* = P_T^{*\beta} P_N^{*\,1-\beta}$ } Foreign Price Index

$$\lambda = \frac{S P^*}{P} = \frac{S P_T^*}{P_T} \cdot \frac{(P_N^*/P_T^*)^{1-\beta}}{(P_N/P_T)^{1-\alpha}}$$ } Real Exchange Rate

Empirical Result: Nearly all of the variation in λ is due to variation in $S P_T^*/P_T$, not $\frac{(P_N^*/P_T^*)^{1-\beta}}{(P_N/P_T)^{1-\alpha}}$!

Potential Explanations

1.) Nominal Rigidities
- Horizon evidence unfavorable (except for Canada)

2.) Measurement Problems

3.) Terms of Trade movements (with different weights)

4.) Only considers rich countries.

Comment on Current Account "Sustainability"

Observation 1: It is often claimed that countries cannot run CA deficits forever. Eventually, a "day of reckoning" must come.

Observation 2: "Equilibrium" Real Ex. Rates often defined by the requirement that eventually, the CA is "in balance".

Do these make sense?

A Simple Example

1.) $\beta(1+r) = 1$ (small country)

2.) $U(C_T, C_N) = \log(C_T^{\alpha} C_N^{1-\alpha})$

$\Rightarrow C_T = \text{constant}$

3.) $Y_T = A_T L_T$ $\quad Y_N = L_N$

Labor Mkt. Equil. $\Rightarrow W_s = A_{Ts}$ $\quad W_s = P_s$

$\Rightarrow P_s = A_{Ts}$ (Balassa-Samuelson)

4.) A_T grows at rate $g < r$

Implications

$$\text{FOC} \Longrightarrow P_t C_{Nt} = \frac{1-\alpha}{\alpha} C_{Tt}$$

$$\Longrightarrow C_{Tt} + P_t C_{Nt} = \bar{C}_T / \alpha$$

Budget Constraint

$$\sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \left(C_{Tt+j} + P_{t+j} C_{Nt+j}\right) = (1+r) B_t + \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \left(Y_{Tt+j} + P_{t+j} Y_{Nt+j}\right).$$

Note, l.h.s $= \frac{1+r}{\alpha r} \bar{C}_T$

$$GDP_t = Y_{Tt} + P_t Y_{Nt} = A_{Tt}(\bar{L} - L_{Nt}) + P_t L_{Nt} = A_{Tt} \bar{L}$$

Thus, r.h.s. $= (1+r) B_t + \frac{1+r}{r-g} \bar{L} A_{Tt}$

Equating, l.h.s = r.h.s., we get

$$B_t = \frac{1}{\alpha r} \bar{C}_T - \frac{1}{r-g} A_{Tt} \bar{L}$$

Let $z_t = B_t / A_{Tt} \bar{L}$: $\frac{\text{Foreign Assets}}{\text{GDP}}$

$$= \frac{1}{\alpha r} \frac{\bar{C}_T}{A_{Tt} \bar{L}} - \frac{1}{r-g}$$

Note that:

1.) Debt/GDP ratio increases forever, asymptoting to $\frac{1}{r-g}$

2.) There is a perpetual, increasing current account deficit $\left[CA_t = B_{t+1} - B_t = \frac{-g}{r-g} A_{T+} \bar{L} \right]$

3.) There is a constant (current acct. deficit)/GDP ratio

4.) The real ex. rate steadily appreciates

$$\left[\text{Suppose } g = .01 \quad r = .11 \Rightarrow \left(\bar{CA}/Y\right) = 10\% \quad \left(\bar{B}/Y\right) = 10\,! \right]$$

Now assume instead labor is immobile between sectors.

Still have $C_{Tt} = \frac{\alpha}{1-\alpha} P_t C_{Nt} = \frac{\alpha}{1-\alpha} P_t Y_{Nt}$

Also, Euler still implies $C_{Tt} = \bar{C}_T$.

Since Y_{Nt} is now constant, P_t = constant

5.) Same debt + CA implications as before, but now real ex. rate is constant!

Real Ex. Rates & the Current Account

Assume linearly homogeneous preferences so that we can separately optimize <u>intratemporally</u> and <u>intertemporally</u>.

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

$$C_s = \Omega(C_{T_s}, C_{N_s}) \quad \} \text{ "real consumption"}$$

$$= \left[\gamma^{\frac{1}{\theta}} C_{T_s}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{N_s}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

Static Demands

$$C_T = \frac{\gamma Z}{\gamma + (1-\gamma) p^{1-\theta}} \qquad C_N = \frac{(1-\gamma) Z p^{-\theta}}{\gamma + (1-\gamma) p^{1-\theta}}$$

Where $Z = C_T + p C_N$ = Total Expenditure measured in traded goods

Can simplify by defining a <u>price index</u>

<u>Definition</u>:

Consumption-Based Price Index ($\mathcal{P}$) = The minimum expenditure, Z, such that $\Omega(c_T, c_N) = 1$

To compute,

a.) Sub demand functions into Ω

b.) Set $Z = \mathcal{P}$, $\Omega = 1$, and solve for $\mathcal{P}$

This yields,

$$\boxed{\mathcal{P} = \left[\gamma + (1-\gamma) p^{1-\theta} \right]^{\frac{1}{1-\theta}}}$$

Note, $\mathcal{P}$ translates total expenditure measured in T goods into "real consumption".

In particular, note $\boxed{C = Z/\mathcal{P}}$

Also, $\boxed{C_T = \gamma \left(\frac{1}{\mathcal{P}}\right)^{-\theta} C}$ and $\boxed{C_N = (1-\gamma)\left(\frac{p}{\mathcal{P}}\right)^{-\theta} C}$

Now the consumer's intertemporal optimization problem becomes

$$\max_{C_s} \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

s.t.

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} P_s C_s = (1+r) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[Y_{T,s} + P_s Y_{N,s} - I_s - G_s \right]$$

Euler Eq.,

$$u'(C_s) = \beta (1+r) \frac{P_s}{P_{s+1}} u'(C_{s+1})$$

$$= \beta (1 + r^c_{s+1}) u'(C_{s+1})$$

$$\boxed{1 + r^c = \text{"consumption-based real interest rate"}}$$

Note,

$$CA_t = B_{t+1} - B_t = rB_t + Y_{T,t} + P_t Y_{N,t} - C_{T,t} - P_t C_{N,t} - I_t - G_t$$

Mkt. clearing $\Rightarrow P_t Y_{N,t} = P_t C_{N,t} \quad \forall t$

Therefore,

$$CA_t = rB_t + Y_{T,t} - C_{T,t} - I_t - G_t$$

Need to get $C_{T,t}$. Assume $u(\cdot) = \dfrac{C_s^{1-1/\sigma}}{1-1/\sigma}$

From Euler,

$$\frac{C_{s+1}}{C_s} = (1+r)^{\sigma}\beta^{\sigma}\left(\frac{P_s}{P_{s+1}}\right)^{\sigma}$$

From demand eqs, $C_T = \gamma C \left(\frac{1}{P}\right)^{-\theta}$

Therefore, $\dfrac{C_{T,s+1}}{C_{T,s}} = \left(\dfrac{P_s}{P_{s+1}}\right)^{\sigma-\theta}(1+r)^{\sigma}\beta^{\sigma}$

Sub into budget constraint,

$$\boxed{C_{T,t} = \frac{(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[Y_{T,s} - I_s - G_s\right]}{\sum \left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}\left(\frac{P_t}{P_s}\right)^{\sigma-\theta}}}$$

If P is constant, or $\sigma = \theta$, then presence of NT goods doesn't matter

More generally, the effects depend on the relative strengths of <u>intertemporal</u> and <u>intratemporal</u> substitution.

Suppose $P_t \uparrow$ (real appreciation), with P_s constant

1.) Consume more T goods (CA def) } θ is the elasticity

2.) r^c rises $\Rightarrow (c_T, c_N) \downarrow$
(rel. price of NT falls over time
rel. price of T rises over time)
(CA surp) } σ is the elasticity

CA surplus if $\sigma > \theta$

CA deficit if $\sigma < \theta$

Application

Can think of labor as NT good, and let leisure enter the utility function.

Temporary (positive) prod. shock

$\Longrightarrow$ 1.) $w \uparrow$ (relative price of NT $\uparrow$)

2.) Substitute from leisure to C_T

3.) However, $r^c \uparrow \Longrightarrow$ less current consumption

CA "improves" if $\sigma > \theta$

Without leisure/labor CA unambiguously improves if shock is purely temporary

Terms of Trade

$$\boxed{TOT = \frac{\text{Price of Exports}}{\text{Price of Imports}}}$$

Consider easy case first, where TOT is exogenous.

Assume country is specialized in "production" for Exportable good, but also wants to consume imports.

$$C = \left[\gamma^{1/\theta} C_M^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta} C_X^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

Let p = Price of exports in terms of imports (Imports are numeraire)

Get the same consumption-based price index P

Assume intl. bonds are denominated in real terms (indexed by P).

Budget Constraint,

$$\sum_{s=t}^{\infty} R_{t,s} C_s = (1+r_t) B_t + \sum_{s=t}^{\infty} R_{t,s} \left(\frac{P_s Y_s}{P_s} \right)$$

Assume: 1.) $r(1+\beta) = 1$

2.) Y_s constant

Then,

$$C_t = r B_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left(\frac{P_s}{P_s} \right) Y$$

The effect of ΔTOT on CA depends, as before, on whether it's temporary or permanent

a.) Permanent fall in TOT has no effect
($C \downarrow$ one-for-one)

b.) Temporary fall in TOT $\Longrightarrow$ CA deficit
(income temporarily low)

Endogenous TOT

Ricardian Model with a continuum of goods. (Dornbusch, Fischer, Samuelson (AER, 1977)).

Now we will see that tempory TOT↓ can be associated with CA <u>surpluses</u>.

<u>Intuition</u>: TOT decline caused by temporary positive productivity shock, which produces temporarily high income.

We will consider 2 cases:

a.) Frictionless goods markets (all goods traded)

b.) Trade costs (some goods Non-traded)
- Interaction between TOT and Real Ex. Rates.

DFS (1977)

Assumptions

1.) Continuum of goods, indexed by $z \in [0,1]$

2.) Two countries, One factor of production (Labor)

3.) Goods are costlessly tradeable (relax later)

4.) Labor is immobile across countries, but mobile within countries

5.) Countries have (exogenously) different labor productivit[y]

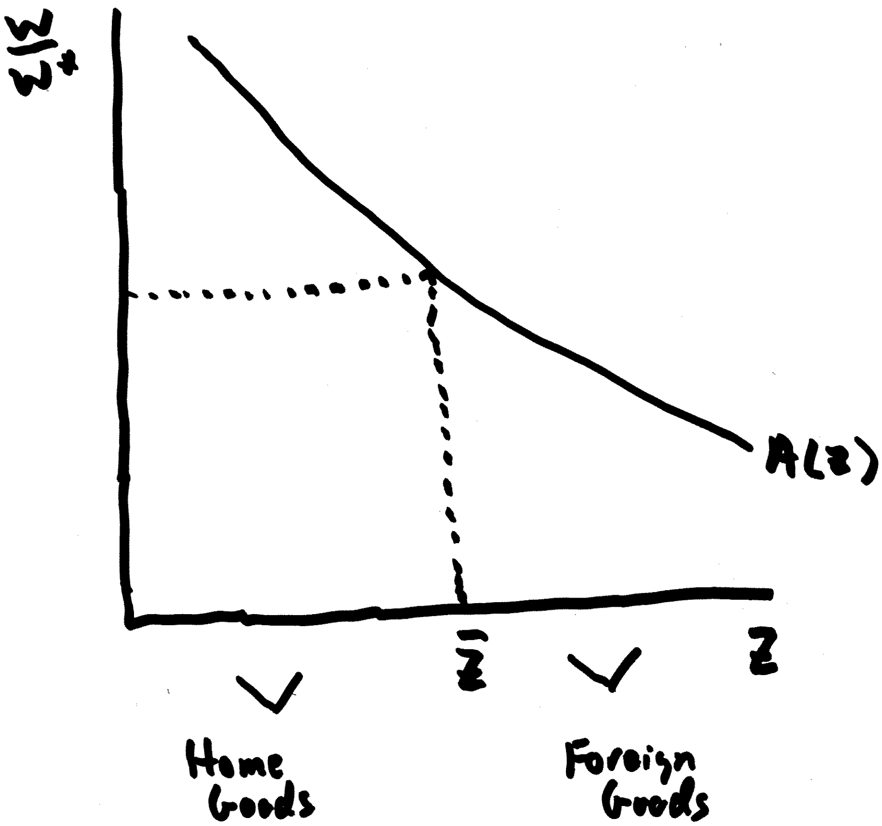
Let $a^*(z)$ = Foreign unit-labor requirement in industry z

$a(z)$ = Home unit-labor requirement in industry z

Define $A(z) = \dfrac{a^*(z)}{a(z)}$ = relative Home productivity

Label goods so that $A'(z) < 0$

Graphically,



<u>Given</u> relative wages, we can determine the production structure + the pattern of intl. specialization

Home Goods: $P^* > P \Rightarrow a^*(z)w^* > a(z)w$

$$\Rightarrow A(z) > w/w^*$$

To jointly determine $\bar{z}$ and w/w^*, we need to derive w/w^* as a function of $\bar{z}$ (relative wages as a function of the production structure).

Demand

Countries have identical homothetic preferences (intl. wealth distribution doesn't matter).

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \log(C_s)$$

$$C = \exp\left[\int_0^1 \log c(z) dz\right]$$

$$c(z) = \left[\frac{p(z)}{P}\right]^{-1} C$$

$$P = \exp\left[\int_0^1 \log p(z) dz\right]$$

Price index (measured in units of good 1)

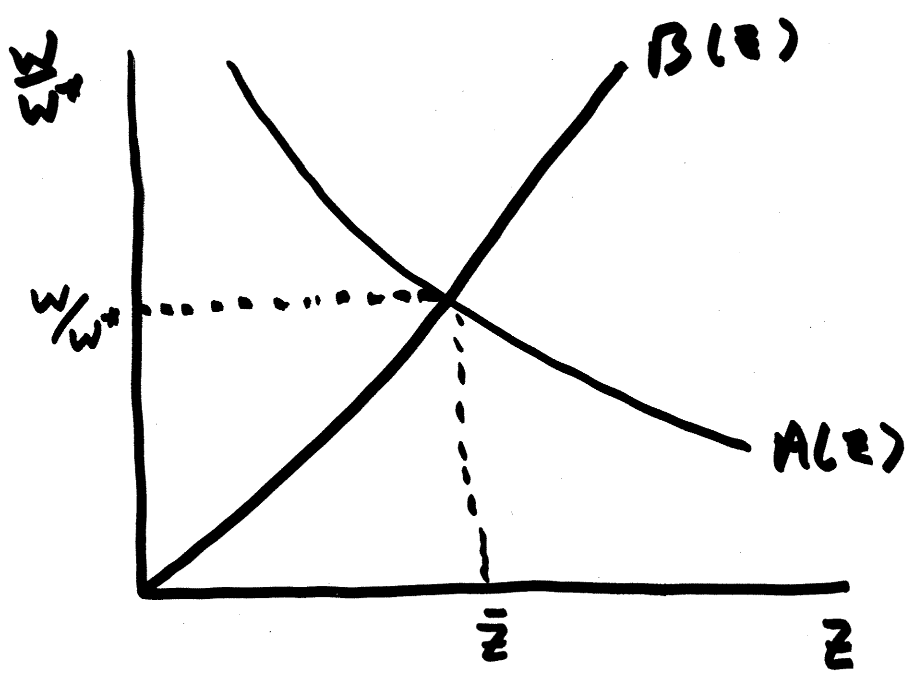
World Mkt. Clearing,

1.) $P(C + C^*) = wL + w^*L^*$

Home Mkt. Clearing,

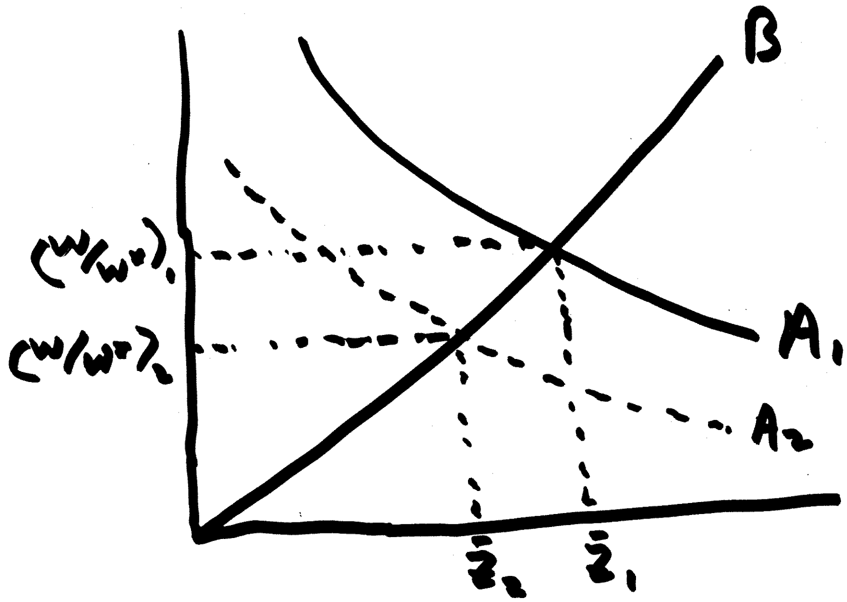
2.) $wL = \underbrace{zPC}_{\text{Home Demand for Home Goods}} + \underbrace{zPC^*}_{\text{Foreign Demand for Home Goods}} = z[wL + w^*L^*]$

Combine, $\dfrac{w}{w^*} = \dfrac{z}{1-z}\left(\dfrac{L^*}{L}\right)$ $\quad B(z; L^*/L)$



Now suppose $a^*(z) \downarrow$ (across the board)

$$a_2^*(z) = \frac{a_1^*(z)}{v} \qquad v > 1$$



1.) H loses industries to F

2.) H's relative wages fall (but by less than relative productivity).

However, H is better off! Why?

Note that H's real wages rise (for <u>all</u> goods)

There are 3 cases:

1.) Consistent H exports (left of $\bar{z}_2$)

$w_2/p_2 = 1/a(z) \implies$ doesn't change

2.) Consistent F exports (right of $\bar{z}_1$)

$$\frac{w_2/p_2}{w_1/p_1} = \frac{w_2/a_2^* w_2^*}{w_1/a_1^* w_1^*} = \frac{w_2/w_2^*}{w_1/w_1^*} \times \frac{a_1^*}{a_2^*} > 1$$

rel. wages fall less than productivity

3.) Switched Industries (New Imports)

- Before switch, H's real wage is just $1/a_1(z)$
- H only abandons an industry when $p_2(z) < w_2 a_1(z) \implies w_2/p_2 > 1/a_1 = w_1/p_1$

The <u>mechanism</u> by which real wages and welfare rise is an improvement in H's Terms of Trade

$$TOT = \frac{\text{Price of Exports}}{\text{Price of Imports}} = \frac{\frac{1}{\bar{z}}\int_0^{\bar{z}} p(z)\,dz}{\frac{1}{1-\bar{z}}\int_{\bar{z}}^{1} p(z)\,dz}$$

$$= \frac{\frac{w}{\bar{z}}\int_0^{\bar{z}} a(z)\,dz}{\frac{w^*}{1-\bar{z}}\int_{\bar{z}}^{1} a^*(z)} = \frac{L^*}{L}\,\frac{\int_0^{\bar{z}} a(z)\,dz}{\int_{\bar{z}}^{1} a^*(z)\,dz}$$

Note $a^* \downarrow$ and $\bar{z} \downarrow$

Can show the first effect dominates. $TOT \uparrow$