Crises and Prices: Information Aggregation, Multiplicity, and Volatility

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Crises are volatile times when endogenous sources of information are closely monitored. We study the role of information in crises by introducing a financial market in a coordination game with imperfect information. The asset price aggregates dispersed private information acting as a public noisy signal. In contrast to the case with exogenous information, our main result is that uniqueness may not obtain as a perturbation from perfect information: multiplicity is ensured with small noise. In addition, we show that: (a) multiplicity may emerge in the financial price itself; (b) less noise may contribute toward nonfundamental volatility even when the equilibrium is unique; and (c) similar results obtain for a model where individuals observe one another's actions, highlighting the importance of endogenous information more generally. (*JEL* D53, D82, D83)

It's a love-hate relationship—economists are at once fascinated and uncomfortable with multiple equilibria. On the one hand, crises can be described as times of high nonfundamental volatility: they involve large and abrupt changes in outcomes, but often lack obvious comparable changes in fundamentals. Many attribute an important role to more or less arbitrary shifts in "market sentiments" or "animal spirits," and models with multiple equilibria formalize these ideas.[1] On the other hand, these models can also be viewed as incomplete theories, which should ultimately be extended along some dimension to resolve the indeterminacy. Stephen Morris and Hyun Song Shin (1998, 2001) argue that this dimension is information, and that multiplicity vanishes once the economy is perturbed away from the perfect-information benchmark.

This result is obtained with an exogenous information structure, but information is largely endogenous in most situations of interest. Financial prices and macroeconomic indicators convey information about what others are doing and thinking. These variables are monitored intensely during times of crisis and appear to be an important part of the phenomena. As an example, consider the Argentine 2001–2002

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[1] Applications range from bank runs, currency attacks, debt crises, and financial crashes to riots, revolutions, and social change. See, for example, Douglas W. Diamond and Philip H. Dybvig (1983); Maurice Obstfeld (1986, 1996); Guillermo A. Calvo (1988); Russell Cooper and Andrew John (1988); Harold L. Cole and Timothy J. Kehoe (1996); Andrés Velasco (1996). Cooper (1998) provides excellent review.

crisis, which included devaluation of the peso, default on sovereign debt, and suspension of bank payments. Leading up to the crisis, the peso-forward rate and bank deposits deteriorated steadily throughout 2001. This was widely reported by news media and investor reports, and closely watched by people making important economic decisions.

The aim of this paper is to understand the role of endogenous information in crises. We focus on two distinct forms of nonfundamental volatility. First, we investigate the existence of multiple equilibria, since sunspots could then create volatility unrelated to fundamentals. Second, for situations with a unique equilibrium, we examine the sensitivity of outcomes to nonfundamental disturbances, namely, aggregate noise in public sources of information. We argue that endogenizing public information is crucial for understanding both sources of volatility.

The backbone of our model is the coordination game that Morris-Shin and others have used to capture applications such as currency crises, bank runs, and financial crashes. We introduce a financial market where individuals trade using their private information. The rational-expectations equilibrium price aggregates disperse private information, while avoiding perfect revelation due to unobservable supply shocks as in Sanford J. Grossman and Joseph E. Stiglitz (1976). This price is our endogenous public signal.

The main insight to emerge is that the precision of *endogenous* public information increases with the precision of *exogenous* private information. When private signals are more precise, individuals' asset demands are more sensitive to their information. As a result, the equilibrium price reacts relatively more to fundamental than to nonfundamental variables, conveying more precise public information.

This has important implications for the determinacy of equilibria. The endogenous increase in the precision of public information permits agents to better forecast one another's actions and thereby makes it easier to coordinate. Consequently, uniqueness need not obtain as a perturbation away from the perfect-information benchmark. Indeed, in our baseline model, multiplicity is *ensured* when noise is small.

This result is illustrated in Figure 1, which displays the regions of uniqueness and multiplicity in the exogenous parameter space of our model. On the vertical axis, σ_x represents the noise in private information; on the horizontal axis, σ_ε represents the noise in the aggregation process, namely, the randomness in asset supply. Multiplicity obtains when either σ_x or σ_ε is sufficiently small.

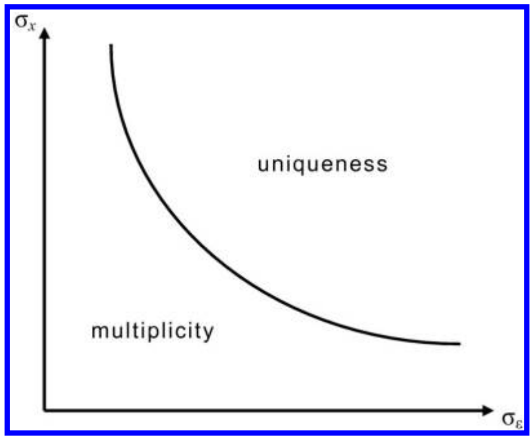

FIGURE 1. REGIONS OF MULTIPLICITY AND UNIQUENESS

Note: σ_x measures the exogenous noise in private information, and σ_ε the exogenous noise in the aggregation of information.

In our baseline model, the asset's dividend depends merely on the exogenous fundamentals. The financial market then provides information relevant for the coordination game, but there is no feedback in the opposite direction. In an extension, we allow for such a feedback by considering the possibility that the dividend depends on the outcome of the coordination game. This may capture, in a stylized fashion, the real rate of return on peso-forwards during currency attacks, or more generally stock-market returns during economic crises. Interestingly, multiplicity then emerges in the equilibrium price.

This is easily explained. In equilibrium, the price affects the coordination outcome; the outcome in turn affects the dividend; hence, the dividend itself is a function of the price. Since a higher price can lead to a higher dividend, the demand for the asset is backward bending, giving rise to multiple intersections with supply.

Motivated by bank runs and riots, we also consider a model where individuals do not trade a financial asset, but instead directly watch over what others are doing: everyone observes a noisy signal of the average action in the population. This introduces endogenous public information in the Morris-Shin framework parsimoniously, without the need for modeling a financial market. It also brings a main element of herding models, the observation of other players' actions, into coordination games. Of course, this framework cannot address multiplicity or volatility in the financial market. However, our results on regime-outcome multiplicity carry over here, highlighting information aggregation as the key mechanism, not any particular form of it.

Results on multiplicity are of interest because nonfundamental volatility may arise if agents use sunspots to coordinate on different equilibria. Our results are not limited, however, to an interpretation of crises as situations with multiple equilibria. We show that a reduction in noise can increase the sensitivity of outcomes to nonfundamental disturbances, thus contributing to volatility, even when the equilibrium is unique.

Related Literature.—Our analysis builds on Morris and Shin (1998, 2001, 2003), underscoring their general theme that the information structure is crucial in coordination games. We also share with V. V. Chari and Kehoe (2003) the perspective that the distinctive feature of crises is nonfundamental volatility, although we focus on the interplay of information and coordination rather than herding.

Andrew Atkeson (2001), in his discussion of Morris and Shin (2001), was the first to highlight the potential role of financial markets as endogenous sources of public information. He noted that fully revealing prices could restore common knowledge. By introducing noise in the aggregation process, we ensure that none of our results is driven by restoring common knowledge.

Closely related is Christian Hellwig et al. (2006), who endogenize interest rates in a currency-crises model. Their model also features information aggregation, but they focus on how the determinacy of equilibria depends on whether the central bank's decision to devalue is triggered by large reserve losses or high interest rates. Nikola A. Tarashev (2003) considers a similar application, but focuses on conditions that deliver a unique equilibrium.

The information structure is also endogenous in Angeletos et al. (forthcoming, 2006), but in different ways. They examine, respectively, signaling effects in a policy game and the interplay between information and crises in a dynamic setting. Amil Dasgupta (forthcoming) introduces signals of others' actions in an investment game, as in Section IV of this paper, but assumes that these signals are entirely private instead of public, thus abstracting from the role of endogenous information for multiplicity of equilibria.

Finally, our paper contributes to models of finance with rational expectations by introducing a coordinating role for prices. In this literature, prices provide information only regarding exogenous dividends. In contrast, in our framework, prices are also useful for predicting one another's actions and hence affect coordination. This novel coordinating role is crucial for our results on price multiplicity and price volatility, offering an entirely different mechanism from those in Gerard Gennotte and Hayne Leland (1990) and Gadi Barlevy and Pietro Veronesi (2003).[2]

The rest of the paper is organized as follows. Section I introduces the basic model and reviews the exogenous information benchmark. Section II incorporates an asset market and examines how the information revealed by prices affects the determinacy of equilibria. Section III examines multiplicity in the price. Section IV studies the model with direct signals on actions. Section V considers the comparative statics of equilibrium volatility in regions with a unique equilibrium. Section VI concludes.

I. The Basic Model: Exogenous Information

Before introducing a financial price or other endogenous public signals, we briefly review the backbone of our model with exogenous information, as in Morris and Shin (2000, 2004).

[2] See Section III for a more detailed discussion.

A. *Setup*

Actions and Payoffs.—There is a status quo and a measure-one continuum of agents, indexed by $i \in [0, 1]$. Each agent i can choose between two actions, either attack the status quo $a_i = 1$, or not attack $a_i = 0$. The payoff from not attacking is normalized to zero. The payoff from attacking is $1 - c$ if the status quo is abandoned and $-c$ otherwise, where $c \in (0, 1)$ parameterizes the cost of attacking. The status quo, in turn, is abandoned if and only if $A > \theta$, where A denotes the mass of agents attacking and θ is the exogenous fundamental representing the strength of the status quo. It follows that the payoff of agent i is

$$U(a_i, A, \theta) = a_i(\mathbf{1}_{A>\theta} - c),$$

where $\mathbf{1}_{A>\theta}$ is an indicator of regime change, equal to 1 if $A > \theta$, and 0 otherwise.

Our normalization that $U(0, A, \theta) = 0$ is irrelevant for equilibrium behavior, and hence for our positive results.[3] The key property of the payoff structure is a coordination motive: $U(1, A, \theta) - U(0, A, \theta)$ increases with A, so the incentive to attack increases with the mass of agents attacking. If θ were commonly observed by all agents, both $A = 1$ and $A = 0$ would be an equilibrium whenever $\theta \in (\underline{\theta}, \overline{\theta}] \equiv (0, 1]$. This interval represents the critical range of fundamentals over which the regime outcome depends on the size of the attack.

Interpretations.—In models of self-fulfilling currency crises, as in Obstfeld (1986, 1996) and Morris and Shin (1998), the central bank is forced to abandon its peg when a sufficiently large group of speculators attacks the currency; θ then parameterizes the amount of foreign reserves or the ability and willingness of the central bank to maintain its peg. In models of bank runs, such as Itay Goldstein and Ady Pauzner (2005) and Jean-Charles Rochet and Xavier Vives (2004), regime change occurs when a large enough number of depositors decide to withdraw their deposits, forcing the banking system to suspend payments. Another possible interpretation is an economy with investment complementarities, as in Cooper and John (1988), Christopher Chamley (1999), and Dasgupta (forthcoming).[4]

Information.—Following Morris-Shin, information is assumed to be imperfect and asymmetric, so that θ is not common knowledge. In the beginning of the game, nature draws θ from a given distribution, which constitutes the agents' common prior about θ. For simplicity, the prior is taken to be the improper uniform over the entire real line. Agent i then receives a private signal $x_i = \theta + \sigma_x \xi_i$, where $\sigma_x > 0$ and $\xi_i \sim \mathcal{N}(0, 1)$ is independent of θ and independently distributed across agents. Agents also observe an *exogenous* public signal $z = \theta + \sigma_z \varepsilon$, where $\sigma_z > 0$ and $\varepsilon \sim \mathcal{N}(0, 1)$ is common noise, independent of both θ and ξ.[5] The information structure is parameterized by the standard deviations σ_x and σ_z, or, equivalently, by $\alpha_x = \sigma_x^{-2}$ and $\alpha_z = \sigma_z^{-2}$, the precisions of private and public information.

B. *Equilibrium Analysis*

Throughout the paper, we focus on *monotone equilibria,* defined as perfect Bayesian equilibria such that, for a given realization z of the public signal, an agent attacks if and only if the realization x of his private signal is less than some threshold $x^*(z)$.[6]

In such an equilibrium, the aggregate size of the attack is

$$A(\theta, z) = \Pr(x < x^*(z)|\theta) = \Phi\left(\sqrt{\alpha_x}(x^*(z) - \theta)\right),$$

where Φ denotes the cumulative distribution function for the standard normal. The status quo

[3] In contrast, welfare analyses would be sensitive to the specification of $U(0, A, \theta)$. One must then take a stand depending on the application. For example, one may wish to assume that $U(0, A, \theta)$ depends on A and θ to capture the idea that crises are undesirable.

[4] Other applications include debt crises, financial crashes, and revolutions (Cole and Kehoe, 1996; Atkeson, 2001; Morris and Shin, 2004; Giancarlo Corsetti, 2006; Chris Edmond, 2005).

[5] Normality makes the analysis of the effects of public information tractable (see Morris and Shin, 1999, 2001, 2003, 2004).

[6] Our main results concerning multiple equilibria are obtained even within this restricted class. Moreover, with exogenous information, uniqueness within this class implies overall uniqueness.

is then abandoned if and only if $\theta \leq \theta^*(z)$, where $\theta^*(z)$ solves $A(\theta, z) = \theta$, or equivalently

$$(1)\quad x^*(z) = \theta^*(z) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(z)).$$

It follows that the expected payoff from attacking is $\Pr(\theta \leq \theta^*(z)|x, z) - c$, and therefore $x^*(z)$ must solve the indifference condition $\Pr(\theta \leq \theta^*(z)|x, z) = c$. Since posteriors about θ are normally distributed with mean $(\alpha_x/\alpha_x + \alpha_z)x + (\alpha_z/\alpha_x + \alpha_z)z$ and precision $\alpha = \alpha_x + \alpha_z$, this indifference condition is

$$(2)\quad \Phi\left(\sqrt{\alpha_x + \alpha_z}\left(\theta^*(z) - \frac{\alpha_x}{\alpha_x + \alpha_z}x^*(z) - \frac{\alpha_z}{\alpha_x + \alpha_z}z\right)\right) = c.$$

Hence, an equilibrium is simply identified with a joint solution to (1) and (2).

Substituting (1) into (2) gives a single equation in θ^*:

$$(3)\quad -\frac{\alpha_z}{\sqrt{\alpha_x}}\theta^* + \Phi^{-1}(\theta^*) = \sqrt{1 + \frac{\alpha_z}{\alpha_x}}\Phi^{-1}(1 - c) - \frac{\alpha_z}{\sqrt{\alpha_x}}z.$$

It is easy to check that this equation always admits a solution and that the solution is unique for every z if and only if $\alpha_z/\sqrt{\alpha_x} \leq \sqrt{2\pi}$, which proves the following result.

PROPOSITION 1 (Morris-Shin): *In the game with exogenous information, the equilibrium is unique if and only if private noise is small relative public noise, so that* $0 < \sigma_x \leq \sigma_z^2\sqrt{2\pi}$.

Figure 2 depicts the regions of (σ_x, σ_z) for which the equilibrium is unique. For any positive level of noise in the public signal ($\sigma_z > 0$), uniqueness in ensured by sufficiently small noise in the private signal (sufficiently small σ_x). The key intuition behind this result is that private information anchors individual behavior and limits the ability to forecast one another's actions. The more precise is private information

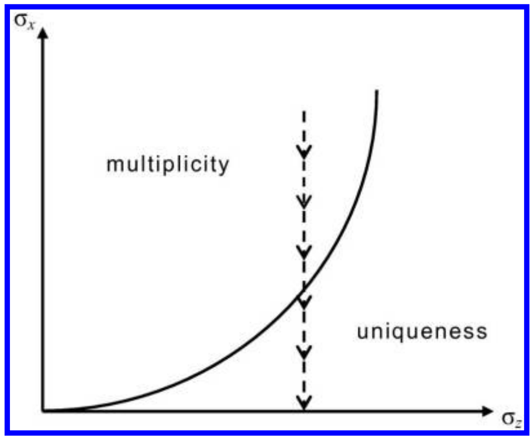

FIGURE 2. EXOGENOUS INFORMATION

Note: σ_x and σ_z parametrize the noise in private and public information; uniqueness is ensured for σ_x small enough.

relative to public information (the lower is σ_x relative to σ_z), the more heavily individuals use their private information. Since private information is diverse, this makes it more difficult for individuals to predict the actions of others, heightening strategic uncertainty. When this effect is strong enough, multiplicity breaks down.

Moreover, as private information becomes arbitrarily precise (as $\sigma_x \to 0$) individuals cease to use the public signal, and hence the equilibrium dependence on the common noise ε vanishes. Indeed, letting $R(\theta, \varepsilon) = \mathbf{1}_{A(\theta,\theta+\sigma_z\varepsilon)>\theta}$ denote the equilibrium regime outcome as a function of the fundamental θ and the nonfundamental disturbance ε, the following limit result holds.

PROPOSITION 2 (Morris-Shin Limit): *In the game with exogenous information, as private noise vanishes so that* $\sigma_x \to 0$, *there is a unique equilibrium in which the dependence of the regime outcome on the nonfundamental variable* ε *vanishes:* $R(\theta, \varepsilon) \to 1$ *if* $\theta < \hat{\theta}$ *and* $R(\theta, \varepsilon) \to 0$ *if* $\theta > \hat{\theta}$, *where* $\hat{\theta} = 1 - c$.

PROOF.
See Appendix.

This limit illustrates a sharp discontinuity of the equilibrium set around perfect information. When information is perfect ($\sigma_x = 0$), any regime outcome is possible for $\theta \in (\underline{\theta}, \bar{\theta}] \equiv (0, 1]$; but an arbitrarily small perturbation away from perfect information (any $\sigma_x > 0$) suffices for the regime outcome to be uniquely pinned down. By implication, crises—defined as situations displaying high nonfundamental volatility—cannot be addressed in the limit as private information becomes arbitrarily precise (as $\sigma_x \to 0$) since then the regime outcome is dictated only by the fundamental θ.

II. Financial Markets: Endogenous Information

The results above presume that the precision of public information remains invariant while varying the precision of private information. We argue that this is unlikely to be the case when public information is endogenous through prices or other macroeconomic indicators.

To investigate the role of prices, we introduce a financial market where agents trade an asset prior to playing the coordination game. Because the dividend depends on the underlying fundamentals or the aggregate attack, the equilibrium price will convey information that is valuable in the coordination game.

A. *Setup*

As before, nature draws θ from an improper uniform distribution over the real line, and each agent receives the exogenous private signal $x_i = \theta + \sigma_x \xi_i$. We avoid direct payoff linkages between the financial market and the coordination game to isolate the role of information aggregation. Agents can be seen as interacting in two separate stages.

In the first stage agents trade over a risky asset with dividend f at price p. We adopt the convenient CARA-normal specification introduced by Grossman and Stiglitz (1976). The utility of agent i is $V(w_i) = -e^{-\gamma w_i}$ for $\gamma > 0$, where $w_i = w_0 - pk_i + fk_i$ is final wealth, w_0 is initial endowed wealth, and k_i investment in the asset. The supply of the asset is uncertain and not observed, given by $K^s(\varepsilon) = \sigma_\varepsilon \varepsilon$, where $\sigma_\varepsilon > 0$ and $\varepsilon \sim \mathcal{N}(0, 1)$ and independent of θ and ξ_i. The role of the unobserved shock ε is to introduce noise in the information revealed by the market-clearing price. In this way, σ_ε parameterizes the exogenous noise in the aggregation process.

The second stage is essentially the same as the benchmark model of the previous section: agents choose whether to attack or not; the status quo is abandoned if and only if the mass of agents attacking, A, exceeds θ; and the payoff from this stage is $U(a_i, A, \theta) = a_i(\mathbf{1}_{A>\theta} - c)$. The only difference is that agents now observe the price that cleared the financial market in stage 1. The regime outcome, the asset's dividend, and the payoffs from both stages are realized at the end of stage 2.

Individual asset demand and attack decisions are functions of x and p, the realizations of the private signal and the price. The corresponding aggregates are then functions of θ and p. We define an equilibrium as follows.

DEFINITION: *An equilibrium is a price function, $P(\theta, \varepsilon)$, individual strategies for investment and attacking, $k(x, p)$ and $a(x, p)$, and their corresponding aggregates, $K(\theta, p)$ and $A(\theta, p)$, such that:*

(4)
$$k(x, p) \in \arg\max_{k \in \mathbb{R}} \mathbb{E}[V(w_0 + (f - p)k)|x, p];$$

(5)
$$K(\theta, p) = \mathbb{E}[k(x, p)|\theta, p];$$

(6)
$$K(\theta, P(\theta, \varepsilon)) = K^s(\varepsilon);$$

(7)
$$a(x, p) \in \arg\max_{a \in \{0,1\}} \mathbb{E}[U(a, A(\theta, p), \theta)|x, p];$$

(8)
$$A(\theta, p) = \mathbb{E}[a(x, p)|\theta, p].$$

The equilibrium regime outcome is $R(\theta, \varepsilon) = \mathbf{1}_{A(\theta, P(\theta,\varepsilon))>\theta}$.

Conditions (4) to (6) define a rational-expectations competitive equilibrium for stage 1. In particular, condition (4) states that individual asset demands are conditioned on all available information, including anything inferable from the price realization $p = P(\theta, \varepsilon)$, while (5) gives aggregate demand and (6) imposes market clearing. Conditions (7) and (8) then define a perfect Bayesian equilibrium for stage 2, much as in Section I, but with the important difference that the endogenous price p replaces the exogenous public signal z.

We first impose that the dividend depends only on the fundamental θ, in which case the only link between the financial market and the coordination game is that the former provides an endogenous public signal that is relevant for the latter. In Section III, we consider the possibility that there is a feedback in the opposite direction as well by letting the dividend depend on the size of the attack, A. The first case isolates the coordinating role of prices; the second shows how this can contribute to volatility in the asset market itself.

B. *Equilibrium Analysis*

For simplicity, we let $f = \theta$ and, following Grossman and Stiglitz (1976), focus on linear price functions that are not perfectly revealing. Observing the price realization is then equivalent to observing a normally distributed signal with some precision $\alpha_p = \sigma_p^{-2} \geq 0$. The posterior of θ conditional on x and p is normally distributed with mean $\delta x + (1 - \delta)p$ and precision α, where $\delta = \alpha_x/\alpha$ and $\alpha = \alpha_x + \alpha_p$. It follows that individual asset demand is

$$k(x, p) = \frac{\mathbb{E}[f|x, p] - p}{\gamma \operatorname{Var}[f|x, p]} = \frac{\delta\alpha}{\gamma}(x - p)$$

$$= \frac{\alpha_x}{\gamma}(x - p),$$

and therefore aggregate demand is $K(\theta, p) = (\alpha_x/\gamma)(\theta - p)$. Market clearing, $K(\theta, p) = \sigma_\varepsilon \varepsilon$, then implies

$$P(\theta, \varepsilon) = \theta - \sigma_p \varepsilon,$$

which verifies the guess of a linear price function with

$$\sigma_p = \gamma \sigma_\varepsilon \sigma_x^2. \tag{9}$$

Thus, public information improves with private information. This is the key observation of the paper and has important implications for the determinacy and characterization of equilibria in the coordination game: agents can use prices to better predict one another's actions.

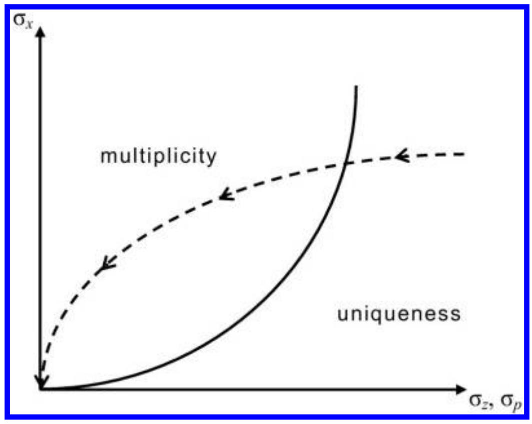

FIGURE 3. ENDOGENOUS INFORMATION

Note: As σ_x decreases, σ_z also decreases; multiplicity is ensured for σ_x small enough.

Indeed, since stage 2 here is equivalent to the benchmark model of Section I, with the price p playing the role of the public signal z, the analysis is completed by replacing σ_z in Proposition 1 with σ_p from equation (9).

PROPOSITION 3: *In the financial market economy with exogenous dividend, there are multiple equilibria if either source of noise is small, so that* $\sigma_\varepsilon^2 \sigma_x^3 < 1/(\gamma^2\sqrt{2\pi})$.

In Proposition 1, the precision of public information was fixed, so that sufficiently precise private information ensured uniqueness. Here, however, better private information improves public information, and at a rate fast enough to ensure multiplicity. The result is illustrated in Figure 3. In contrast to Figure 2, as the private noise σ_x decreases, the public noise σ_z also decreases, eventually pushing the economy into the multiplicity region.[7]

An immediate implication is that uniqueness can no longer be seen as a small perturbation away from prefect information: multiplicity is ensured when either the noise σ_x in private infor-

[7] Adding an exogenous source of public information in our model would only strengthen the case for multiplicity, which would then obtain for either low or high private noise σ_x.

mation or the noise σ_ε in the aggregation of this information through prices is small, as illustrated in Figure 1. Indeed, both extreme common-knowledge outcomes can be recovered as *either* noise vanishes: for any $\theta \in (\underline{\theta}, \bar{\theta})$, the regime can be either abandoned ($R = 1$) or maintained ($R = 0$), regardless of the fundamental θ.

PROPOSITION 4: *For the financial market economy with exogenous dividend, consider the limit as either source of noise vanishes so that $\sigma_x \to 0$ for given σ_ε, or $\sigma_\varepsilon \to 0$ for given σ_x. There exists a passive equilibrium in which $R(\theta, \varepsilon) \to 0$ whenever $\theta \in (\underline{\theta}, \bar{\theta})$, as well as an aggressive equilibrium in which $R(\theta, \varepsilon) \to 1$ whenever $\theta \in (\underline{\theta}, \bar{\theta})$.*

In this sense, nonfundamental volatility is maximized in the regime outcome as noise vanishes. This is in sharp contrast to the economy with exogenous information, where nonfundamental volatility disappears when private noise vanishes (Proposition 2).

Our results highlight the coordinating role of prices. Because agents interact in the financial market, they can use prices to predict what others will do in the coordination game. Indeed, the better informed agents are when entering the financial market, the better able they are to predict one another's actions when leaving. Thus, improving private information *reduces* strategic uncertainty and recovers multiplicity.

This argument relies on σ_p, the endogenous public noise, falling at a rate faster than does the square root of σ_x, the exogenous private noise. This property holds here and in the cases considered below, but is sensitive to the details of the aggregation channel. In the Appendix, we discuss and analyze an extension designed to highlight this point. There, the dividend of the asset is imperfectly correlated with the exogenous fundamentals that are relevant for the coordination game. The idea is to introduce additional noise in the aggregation process. If one assumes that this noise remains bounded away from zero as σ_x goes to zero, then σ_p also remains bounded away from zero and hence uniqueness obtains in the limit. Nevertheless, more precise private information continues to generate more precise public information, and contributes to multiplicity over some range of parameters. Moreover, the limit result turns out to be robust to this extension for the case with an endogenous dividend, which we turn to next.

III. Price Multiplicity

Motivated by the fact that crises are likely to affect asset market returns, we now consider the case where the asset's dividend is endogenously determined by the coordination game. This may capture, in a stylized fashion, the real rate of return on peso-forwards during currency attacks, or more generally stock-market returns during economic crises. As in the case with an exogenous dividend, the precision of the information conveyed endogenously by the price increases with the precision of exogenous private information. Again, this guarantees multiplicity for small levels of noise. The novel implication here is that multiplicity emerges also in the financial price.

The model is exactly as in the previous section, except for the endogeneity of the dividend. In particular, we let the dividend be a function of the aggregate size of attack in the coordination game, $f = f(A)$. To preserve normality of the information structure, we take $f(A) = -\Phi^{-1}(A)$.

In monotone equilibria, agents attack if and only if their private signal is below some threshold $x^*(p)$, so the aggregate attack is $A(\theta, p) = \Phi(\sqrt{\alpha_x}(x^*(p) - \theta))$ and the realized dividend is $f = \sqrt{\alpha_x}(\theta - x^*(p))$. Since p is observed, agents can calculate $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$, which represents the price of an asset that pays $\tilde{f} = f/\sqrt{\alpha_x} + x^*(p) = \theta$. We focus on equilibria with a one-to-one mapping between p and $\tilde{p}$, so that the observation of p is equivalent to the observation of $\tilde{p}$.

We guess and verify that the posterior for θ is normally distributed with mean $\delta x + (1 - \delta)\tilde{p}$ and precision α, where $\delta = \alpha_x/\alpha$ and $\alpha = \alpha_x + \alpha_p$, for some $\alpha_p = \sigma_p^{-2} \geq 0$. Individual asset demands are then given by

$$k(x, p) = \frac{\mathbb{E}[f|x, p] - p}{\gamma \operatorname{Var}[f|x, p]} = \frac{\sqrt{\alpha_x}}{\gamma}(x - \tilde{p})$$

and aggregate demand by

$$K(\theta, p) = \frac{\sqrt{\alpha_x}}{\gamma}(\theta - \tilde{p})$$

$$= \frac{\sqrt{\alpha_x}}{\gamma}\left(\theta - \frac{p}{\sqrt{\alpha_x}} - x^*(p)\right).$$

Market clearing thus implies $\tilde{p} = \theta - \sigma_p \varepsilon$ with

$$\sigma_p = \gamma \sigma_\varepsilon \sigma_x.$$

Once again, public information improves with private information.

Since stage 2 is identical to the benchmark model, except for the endogeneity of the public signal, the thresholds $\theta^*(p)$ and $x^*(p)$ must solve versions of equations (1) and (2), but with $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$ replacing z, and with α_p replacing α_z:

$$\theta^*(p) = \Phi\left(\sqrt{\frac{\alpha_x}{\alpha_x + \alpha_p}}\,\Phi^{-1}(1-c) - \frac{\alpha_p}{\alpha_x + \alpha_p}\,p\right) \tag{10}$$

and

$$x^*(p) = \theta^*(p) + \frac{1}{\sqrt{\alpha_x}}\,\Phi^{-1}(\theta^*(p)).$$

It follows that the thresholds $\theta^*(p)$ and $x^*(p)$, and hence the asset demand $K(\theta, p)$, are uniquely determined.

Also, $K(\theta, p)$ is continuous in p with $\lim_{p\to-\infty} K(\theta, p) = \infty$ and $\lim_{p\to\infty} K(\theta, p) = -\infty$. Thus, the market clearing condition $K(\theta, p) = K^s(\varepsilon)$ always admits at least one equilibrium price. Since, however, the dividend $f = \sqrt{\alpha_x}(\theta - x^*(p))$ is increasing in p, asset demand is not necessarily decreasing in p. Indeed,

$$\operatorname{sign}\left(\frac{\partial K(\theta, p)}{\partial p}\right) = -\operatorname{sign}\left(\frac{\sqrt{\alpha_x}}{\alpha_p} - \phi(\Phi^{-1}(\theta^*))\right),$$

so that demand is nonmonotone if and only if $\sqrt{\alpha_x}/\alpha_p < \sqrt{2\pi}$, or equivalently $\sigma_\varepsilon^2 \sigma_x < 1/(\gamma^2\sqrt{2\pi})$, that is, when either noise is small.

A backward-bending demand curve is possible here because of the two-way feedback between the financial market and the coordination game. A higher price realization makes agents in the second stage less inclined to attack. A smaller attack raises the asset dividend. Provided that this effect is strong enough, the demand for the asset can increase with its price over some region.

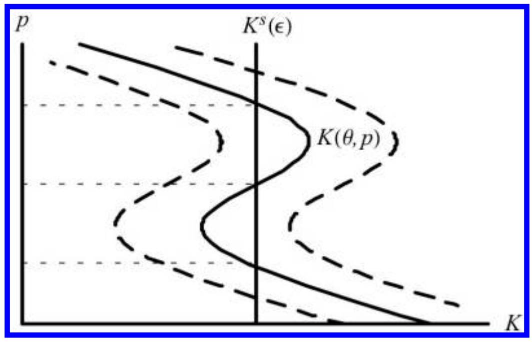

FIGURE 4. BACKWARD-BENDING ASSET DEMAND AND PRICE MULTIPLICITY

The solid line in Figure 4 illustrates a case where a backward-bending demand meets supply three times. The dashed lines show parallel shifts with changes in θ; only the low (high) price equilibrium remains for low (high) enough values of θ relative to ε. Thus, when demand is nonmonotone, there is a nonempty set of (θ, ε) for which there are three market clearing prices. Multiplicity in the price function then feeds into multiplicity in the coordination game, by composing $x^*(p)$ and $\theta^*(p)$ with $P(\theta, \varepsilon)$.

PROPOSITION 5: *In the financial market economy with endogenous dividend, there are multiple equilibria if either source of noise is small, so that* $\sigma_\varepsilon^2\sigma_x < 1/(\gamma^2\sqrt{2\pi})$. *Multiplicity then emerges in both the regime outcome* $R(\theta, \varepsilon)$ *and the price function* $P(\theta, \varepsilon)$.

Note that multiplicity does not emerge in individual strategies for given price realization. In this sense, price multiplicity is crucial for equilibrium multiplicity. To gain some intuition for this result, consider the common-knowledge case with $\sigma_x = 0$. Then $x = \theta$, $p = f = -\Phi^{-1}(A)$, and, therefore, $\theta < A$ if and only $x < \Phi(-p)$; so it is optimal to attack if and only if $x < \Phi(-p)$ and individual strategies are uniquely determined as functions of (x, p). Indeed, these common-knowledge outcomes are approached as noise vanishes.

PROPOSITION 6: *For the financial market economy with endogenous dividend, consider* *the limit as either source of noise vanishes, so that $\sigma_x \to 0$ for given σ_ε, or $\sigma_\varepsilon \to 0$ for given σ_x. There is a passive equilibrium in which $R(\theta, \varepsilon) \to 0$ and $P(\theta, \varepsilon) \to \infty$ whenever $\theta \in (\underline{\theta}, \bar{\theta})$, as well as an aggressive equilibrium in which $R(\theta, \varepsilon) \to 1$ and $P(\theta, \varepsilon) \to -\infty$ whenever $\theta \in (\underline{\theta}, \bar{\theta})$.*

PROOF.
See Appendix.

Comparing this result with Proposition 4, the novelty is that nonfundamental volatility is now maximized, as noise vanishes, not only in the regime outcome, but also in the financial price.

In our economy, the financial price plays three roles for market participants. First, it affects the cost of acquiring a given asset—the standard substitution effect present in any model. Second, it signals the dividend of the asset—the usual information-aggregation role highlighted by the rational expectations literature. Third, it affects the outcome in the coordination game and thereby *changes* the dividend of the asset itself—the novel coordination role for prices identified in this paper.

This third effect is the source of price multiplicity in our model. Indeed, somewhat paradoxically, this effect is highest in situations of low exogenous noise. This is in contrast to standard Grossman-Stiglitz environments where lower noise (e.g., less variable supply shocks or noisy traders) leads to lower volatility.

To the best of our knowledge, our result on price multiplicity is new. Gennotte and Leland (1990) and Barlevy and Veronesi (2003) find multiple equilibrium prices in noise rational-expectation models, but the source of multiplicity there is entirely different. In these papers, the dividend is exogenous and the price does not play any coordinating role. Instead, multiplicity obtains from nonlinearities in information aggregation.[8]

Also, our result on price multiplicity was obtained in a particular context, but is likely to apply more generally in environments where coordination impacts asset returns. Indeed, Hellwig et al. (2006) and Emre Ozdenoren and Kathy Yuan (2006) verify that a similar multiplicity result obtains in models where the coordination game is embedded in the financial market.

IV. Observing One Another

In this section, we remove the financial market and examine instead situations where information originates *within* the coordination game itself: agents observe a public signal about the aggregate attack. Such a feature seems relevant for thinking about bank runs, where widespread news coverage of a panic may spur other depositors to draw on their own accounts. More generally, during times of crises, it is unlikely that individuals are in the dark about what others are doing. Quite the contrary, they are most likely looking avidly over their shoulders. Indeed, in coordination models, the desire for such direct information is most natural—agents are keen to learn about the actions of others since this affects their payoffs, $U(a, A, \theta)$, directly.

An additional benefit is that this framework allows us to study information aggregation with a minimal modification of the Morris-Shin exogenous-information benchmark. It also bridges a gap between coordination models—which stress complementarities in actions—and herding models—which stress the observation of others' actions.

The model is identical to the benchmark model from Section I, except the public signal z is replaced with

$$y = S(A, \varepsilon),$$

where ε is noise independent of θ and ξ. To preserve normality of the information structure and obtain closed-form solution, we take $S(A, \varepsilon) = \Phi^{-1}(A) + \sigma_\varepsilon \varepsilon$ and $\varepsilon \sim \mathcal{N}(0, 1)$.[9] The information structure is parameterized by σ_x and σ_ε.

We assume that agents can condition their decision to attack on this indicator of *contemporaneous* aggregate behavior. Taken literally,

[8] In particular, informed traders interact with uninformed traders, and multiplicity originates from the inference problem faced by the latter: the uninformed agents' demand for the asset can turn backward when they interpret an increase in the price as an indication of high demand from the informed agents.

[9] This convenient specification was introduced by Dasgupta (forthcoming) in a different environment.

this clashes with standard game theory; but we do not take this literally. Rather, we think this captures in a parsimonious way the idea that many act based on some information about others' actions, or are able to revise their actions based on such information.[10] To substantiate these ideas, in Angeletos and Werning (2004) we developed a sequential variant that delivers similar results, while allowing standard game-theoretic equilibrium concepts.[11]

DEFINITION: *An equilibrium consists of an endogenous signal $y = Y(\theta, \varepsilon)$, an individual attack strategy $a(x, y)$, and an aggregate attack $A(\theta, y)$, which satisfy:*

$$a(x, y) \in \arg\max_{a \in \{0,1\}} \mathbb{E}[U(a, A(\theta, y), \theta) | x, y]; \tag{11}$$

$$A(\theta, y) = \mathbb{E}[a(x, y) | \theta, y]; \tag{12}$$

$$y = S(A(\theta, y), \varepsilon). \tag{13}$$

Just as in the asset market model of Section II, our equilibrium definition is a hybrid of rational-expectations and perfect Bayesian equilibrium concepts. Equation (11) requires the attack choice to be optimal given all available information, including the realized signal y of the aggregate attack. Equation (12) aggregates. Equation (13) imposes the rational-expectations consistency requirement; the signal must be generated by individual actions that are, in turn, dependent on it.

In monotone equilibria, an agent attacks if and only if $x \leq x^*(y)$, and the status quo is abandoned if and only if $\theta \leq \theta^*(y)$, so an equilibrium is identified with a triplet of functions $x^*(y)$, $\theta^*(y)$, and $Y(\theta, \varepsilon)$. As before, we focus on equilibria that preserve normality of the information structure.[12]

The model behaves in a similar way to the endogenous dividend model from Section III. Here, agents receive a direct signal of the attack A, while, there, the price was an indirect signal of the attack A; both y and p convey the same information in equilibrium. Indeed, the noise in the endogenous public information generated by y turns out to be

$$\sigma_y = \sigma_x \sigma_\varepsilon,$$

implying that multiplicity once again survives for small levels of noise.

PROPOSITION 7: *In the economy with observable actions, an equilibrium always exists. There are multiple equilibria if either noise is small so that $\sigma_\varepsilon^2 \sigma_x < 1/\sqrt{2\pi}$.*

PROOF.
See Appendix.

When multiplicity arises, it is with respect to aggregate outcomes and not individual strategies; the intuition for this is the same as in Section III. Extreme common-knowledge outcomes can be obtained as either noise vanishes, so that nonfundamental volatility is greatest near perfect information (as in Proposition 4 and Proposition 6).

V. Nonfundamental Volatility

We now investigate the role of the information structure for *nonfundamental* volatility, that is, volatility conditional on θ. We are interested in two sources of nonfundamental volatility. First, when there are multiple equilibria, sunspot variables may be used as coordination devices and thus contribute to volatility. Second, when the equilibrium is unique, its dependence on the noise shock ε generates volatility.

[10] Enrico Minelli and Hercules Polemarchakis (2003) develop a similar theme and argue, "At a Nash equilibrium of a game with uncertainty and private information [···] individuals do not extract information from the acts of other individuals in the same round of play; this takes literally the simultaneity of moves. But it is naive."

[11] The population is divided into two groups, "early" and "late" agents. Neither group observes contemporaneous activity. Early agents move first, on the basis of their private information alone. Late agents move second, on the basis of their private information, as well as a noisy public signal about the aggregate actions of early agents. Moreover, the case with simultaneous moves studied here is approached in the sequential variant, as the fraction of early agents goes to zero.

[12] Formally, we consider equilibria such that $G(Y(\theta, \varepsilon)) = \lambda_1 \theta + \lambda_2 \varepsilon$ for some strictly monotone function G and nonzero coefficients λ_1, λ_2.

Recall that with *exogenous* information, multiplicity disappears when agents observe the fundamentals with small private noise (Proposition 1). Thus, there is no sunspot volatility when σ_x is small enough. Moreover, as private noise vanishes ($\sigma_x \to 0$), the size of the attack and the regime outcome become independent of ε (Proposition 2). Thus, all nonfundamental volatility also vanishes.[13]

With *endogenous* information, the impact of private noise is quite different. We first summarize the implications of our results for the sunspot source of nonfundamental volatility. A reduction in σ_x may take the economy from the uniqueness to the multiplicity region, introducing sunspots (Proposition 3, Proposition 5, and Proposition 7). Indeed, potential sunspot volatility is greatest when either noise vanishes, $\sigma_x \to 0$ or $\sigma_\varepsilon \to 0$, in the sense that the regime's fate can become entirely dependent on the sunspot realization (Proposition 4 and Proposition 6). Moreover, when the dividend is endogenous, sunspot volatility also emerges in prices (Proposition 5), and again becomes most extreme as noise vanishes (Proposition 6).

We now turn to the second source of nonfundamental volatility and argue that, with endogenous information, less noise may increase volatility even without entering the region of multiple equilibria: when the equilibrium is unique, a reduction in σ_x or σ_ε can increase the sensitivity of equilibrium outcomes to the exogenous shock ε.

To show this result, we focus on the two financial-market models and proceed as follows. The regime is abandoned if and only if $\theta \leq \theta^*(p)$, where $p = P(\theta, \varepsilon)$. As long as the equilibrium is unique, $\theta^*(p)$ is continuously decreasing in p, and the price function $P(\theta, \varepsilon)$ is continuously increasing in θ. Hence, the regime is abandoned if and only if $\theta \leq \hat{\theta}(\varepsilon)$, where $\hat{\theta}(\varepsilon)$ is the unique solution to $\hat{\theta}(\varepsilon) = \theta^*(P(\hat{\theta}(\varepsilon), \varepsilon))$. Solving for $\hat{\theta}(\varepsilon)$ in this way we obtain

$$\hat{\theta}(\varepsilon) = \Phi\left(\psi + \frac{\sigma_p}{\sigma_x}\varepsilon\right),$$

[13] Morris and Shin (2003, 2004) study the volatility of unique equilibria further in coordination games with exogenous information.

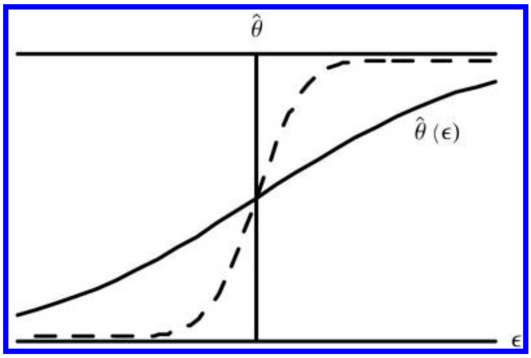

FIGURE 5. THE REGIME-CHANGE THRESHOLD $\hat{\theta}$ AS A FUNCTION OF THE SHOCK ε

Note: The dashed line corresponds to a lower level of noise than the solid one.

where $\psi \equiv (1 + 1/\sigma_p^2)^{1/2}\Phi^{-1}(1 - c)$. It follows that

$$\frac{\partial \hat{\theta}}{\partial \varepsilon} = \frac{\sigma_p}{\sigma_x}\phi(\Phi^{-1}(\hat{\theta})),$$

and therefore $\hat{\theta}(\varepsilon)$ satisfies a single-crossing property with respect to σ_p/σ_x. In this sense, the sensitivity of the regime outcome to the nonfundamental shock ε increases with the ratio of the noise in prices to the noise in private information σ_p/σ_x.

With exogenous dividend, $\sigma_p/\sigma_x = 1/(\gamma\sigma_\varepsilon\sigma_x)$, and therefore the sensitivity of $\hat{\theta}(\varepsilon)$ to ε increases with a reduction in either noise. This result is illustrated in Figure 5, which depicts the threshold $\hat{\theta}(\varepsilon)$, with the dashed line corresponding to a lower σ_x or σ_ε than the solid one.

With endogenous dividend, $\sigma_p/\sigma_x = 1/(\gamma\sigma_\varepsilon)$. The impact of aggregate noise is identical to the exogenous dividend case: sensitivity increases with σ_ε. In contrast, the sensitivity is now invariant to the amount of private noise σ_x. This result still contrasts with the case of exogenous information, where one can show that sensitivity is reduced when private information improves. (The result in Proposition 2 can be seen as the extreme case.)

Consider next the implications for price volatility. With exogenous dividend we have $p = \theta - \gamma\sigma_\varepsilon\sigma_x^2\varepsilon$. The impact of noise on the sensitivity of the price to ε is then exactly as in

Grossman-Stiglitz: a reduction in either σ_x or σ_ε reduces price volatility.

In contrast, when the dividend is endogenous, we have $p = f(A) - \gamma\sigma_\varepsilon\varepsilon$. Conditional on the size of the attack—or, equivalently here, on the dividend—the volatility of the price decreases with a reduction in σ_ε and is independent of σ_x. But since the attack A is a function of ε, a reduction in σ_ε may have an ambiguous overall effect on price volatility. Indeed, we have verified numerically that price volatility can increase with a reduction in σ_ε. Thus, the coordinating role of prices identified in Section III can generate volatility in asset markets even without multiplicity.

We conclude that less noise may increase volatility in both the regime outcome and the asset price, even when the equilibrium is unique. The results on equilibrium multiplicity may thus be viewed as extreme versions of this effect.

VI. Discussion

This paper emphasizes the importance of endogenous public information for understanding multiplicity and volatility in situations where coordination is important. We model this by letting agents observe either (a) the price of a financial asset, or (b) a direct noisy signal of others' activity in the coordination game.

Our key result is that the precision of endogenous public information increases with the precision of exogenous private information. This feature is likely to be very robust and carries with it the important implication that lower levels of private noise do not necessarily contribute toward uniqueness.

Whether this effect is strong enough to ensure multiplicity in the limit is sensitive to the details of the aggregation process, for it depends on whether the precision of public information increases faster than the square root of the precision of private information. Although this turns out to hold in all the cases studied above, it need not obtain in some variations of our asset-market model that introduce additional noise in the aggregation process.

Nevertheless, we believe that the cases presented here, and the result that information aggregation ensures multiplicity for small enough noise, provide an important benchmark. Indeed, the simplest model featuring information aggregation selects N individuals at random to be on a "talk show." Those on the show broadcast their signals to the rest of the population. This amounts to generating a public signal $z = \theta + \sigma_z\varepsilon$ with $\sigma_z = \sigma_x/\sqrt{N}$. In this case, public communication links the precision of private and public information in such a way that multiplicity is once again ensured for small enough noise. We conclude that, while some extensions may qualify our limit results, they are unlikely to modify our main point that endogenizing public information in coordination games is crucial for understanding the determinants of nonfundamental volatility, and thus of crises.

Appendix

PROOF OF PROPOSITION 2:

Consider the limit as $\sigma_x \to 0$ for given σ_z, or $\sigma_z \to \infty$ for given σ_x. In either case, $\alpha_z/\sqrt{\alpha_x} \to 0$ and $\alpha_z/\alpha_x \to 0$. Condition (3) then implies that $\theta^*(z) \to \hat{\theta} = 1 - c$ for any z, meaning that the regime outcome is unique and independent of the nonfundamental shock ε. Similarly, $x^*(z) \to \hat{x}$, where $\hat{x} = \hat{\theta}$ if we consider the limit $\sigma_x \to 0$, whereas $\hat{x} = \hat{\theta} + \sigma_x\Phi^{-1}(\hat{\theta})$ if we instead consider the limit $\sigma_z \to \infty$.

PROOF OF PROPOSITION 4:

In direct analogy to (3), the equilibrium correspondence here is given by

$$\Theta^*(p) = \{\theta^* \in (0, 1) : p = Q(\theta^*)\},$$

where

$$Q(\theta^*) \equiv \theta^* - \frac{\sqrt{\alpha_x}}{\alpha_p}\Phi^{-1}(\theta^*) + \sqrt{\frac{\alpha_x + \alpha_p}{\alpha_p^2}}\Phi^{-1}(1 - c). \tag{14}$$

Note that $\lim_{\theta^* \to 0} Q(\theta^*) = \infty$ and $\lim_{\theta^* \to 1} Q(\theta^*) = -\infty$. Moreover, whenever $\alpha_p/\sqrt{\alpha_x} > 1/\sqrt{2\pi}$, there exists a nonempty interval $(\theta_1, \theta_2) \subset (0, 1)$ such that Q is decreasing outside this interval, and increasing inside it, as illustrated by the dashed line in Figure 6. It follows that $\Theta^*(p)$ is nonempty and has at most three elements.

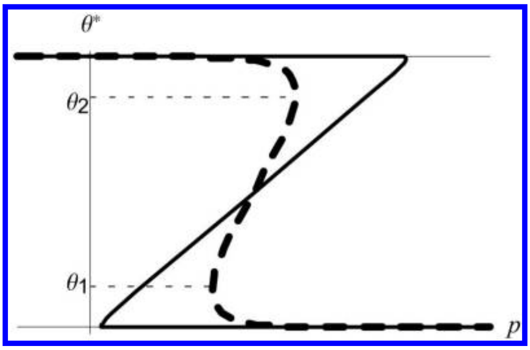

FIGURE 6. THE EQUILIBRIUM CORRESPONDENCE AS THE NOISE VANISHES

Any monotone selection from $\Theta^*(p)$ defines an equilibrium. Let $\theta_l^*(p) = \min\ \Theta^*(p)$ and $\theta_h^*(p) = \max\ \Theta^*(p)$; these represent the least and most aggressive equilibria. Consider now the limit as $\sigma_x \to 0$ or $\sigma_\varepsilon \to 0$. Using (9), we have $\sigma_p = \gamma\sigma_\varepsilon\sigma_x^2 \to 0$, $\sqrt{\alpha_x}/\alpha_p = \gamma^2\sigma_\varepsilon^2\sigma_x^3 \to 0$, and therefore $Q(\theta^*) \to \theta^*$ for all $\theta^* \in (0, 1)$. It follows that $\Theta^*(p)$ converges to $\{1\}$ for $p \leq 0$, to $\{0, p, 1\}$ for $p \in (0, 1)$, and to $\{0\}$ for $p \geq 1$, as illustrated by the solid line in Figure 6. By implication,

$$\theta_l^*(p) \;\to\; \begin{cases} 1 & \text{for } p < 0 \\ 0 & \text{for } p > 0 \end{cases}$$

$$\text{and} \qquad \theta_h^*(p) \;\to\; \begin{cases} 1 & \text{for } p < 1 \\ 0 & \text{for } p > 1 \end{cases}.$$

At the same time, $\sigma_p \to 0$ implies that, for any (θ, ε), $P(\theta, \varepsilon) \to \theta$. It follows that, for any $\theta \in (0, 1)$ and any ε, $\theta - \theta_l^*(P(\theta, \varepsilon)) \to \theta > 0$ and $\theta - \theta_h^*(P(\theta, \varepsilon)) \to \theta - 1 < 0$, which completes the proof.

PROOF OF PROPOSITION 6:

Market clearing requires $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$. Using (10), this reduces to $\tilde{p} = F(p)$, where

$$F(p) \equiv \Phi\left(\psi - \frac{\alpha_p}{\alpha_x + \alpha_p}\,p\right) + \frac{1}{\sqrt{\alpha_x}}\,\psi + \frac{\sqrt{\alpha_x}}{\alpha_x + \alpha_p}\,p$$

and $\psi \equiv \sqrt{\alpha_x/\alpha_x + \alpha_p}\,\Phi^{-1}(1 - c)$ and $\alpha_p = \alpha_x\alpha_\varepsilon/\gamma^2$. Consider the correspondence

$$\mathcal{P}(\tilde{p}) = \{p : \tilde{p} = F(p)\}.$$

Any monotone selection P^* from this correspondence defines an equilibrium price function by letting $P(\theta, \varepsilon) = P^*(\theta - \sigma_p\varepsilon)$. Note that $\lim_{p\to-\infty} F(p) = -\infty$ and $\lim_{p\to\infty} F(p) = \infty$, which together with the continuity of F ensures that $\mathcal{P}(\tilde{p})$ is always nonempty. Moreover, whenever $\alpha_p/\sqrt{\alpha_x} > 1/\sqrt{2\pi}$, there exists a nonempty interval $(p_1, p_2) \subset \mathbb{R}$ such that F is increasing outside this interval, and decreasing inside it. (Note that $F(p) = \theta - \gamma\sigma_x K(\theta, p)$ and hence the nonmonotonicity of F simply reflects the nonmonotonicity of asset demand.)

Take $P_l^*(\tilde{p}) = \min\ \mathcal{P}^*(\tilde{p})$ and $P_h^*(\tilde{p}) = \max\ \mathcal{P}^*(\tilde{p})$; let $P_l(\theta, \varepsilon) = P_l^*(p - \sigma_p\varepsilon)$ and $P_h(\theta, \varepsilon) = P_h^*(p - \sigma_p\varepsilon)$; consider the limit as $\sigma_x \to 0$ or $\sigma_\varepsilon \to 0$. It can be shown that

$$P_l^*(\tilde{p}) \;\to\; \begin{cases} +\infty & \text{for } \tilde{p} < 1 \\ -\infty & \text{for } \tilde{p} > 1 \end{cases}$$

$$\text{and} \qquad P_h^*(\tilde{p}) \;\to\; \begin{cases} +\infty & \text{for } \tilde{p} < 0 \\ -\infty & \text{for } \tilde{p} > 0 \end{cases}.$$

At the same time, $\sigma_p \to 0$ implies that, for any (θ, ε), $\tilde{p} \to \theta$. It follows that, for any $\theta \in (0, 1)$ and any ε, $P_l(\theta, \varepsilon) \to -\infty$ and $\theta - \theta^*(P_l(\theta, \varepsilon)) \to \theta > 0$, while $P_h(\theta, \varepsilon) \to +\infty$ and $\theta - \theta^*(P_h(\theta, \varepsilon)) \to \theta - 1 < 0$, which completes the proof.

PROOF OF PROPOSITION 7:

Given that an agent attacks if and only if $x \leq x^*(y)$, the aggregate attack is $A(\theta, y) = \Phi(\sqrt{\alpha_x}(x^*(y) - \theta))$. Condition (13) then implies that the signal satisfies

$$x^*(y) - \sigma_x y = \theta - \sigma_x\sigma_\varepsilon\varepsilon. \tag{15}$$

Note that (15) is a mapping between y and $z = \theta - \sigma_x\sigma_\varepsilon\varepsilon$. Define the correspondence

$$\mathcal{Y}(z) = \{y \in \mathbb{R} | x^*(y) - \sigma_x y = z\}.$$

We will later show that $\mathcal{Y}(z)$ is nonempty and examine when it is single- or multi-valued.

Take any function $\tilde{Y}(z)$ that is a selection from this correspondence, $\tilde{Y}(z) \in \mathcal{Y}(z)$ for all z, and let $Y(\theta, \varepsilon) = \tilde{Y}(\theta - \sigma_x\sigma_\varepsilon\varepsilon)$. The observation of $y = Y(\theta, \varepsilon)$ is then equivalent to the observation of $z = \theta - \sigma_z\varepsilon = Z(y)$, where $Z(y) \equiv x^*(y) - \sigma_x y$ and

$$\sigma_z = \sigma_x\sigma_\varepsilon. \tag{16}$$

That is, it is as if agents observe a normally distributed public signal with precision proportional to precision of exogenous private information.

An agent attacks if and only if $x \leq x^*(y)$, where $x^*(y)$ solves the indifference condition

$$\Phi\left(\sqrt{\alpha_x + \alpha_z} \times \left(\theta^*(y) - \frac{\alpha_x}{\alpha_x + \alpha_z} x^*(y) - \frac{\alpha_z}{\alpha_x + \alpha_z} Z(y)\right)\right) = c. \tag{17}$$

The regime in turn is abandoned if and only if $\theta \leq \theta^*(y)$, where $\theta^*(y)$ solves $A(\theta, y) = \theta$, or equivalently

$$x^*(y) = \theta^*(y) + \frac{1}{\sqrt{\alpha_x}} \Phi^{-1}(\theta^*(y)). \tag{18}$$

Using $Z(y) = x^*(y) - \sigma_x y$ and substituting $x^*(y)$ from (18) into (17), we get

$$\theta^*(y) = \Phi\left(\sqrt{\frac{\alpha_x}{\alpha_x + \alpha_z}} \Phi^{-1}(1 - c) + \frac{\alpha_z}{\alpha_x + \alpha_z} y\right), \tag{19}$$

which together with (18) determines a unique pair $\theta^*(y)$ and $x^*(y)$. The strategy $a(x, y)$ and the corresponding aggregate $A(x, y)$ are thus uniquely determined.

We return to the equilibrium correspondence $\mathcal{Y}(z)$. Using (18) and (19), this reduces to $\mathcal{Y}(z) = \{y : F(y) = z\}$, where

$$F(y) \equiv \Phi\left(\frac{\alpha_z}{\alpha_x + \alpha_z} y + q\right) + \frac{1}{\sqrt{\alpha_x}}\left(-\frac{\alpha_x}{\alpha_x + \alpha_z} y + q\right)$$

and $q \equiv \sqrt{\alpha_x/(\alpha_x + \alpha_z)}\Phi^{-1}(1 - c)$. Note that $F(y)$ is continuous in y, $F(y) \to -\infty$ as $y \to \infty$, and $F(y) \to \infty$ as $y \to -\infty$, which guarantees that $\mathcal{Y}(z)$ is nonempty and therefore an equilibrium always exists. Next, note that

$$\text{sign}(F'(y)) = -\text{sign}\left(1 - \frac{\alpha_z}{\sqrt{\alpha_x}} \phi\left(\frac{\alpha_z}{\alpha_x + \alpha_z} y + q\right)\right)$$

and therefore $F(y)$ is globally monotonic if and only if $\alpha_z/\sqrt{\alpha_x} \leq \sqrt{2\pi}$, in which case $\mathcal{Y}(z)$ is single-valued. If, instead, $\alpha_z/\sqrt{\alpha_x} > \sqrt{2\pi}$, there is a nonempty interval $(\underline{z}, \overline{z})$ within which $\mathcal{Y}(z)$ takes three values. Different (monotone) selections then sustain different equilibria. Using $\alpha_z = \alpha_\varepsilon\alpha_x$ from (16) completes the proof.

Extension with Noisy Dividend

Multiplicity obtains in the limit if the precision of public information increases at a faster rate than the square root of the precision of private information. Here we show that this property need not obtain in some variations of our asset-market model that introduce additional noise in the aggregation process.

The model is as in Section II or Section III, except that the dividend is not perfectly correlated with the fundamental or the coordination outcome: $f = \theta + \eta$ in the one case, and $f = f(A) + \eta$ in the other, where $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$ is independent of $(\theta, \xi, \varepsilon)$.

The equilibrium price continues to aggregate information, but the risk introduced by η limits the sensitivity of asset demands to changes in expected excess returns. With exogenous dividend, this effect implies an upper bound on the precision of the information revealed by the price. As a result, for any given $(\sigma_\eta, \sigma_\varepsilon) > 0$, multiplicity holds for an intermediate range of

σ_x, but not in the limit as $\sigma_x \to 0$. With endogenous dividend, however, the sensitivity of the dividend itself to θ increases with the precision of private information, overturning the previous dampening effect. As a result, multiplicity now obtains even in the limit as $\sigma_x \to 0$.

Finally, with either exogenous or endogenous dividend, less noise in the form of smaller σ_ε or σ_η contributes to multiplicity. In particular, for any $(\sigma_x, \sigma_\varepsilon)$ for which multiplicity was obtained when $\sigma_\eta = 0$, multiplicity is again ensured as long as σ_η is positive but small enough.

PROPOSITION A:

(*i*) *When $f = \theta + \eta$, a unique equilibrium survives for sufficiently small σ_x, given $(\sigma_\eta, \sigma_\varepsilon)$.*
(*ii*) *When $f = f(A) + \eta$, multiple equilibria exist for sufficiently small σ_x, given $(\sigma_\eta, \sigma_\varepsilon)$.*
(*iii*) *In either case, the region of $(\sigma_x, \sigma_\varepsilon)$ for which the equilibrium is unique vanishes as $\sigma_\eta \to 0$.*

PROOF.

Part (*i*). Postulating that the posterior for θ conditional on (x, p) is normally distributed with mean $\delta x + (1 - \delta)p$ and precision α, where $\delta = \alpha_x/\alpha$ and $\alpha = \alpha_x + \alpha_p$, we have that individual asset demands are given by

$$k(x, p) = \frac{\mathbb{E}[f|x, p] - p}{\gamma \operatorname{Var}[f|x, p]} = \frac{\delta(x - p)}{\gamma(\alpha^{-1} + \sigma_\eta^2)}.$$

It follows that the equilibrium price is $p = \theta - \sigma_p \varepsilon$, where $\sigma_p = \gamma(\sigma_x^2 + \sigma_\eta^2/\delta)\sigma_\varepsilon$. Since $\delta \in [0, 1]$ and $\sigma_x > 0$, σ_p is bounded from below by $\gamma\sigma_\eta^2\sigma_\varepsilon > 0$ and hence $\sigma_x < (\gamma\sigma_\eta^2\sigma_\varepsilon)^2\sqrt{2\pi}$ suffices for the equilibrium to be unique.

Part (*ii*). We now postulate that the posterior for θ is normally distributed with mean $\delta x + (1 - \delta)\tilde{p}$ and precision α, where $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$, $\delta = \alpha_x/\alpha$, and $\alpha = \alpha_x + \alpha_p$. It follows that

$$k(x, p) = \frac{\mathbb{E}[f|x, p] - p}{\gamma \operatorname{Var}[f|x, p]} = \frac{\sqrt{\alpha_x}\delta(x - \tilde{p})}{\gamma(\alpha_x\alpha^{-1} + \sigma_\eta^2)}$$

and therefore $\tilde{p} = \theta - \sigma_p \varepsilon$, where

$$\sigma_p = \frac{1 + \sigma_\eta^2}{1 - \gamma\sigma_\eta^2\sigma_\varepsilon}\gamma\sigma_\varepsilon\sigma_x.$$

Hence, a higher σ_η again makes it harder for multiple equilibria to exist; nevertheless, multiplicity is ensured by a sufficiently small σ_x or σ_ε.

Part (*iii*). This follows immediately from the fact that, for any given $(\sigma_x, \sigma_\varepsilon)$, σ_p is decreasing in σ_η, with $\sigma_p \to 0$ as $\sigma_\eta \to 0$.

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