Topics

1.) Real vs. Nominal Returns
 - "Siegel's Paradox"

2.) Risk Premia in the FX Market
 - Fama (JME, 1984) Regressions

3.) General Equilibrium Models of the FX Risk Premium
 - Lucas (JME, 1982)
 - GMM Tests

4.) Higher-Order Beliefs
 - Bacchetta & VanWincoop (AER, 2006)

① Jensen's Inequality

Our log approx. actually implied

$$\Delta s_{t+1} = -\tfrac{1}{2}\, var_t(s_{t+1}) + (f_t - s_t) + \varepsilon_{t+1}$$

In principle, $var_t(s_{t+1})$ could be time-varying and correlated with $(f_t - s_t) \Rightarrow$ omitted variable bias.

Doesn't work empirically, however.

Interestingly, note that Jensen's $\neq$ implies that forward rates cannot simultaneously be unbiased in terms of both currencies.

Example:

Suppose $F_t = E_t S_{t+1}$ [Forward rate unbiased in terms of \$]

What about from euro perspective?

Define $\hat{F}_t = \frac{1}{F_t}$ and $\hat{S}_t = \frac{1}{S_t}$ $\left[\frac{\text{euro}}{\$}\right]$

Unbiasedness in terms of euro implies,

$$\hat{F}_t = E_t \hat{S}_{t+1} \Rightarrow \frac{1}{F_t} = E_t\left(1/S_{t+1}\right)$$

But Jensen's implies $E(1/S) > \frac{1}{E(S)}$

$\Rightarrow F_t < E_t S_{t+1}$! Can't have it both ways.

This is known as "Siegel's Paradox"

(2.) Real vs. Nominal Returns

Note, people care about real returns, not nominal returns.

Real return on Home bond : $(1+i_t)\frac{P_t}{P_{t+1}}$

Real return on Foreign bond : $(1+i_t^*)\frac{S_{t+1}}{S_t}\cdot\frac{P_t}{P_{t+1}}$

Euler Eqs.,

$$u'(C_t) = \beta E_t\left[u'(C_{t+1})(1+i_t)\frac{P_t}{P_{t+1}}\right]$$

$$u'(C_t) = \beta E_t\left[u'(C_{t+1})(1+i_t^*)\frac{S_{t+1}}{S_t}\cdot\frac{P_t}{P_{t+1}}\right]$$

Equating, and using CIP to sub-out $1+i_t^*$

$$\boxed{F_t = \frac{E_t\left[u'(C_{t+1})\frac{S_{t+1}}{P_{t+1}}\right]}{E_t\left[u'(C_{t+1})\frac{1}{P_{t+1}}\right]}}$$

With risk-neutrality, u' is constant,

$$F_t = \frac{E_t\left(\frac{S_{t+1}}{P_{t+1}}\right)}{E_t\left(1/P_{t+1}\right)} \neq E_t S_{t+1} \quad \text{if } P_{t+1} \text{ is random}$$

Note, if PPP holds, this provides an explanation of "Siegel's Paradox".

With PPP, $P_t = S_t P_t^* \implies \frac{S_{t+1}}{P_{t+1}} = \frac{1}{P^*_{t+1}}$

$$\frac{1}{P_{t+1}} = \frac{1}{S_{t+1} P^*_{t+1}} = \frac{\hat{S}_{t+1}}{P^*_{t+1}}$$

Therefore, if

$$F_t = \frac{E_t\left(\frac{S_{t+1}}{P_{t+1}}\right)}{E_t\left(1/P_{t+1}\right)}$$

Then,

$$\hat{F}_t = \frac{E_t\left(\frac{\hat{S}_{t+1}}{P^*_{t+1}}\right)}{E_t\left(1/P^*_{t+1}\right)}$$

That is, expected <u>real</u> profits are zero for both investors.

Of course, if PPP ~~is not~~ does not hold, then this need not be true, but then investors residing in different countries define "real" returns differently, so it is unsurprising that they evaluate UIP differently.

Suppose (s_{t+1}, p_{t+1}) are jointly log-Normal, then with risk-neutrality we get the log-linear expression,

$$f_t = E_t\, s_{t+1} + \tfrac{1}{2}\, var_t(s_{t+1}) - \underbrace{cov_t(s_{t+1}, p_{t+1})}_{\text{Jensen's inequality with real returns}}$$

Unfortunately, again this does not work empirically.

3. So let's now incorporate risk aversion,

Suppose $\quad u(\cdot) = \dfrac{C^{1-\rho}}{1-\rho} \qquad \rho = \text{CRRA}$

Plugging into our earlier expression,

$$F_t = \frac{E_t\left[S_{t+1}\dfrac{C_{t+1}^{-\rho}}{P_{t+1}}\right]}{E_t\left[\dfrac{C_{t+1}^{-\rho}}{P_{t+1}}\right]}$$

Take logs of both sides,

$$\boxed{f_t = E_t(s_{t+1}) + \tfrac{1}{2}\, var_t(s_{t+1}) - cov_t(s_{t+1}, p_{t+1}) - \underbrace{\rho\, cov_t(s_{t+1}, c_{t+1})}_{\text{risk premium}}}$$

Suppose $e_{t+1} \downarrow$ when $c_{t+1} \downarrow$

$\Rightarrow$ Low fx returns when consumption is low

$\Rightarrow$ fx is risky

$\Rightarrow f_t < E_t e_{t+1} \quad [\, i_t^* + E_t e_{t+1} - e_t > i_t \,]$

$E_t e_{t+1} - f_t$ is positive expected profit from buying fx forward.

Of course, $(s_{t+1}, p_{t+1}, c_{t+1})$ are all endogenous, so their correlations will depend on the correlation of exogenous shocks and the underlying asset market structure.

Consider the following complete markets benchmark,

$$\left.\begin{aligned} c_{t+1} &\sim y^{w}_{t+1} \\ p_{t+1} &\sim m_{t+1} - \rho\, y^{w}_{t+1} \\ s_{t+1} &\sim m_{t+1} - m^{*}_{t+1} \end{aligned}\right\} \text{[In Logs]}$$

$$\Rightarrow \operatorname{cov}_t(s_{t+1}, c_{t+1}) = \operatorname{cov}_t(m_{t+1} - m^{*}_{t+1}, y^{w}_{t+1})$$

What about the Jensen's Inequality term?

$$\frac{1}{2}\operatorname{var}_t(s_{t+1}) - \operatorname{cov}_t(s_{t+1}, p_{t+1}) = \frac{1}{2}\left[\operatorname{var}_t(m^{*}_{t+1}) - \operatorname{var}_t(m_{t+1})\right] + \rho \operatorname{cov}_t(m_{t+1} - m^{*}_{t+1}, y^{w}_{t+1})$$

[Assuming independent money supply processes]

Substituting in,

$$\boxed{f_t = E_t\, s_{t+1} + \frac{1}{2}\left[\operatorname{var}_t(m^{*}_{t+1}) - \operatorname{var}_t(m_{t+1})\right]}$$

2 comments:

1.) Bias in forward rate related to <u>difference</u> in relative (conditional) variances of national money supplies. Empirically, this is too small to account for observed bias.

2.) Bias here derives from Jensen's inequality, not risk aversion. When $var(m^*) > var(m)$, then foreign price level is more volatile. Ceteris paribus, this <u>raises</u> expected real profits on fx. To maintain equal expected real returns, must have $f_t > E_t e_{t+1}$ (i.e., lower expected nominal return on foreign asset).

Fama Regressions

Let's go back to the UIP regression,

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1}$$

Empirical evidence suggests $\beta < 0$. What does this imply about the nature of risk premia?

Note,

$$\text{plim}(\hat{\beta}) = \frac{\text{cov}_t(\Delta s_{t+1}, f - s_t)}{\text{var}_t(f_t - s_t)}$$

Define,

$$rp_t = f_t - E_t(s_{t+1})$$ } risk premium on \$
$rp > 0 \Rightarrow$ \$ is relatively risky

Therefore,

(1.) $$f_t - s_t = E_t(\Delta s_{t+1}) + rp_t$$

With Rational Expectations,

$$\text{cov}_t(f_t - s_t, \Delta s_{t+1}) = \text{cov}_t(f_t - s_t, E_t(\Delta s_{t+1}))$$

Using this along with eq. (1.)

$$\text{cov}_t(f_t - s_t, E_t \Delta s_{t+1}) = \text{var}_t[E_t(\Delta s_{t+1})] + \text{cov}_t(rp_t, E_t(\Delta s_{t+1}))$$

We can therefore conclude,

$$\boxed{\text{plim}(\hat{\beta}) < 0 \implies \text{cov}_t(rp_t, E_t(\Delta s_{t+1})) < 0}$$

In other words, The \$ gets riskier (on average) when it's expected to <u>appreciate</u>!

Next, multiply both sides of the $\text{plim}(\hat{\beta})$ eq. by $\text{var}(f_t - s_t)$,

$$\text{var}_t(f_t - s_t)\,\text{plim}(\hat{\beta}) = \text{var}_t[E_t(\Delta s_{t+1})] + \text{cov}_t(rp_t, E_t\Delta s_{t+1})$$

Sub-in for $\text{var}_t(f_t - s_t)$,

$$\{\text{var}_t[E_t(\Delta s_{t+1})] + 2\,\text{cov}_t(E_t\Delta s_{t+1}, rp_t) + \text{var}_t(rp_t)\}\,\text{plim}(\hat{\beta})$$

$$= \text{var}_t[E_t\Delta s_{t+1}] + \text{cov}_t(rp_t, E_t\Delta s_{t+1})$$

Note, as long as $\text{plim}(\hat{\beta}) < 1/2$,

$$\frac{1}{2}[\text{var}_t(E_t\Delta s_{t+1}) + \text{var}_t(rp_t)] > \text{var}_t(E_t\Delta s_{t+1})$$

$$\implies \boxed{\text{var}_t(rp_t) > \text{var}_t(E_t\Delta s_{t+1})}$$

Thus, risk premium is more volatile than (predictable component of) macro fundamentals!

Although Fama regressions provide useful information, "a picture is worth a thousand moments", so it is desirable to construct observable proxies for rp_t.

With Ratl. Expectations we have

$$f_t - s_{t+1} = rp_t - \varepsilon_{t+1}$$

Therefore, we can get estimates of the conditional mean of rp_t by running the following regressions,

$$f_t - s_{t+1} = Z_t' \beta + u_{t+1}$$

where the fitted value delivers a consistent estimate of the conditional mean

$$E(rp_t \mid Z_t) = Z_t' \hat{\beta}$$

Note, we get different measures, depending on our conditioning info. set.

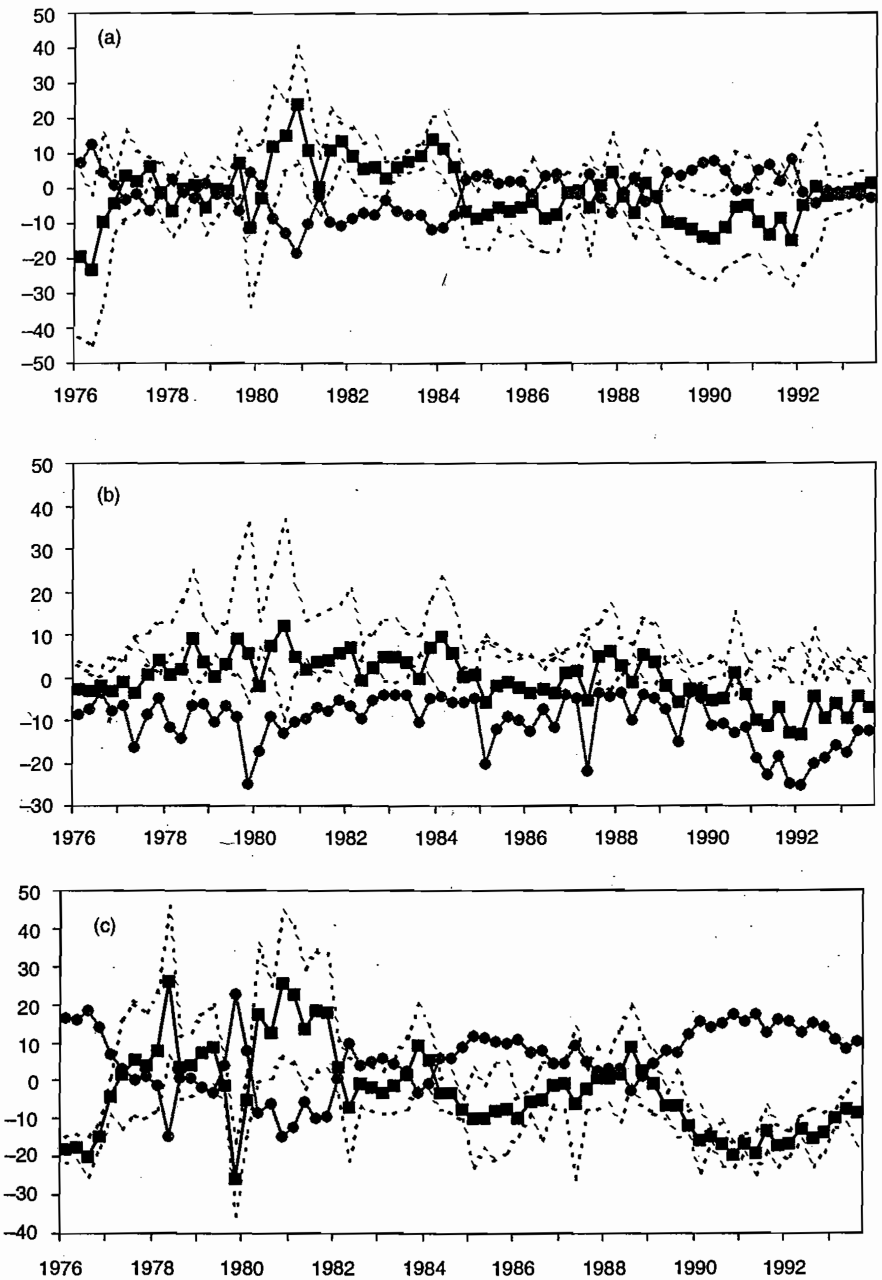

Figure 6.1 Time-series point estimates of p_t (boxes), plus and minus two standard errors (dotted) and point estimates of $E_t(\Delta s_{t+1})$ (circles). (a) US–UK; (b) US–Germany; (c) US–Japan.

General Equilibrium Models

There are as many different GE models of fx risk premia as there are GE theories of money. (Note: Arrow-Debreu has no role for money!)

Money Demand Models

1.) Money-in-Utility

2.) Cash-in-Advance

3.) Overlapping Generations

4.) Shopping time

5.) Search/Matching

Unfortunately, none of these is entirely satisfactory, so practitioners & empirical researchers must adopt an eclectic approach.

Despite its shortcomings (e.g., it is not entirely immune to the Lucas Critique!), the Lucas (JME, 1982) cash-in-advance model is one of the most widely studied models, so it is a good place to start.

Lucas (1982)

Assumptions

1.) 2 countries (H + F), each with a rep. agent.
H + F residents have identical preferences.

2.) 2 goods (X and y). Exogenous production
(Endowment Economy).

Countries receive endowments of different goods.

3.) 2 monies (M + N).
H agents receive transfers of M : $M_t = \lambda_t M_{t-1}$
F agents receive transfers of N : $N_t = \lambda_t^* N_{t-1}$
(λ_t and λ_t^* are, in general, random).

4.) Cash-in-Advance.
X must be purchased with M
y " " " N

5.) Timing
1.) Period-t shocks realized
2.) Period-t asset markets open
3.) Shoppers buy goods. Producers sell output.

Objective Function

$$\max E_t \sum_{j=0}^{\infty} \beta^j U(C_{x,t+j}, C_{y,t+j})$$

s.t.

1.) $P_t C_{x,t} \leq m_t$

2.) $P_t^* C_{y,t} \leq n_t$

Key Point

As long as $i, i^* > 0$, Cash-in-Advance constraint always binds, e.g., $P_t C_{x,t} = m_t$ (Depends on timing assumptions)

3.) $$\omega_{x,t} e_t + \omega_{y,t} e_t^* + \psi_{M,t} r_t + \psi_{N,t} r_t^* + \frac{M_t}{P_t} + \frac{n_t S_t}{P_t}$$

$$\leq \frac{P_{t-1}}{P_t} \omega_{x,t-1} X_{t-1} + \frac{S_t P_{t-1}^*}{P_t} \omega_{y,t-1} Y_{t-1} + \frac{\psi_{M,t-1} \Delta M_t}{P_t}$$

$$+ \frac{\psi_{N,t-1} \Delta N_{t-1}}{P_t} + \omega_{x,t-1} e_t + \omega_{y,t-1} e_t^* + \psi_{M,t-1} r_t + \psi_{N,t-1} r_t^*$$

Where,

ω_x : shares of domestic firm

ω_y = " " foreign firm

e_t = price of Home equity

e_t^* = " " Foreign equity

ψ_M = shares in domestic money supply process

ψ_N = " " Foreign " " "

r_t = price of claim to future H money transfers

r_t^* = " " " " " F " "

FOCs

$$c_{y,t} : \quad \frac{S_t P_t^*}{P_t} U_1(t) = U_2(t)$$

$$w_{x,t} : \quad e_t U_1(t) = \beta E_t\left[U_1(t+1)\left(\frac{P_t}{P_{t+1}} x_t + e_{t+1}\right)\right]$$

$$w_{y,t} : \quad e_t^* U_1(t) = \beta E_t\left[U_1(t+1)\left(\frac{S_{t+1} P_t^*}{P_{t+1}} y_t + e^*_{t+1}\right)\right]$$

$$\psi_{M,t} : \quad r_t U_1(t) = \beta E_t\left[U_1(t+1)\left(\frac{\Delta M_{t+1}}{P_{t+1}} + r_{t+1}\right)\right]$$

$$\psi_{N,t} : \quad r_t^* U_1(t) = \beta E_t\left[U_1(t+1)\left(\frac{\Delta N_{t+1} S_{t+1}}{P_{t+1}} + r^*_{t+1}\right)\right]$$

Market Clearing

$$\psi_{M,t} + \psi^*_{M,t} = 1$$

$$\psi_{N,t} + \psi^*_{N,t} = 1$$

$$w_{x,t} + w^*_{x,t} = 1$$

$$w_{y,t} + w^*_{y,t} = 1$$

$$\left.\begin{aligned} c_{x,t} + c^*_{x,t} &= x_t \\ c_{y,t} + c^*_{y,t} &= y_t \\ m_t + m_t^* &= M_t \\ n_t + n_t^* &= N_t \end{aligned}\right\} \xRightarrow{\text{Use CIA constraint}} \begin{aligned} M_t &= P_t x_t \\ N_t &= P_t^* y_t \end{aligned} \quad \left(\begin{matrix}\text{Simple} \\ \text{Quantity} \\ \text{Theory}\end{matrix}\right)$$

Key Point

Rather than attempt to solve the system of FOCs directly, we're going to follow Lucas's "guess & verify" strategy. That is, we're going to guess an equilibrium allocation, and then verify that it solves the FOCs. This is much simpler!

Conjectured Perfectly Pooled Equilibrium

$$\omega_{x,t} = \omega^*_{x,t} = \omega_{y,t} = \omega^*_{y,t} = \psi_{M,t} = \psi^*_{M,t} = \psi_{N,t} = \psi^*_{N,t} = 1/2$$

$$\Longrightarrow C_{x,t} = C^*_{x,t} = \tfrac{1}{2} X_t \qquad C_{y,t} = C^*_{y,t} = \tfrac{1}{2} Y_t$$

That the above conjectured portfolios support the conjectured consumption allocations is pretty obvious. The fact that each country consumes one-half of the endowments is based on the implicit assumption of equal initial wealth.

The only subtlety is that we earlier assumed the CIA constraints always were binding. At this point, we should go back and veryfy that nominal interest rates are always positive. This will place restrictions on the permissable money & output processes.

Let's now use these results to solve for the exchange rate.

From the FOCs,

$$\frac{U_2(t)}{U_1(t)} = \frac{S_t P_t^*}{P_t} = \frac{N_t}{M_t} \cdot \frac{X_t}{Y_t} \cdot S_t$$

$$\Rightarrow \boxed{S_t = \frac{U_2(\tfrac{1}{2}X_t, \tfrac{1}{2}Y_t)}{U_1(\tfrac{1}{2}X_t, \tfrac{1}{2}Y_t)} \cdot \frac{M_t}{N_t} \cdot \frac{Y_t}{X_t}}$$

Suppose, $U = \dfrac{C^{1-\rho}}{1-\rho}$

where $C = \left[\gamma^{1/\theta} X^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} Y^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$

θ = elast. of subst.

Subbing in,

$$S_t = \left(\frac{1-\gamma}{\gamma}\right)^{1/\theta} \left(\frac{X_t}{Y_t}\right)^{\frac{1-\theta}{\theta}} \frac{M_t}{N_t}$$

1.) $\underline{\theta < 1 \text{ (complements)}}$: Home currency depreciates when Home output rises, i.e., strong ~~#~~adverse TOT effect.

2.) $\underline{\theta > 1 \text{ (substitutes)}}$: Home currency appreciates when Home output rises.

3.) <u>$\theta = 1$ (log util.)</u> : Nominal exchange rate is independent from relative output levels.

<u>Exercise</u> : Show that when $\theta = 1$ the consumption allocations are supported under financial autarky (i.e., there is no need to trade claims on each other's output processes)! What is the intuition? (See Cole + Obstfeld (1991) for more discussion).

What about the fx risk premium?

This will depend on the stochastic specification of the money + output processes, along with the degree of risk aversion.

The following plots are based on a calibrated Markov chain for the joint money / output processes, and a utility function with $\theta = 1$, $\rho = 10$.

Interestingly, the model can generate a negative UIP regression coefficient, although it fails on other dimensions.

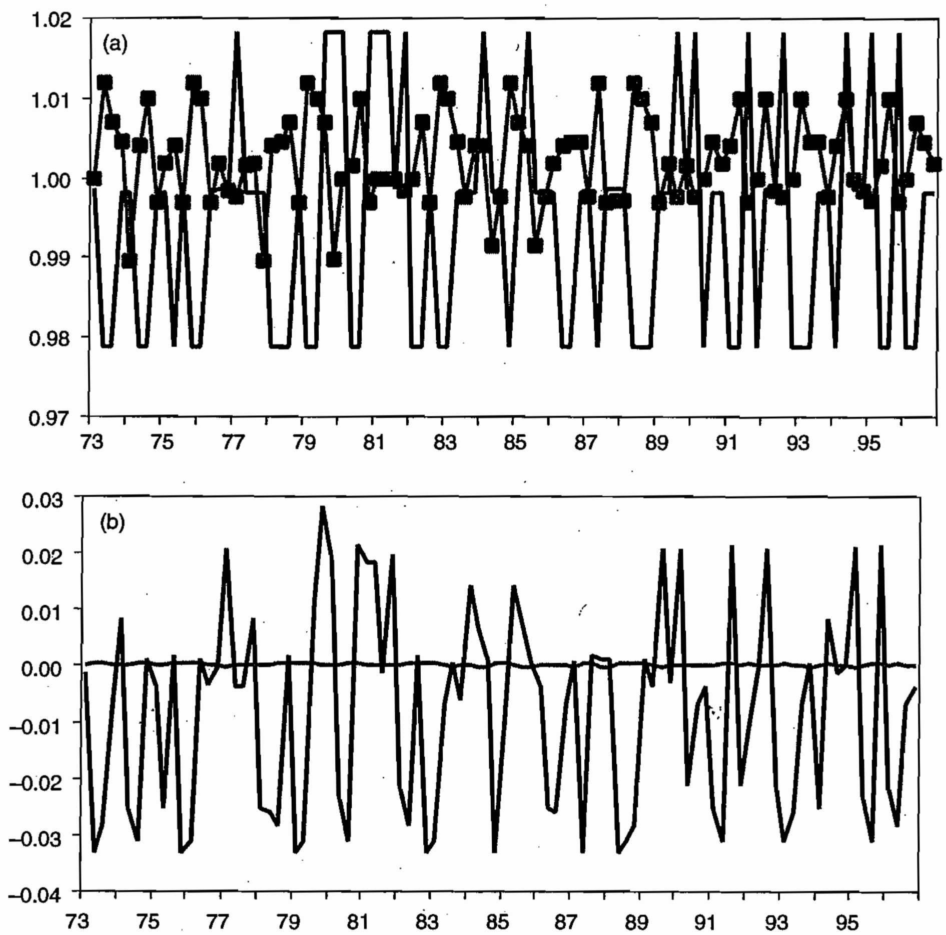

Figure 4.1 Simulated observations from the Lucas model. (a) The implied gross one-period-ahead change in nominal exchange rate S_{t+1}/S_t and current forward premium F_t/S_t (in boxes). (b) The implied *ex post* forward payoff $(S_{t+1} - F_t)/S_t$ (jagged line) and risk premium $E_t(S_{t+1} - F_t)/S_t$ (smooth line).

Table 4.2 Measured and implied moments, US–Germany

		Volatility			*Autocorrelation*		
	Slope	S_{t+1}/S_t	F_t/S_t	$(F_t - S_{t+1})/S_t$	S_{t+1}/S_t	F_t/S_t	$(F_t - S_{t+1})/S_t$
Data	−0.293	0.060	0.008	0.061	0.007	0.888	0.026
Model	−1.444	0.014	0.006	0.029	0.105	0.006	0.628

Model values generated with $\gamma = 10$, $\theta = 0.5$.

GMM Tests

A potential problem with GE models of fx risk premia is their reliance on specific specifications for exogenous stochastic processes. Misspecification of these processes could produce misleading results.

A more robust (but less powerful) approach is to examine the validity of a model's FOCs, without solving for a full competitive equilibrium. Rejection of a model's FOCs implies rejection of the model. This approach leads to so-called GMM (Generalized Method of Moments) tests.

A wide class of fx risk premium models feature the following FOC:

$$E_t\left\{U'(C_{t+1})\left[\frac{F_t - S_{t+1}}{P_{t+1}}\right]\right\} = 0$$

or, with $U(\cdot) = \frac{1}{1-\rho}C^{1-\rho}$,

$$E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left[\frac{F_t - S_{t+1}}{P_{t+1}}\right]\right\} = 0$$

Given data on (C_t, F_t, S_t), and a set of instruments (variables in the time-t info set), this orthogonality restriction can be tested using the procedures in Hansen's

__Findings__

1.) Over-identifying restrictions rejected

2.) $\hat{\rho}$ very big

__Basic Problem__

C_t is too smooth to account for volatility of S_t

Higher-Order Beliefs

A Key implicit assumption thus far has been homogeneous expectations, either due to homogeneous info. sets, or a fully revealing Ratl. Expectations Equilibrium. What if this isn't the case?

Suppose agents receive private signals about future fundamentals.

Warning: Info. Heterogeneity $\not\Rightarrow$ Belief Heterogeneity, since asset prices potentially reveal private info.

Consider the following simple monetary model, extended to allow for heterogeneous beliefs.

$$m_t - p_t = -\alpha i_t$$

$$p_t = s_t + p_t^*$$

$$i_t = i_t^* + \bar{E}_t s_{t+1} - s_t + \chi_t$$

Aggregate Hedging Demand

where $\bar{E}_t s_{t+1} = \int_0^1 E_t^i s_{t+1} \, di$

Average (first-order) Beliefs

Define,

$$\bar{E}_t^K(f_{t+K}) = \bar{E}_t \bar{E}_{t+1} \cdots \bar{E}_{t+K-1}(f_{t+K})$$

Then we can write the solution as,

$$s_t = \frac{1}{1+\alpha} \sum_{K=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^K \bar{E}_t^K (f_{t+K} + x_{t+K})$$

Key Point

$$\bar{E}_t \bar{E}_{t+1} \cdots \bar{E}_{t+K-1}(f_{t+K}) \neq \bar{E}_t f_{t+K}$$

Although it is rational for an individual not to expect to revise his future beliefs, this "law of iterated expectations" does not apply to the average beliefs operator. That is, it is quite rational for individuals to expect average opinion to change!

The basic problem now is that we confront an infinite-dimensional fixed point problem. The ex. rate depends on a cascade of higher order beliefs, but these beliefs in turn depend on the info. revealed by the ex. rate.

Bacchetta + Van Wincoop (AER, 2006) construct a truncation solution strategy. Kasa, Walker + Whiteman (2007) show how to solve this problem without truncation.