next to nothing about the fact that the ADM model imagines trade as a one-shot affair, not at all like the purchases and sales we see and engage in all the time. So at this point you may be asking, "How can the ADM model possibly play *any* role at all in organizing the thoughts of macroeconomists?"

In chapter 2, I noted (though rather obliquely) that even though the ADM model looked awfully incapable of handling time and uncertainty, a full accommodation of both features was actually possible, thanks to the existence of a much more "realistic" version of the ADM model, called the Radner model. This version of the ADM model, and variants of it, are the bedrock on which macroeconomics sits. In the Radner version of the ADM model, time and uncertainty are modeled explicitly, but—and this is crucial—under some standard assumptions, Radner outcomes are absolutely *identical* to those coming from the ADM model! Thus, in many instances, nothing is gained by modeling the many complications one might imagine arising from the presence of uncertainty and time, especially when the goal is to understand the relationship between Walrasian outcomes and efficiency.

As we will see, versions of Radner models are used by the profession to address many of the major macroeconomic phenomena you might read about. Examples include economic growth, unemployment, the consumption of households, the relative returns on various classes of assets, and fiscal policy and monetary policy. The goal of this chapter is to get you to the Radner model in two steps. First, I'll show you how the ADM model deals with time and uncertainty. Then, I'll describe the far more "realistic" trading arrangement of the Radner model, and some classic benchmark models that employ versions of it.[1]

5.2 Time, Uncertainty, and the ADM Model

It was noticed by Debreu (1959) and Arrow (1953, 1964), among others, that the notion of whether any two goods or services are different from each other should depend fundamentally on whether consumers or producers view them this way, and not inherently on any purely *physical* characteristics of the good or service in question.[2] Think of an umbrella. This physical object provides different services when it is raining than when it is not. Therefore, the interaction of uncertainty and the physical good we know as an "umbrella" together imply that there are really *two* goods that consumers care about: "umbrellas on sunny days" and "umbrellas on rainy days." This is intuitive: prior

to the realization of whether a day is sunny or rainy, an individual would value having umbrellas differently in these two eventualities. Conversely, in a world where all people were color-blind to red and green, red umbrellas and green umbrellas would be equally valuable and viewed (literally) as identical, though "physically" they are not. Even more generally, imagine a contract that promises you "canned radishes delivered at your front door one year from now, but only if you are feeling well." The eventual value of such a contract may well depend on the weather that prevails now and in the interim in radish-growing areas of China. This example also makes it clear that even radishes aren't just radishes: the fact that, in this case, they will be delivered under a given set of circumstances differentiates them from, among other things, radishes to be delivered under other circumstances.

A **contingency** in the Arrow-Debreu sense is a *complete* description of the environment prevailing at some future date. It is as finely detailed as is relevant to buyers and sellers. In his landmark work, Gerard Debreu (1959) introduced the idea of a **contingent commodity,** whereby a given "physical" good, say, radishes, would be differentiated by *whatever* circumstances were deemed relevant by consumers and producers. In settings with uncertainty, the notion of complete markets is then simply one in which the markets required are expanded so that there will be markets for every single contingent commodity. A Walrasian equilibrium for a model allowing for trade in a full set of contingent commodities *prior to the realization of any uncertainty* is usually referred to as an **Arrow-Debreu equilibrium**, rather than an ADM model, which usually connotes an economy without uncertainty. I will use the term "ADM model" to refer to the ADM both without uncertainty and with it, with the context making matters obvious.

To see another example of an ADM economy that includes both time and uncertainty, first imagine the same simple agrarian society we laid out in chapter 1, in which there were two "physical" products, corn and wheat. Now add a twist: let the economy be subject to three kinds of uncertain weather: sunny, cloudy, and rainy. Now, recall first the market structure imagined by the ADM model: a WCH would open in the town square and establish Walrasian prices for all commodities. Because of the uncertainty present in this economy, the Arrow-Debreu WCH sets up trade in not just two markets as before (i.e., not just markets for wheat and corn alone) but *six* markets, one for each physical good in each contingency. These are: corn in sunny weather, corn

in cloudy weather, corn in rainy weather, wheat in sunny weather, wheat in cloudy weather, and wheat in rainy weather.

While, in the original "no-uncertainty" case, a household's endowments of these goods was simply a listing of how much corn and wheat they had, now we must distinguish between the amount of the goods in each of the three possible weather conditions. Households would then take their endowments of these goods—which now includes "titles" to receive possibly varying amounts of corn and what depending on the weather—and sell them to the WCH. They would then turn around and buy the bundles of these six goods that they like most, subject to the budget determined by their endowments and prices.

The markets just described are called "complete *forward* markets" as they are markets in *promises*. Be clear on this: the *only* things that are actually bought and sold in an Arrow-Debreu market are promises to deliver, or take delivery of, the amounts agreed upon in the WCH prior to the realization of any uncertainty, under the various contingencies. For example, a trader in the ADM world may have agreed to deliver 1 ton of red winter wheat in rainy weather, but expects to receive 0.5 ton of the same in sunny weather, with analogous agreements for corn.

As a consequence of the First Welfare Theorem, we also know that such an outcome is Pareto-optimal, which immediately means that no further mutually beneficial trading opportunities exist—so no new agreements would be struck after the initial round of trading *even if markets reopened* once the uncertainty had resolved itself. This restates what we learned earlier: ex-ante Pareto-optimal outcomes are ex-post Pareto-optimal.

5.2.1 The Long Arm Attached to the Invisible Hand

The broad view of a commodity imagined by the Arrow-Debreu setting is of enormous importance. It tells us that, in principle, real-life aspects such as time and uncertainty are *fully accommodated* by Walrasian theory: simply differentiate physical goods and services by the exact time and circumstances under which they will be available. A textbook rendition will be something like the following.

Think of a world with H "basic physical commodities" (apples, oranges, and motor oil, say), that lasts T periods. T and H are just round numbers, e.g., 20 and 2000. Next, think of a **state of the world** as being a description of the complete particular unfolding of history over the entire (T-period) life of the economy. If $T = 3$, and the weather was the

only uncertain thing, and it could either be cloudy or sunny, a "state of the world" would be a full listing of the entire history of the weather in this economy, e.g., "sunny, sunny, cloudy." Of course, at time 0, one doesn't know which state (i.e., history) will unfold.

To see that an economy like this has the same fundamental structure as the ADM model, now just redefine the set of commodities by the *date*. In this case, each basic physical commodity is differentiated by the date on which it is consumed by households, produced by firms, or becomes available (as an endowment). This means we have $L = H \times T$ dated physical goods.

Finally, we impose the commonsense restriction that these goods cannot appear in different amounts in the endowments or consumption plans of households or in the production plans of firms across any two states that the economy's participants cannot themselves distinguish at any date. These are called **measurability** restrictions. With this redefining of the goods and services in the economy and the imposition of the measurability restrictions, the model immediately becomes mathematically identical to the ADM, and we're done.

This equivalence is of supreme importance: it immediately means that the First and Second Welfare Theorems are true. This teaches us that the ability of Walrasian prices under complete markets to exhaust all gains from trade between self-interested rational price takers is thus in no way dependent on the economy being a one-date affair (what economists call "static"). Instead, Walrasian prices can efficiently coordinate activity in economies that are almost arbitrarily rich in their spatial, temporal, or stochastic structure.[3] Moreover, it implies that we can invoke the existence theorems as well!

To sum up, we know that when a full set of contingent commodities is available, Walrasian equilibrium exists and is Pareto-optimal in a setting where most (nearly all) allocations are not. We'll also see later that under mild conditions, Walrasian equilibria will also be (almost) unique—i.e., the model has a definite prediction for prices and allocations given the primitives of preferences, endowments, and technology.

To echo a point I made in chapter 2, entirely apart from the practicality of whether private trade will give rise to a full set of Arrow-Debreu contingent claims, it should strike the reader as astonishing that an object as impersonal and "small" as a set of Walrasian prices is capable of leading self-interested parties to Pareto-efficient outcomes with no direct communication between them whatsoever—even in the presence of uncertainty that only resolves over time.

5.2.1.1 The Impossibility of Literal Arrow-Debreu Market Completeness

The ADM model with contingent commodities is a fantastic illustration of the power of mathematics to demonstrate the logical "sameness" of seemingly different objects. In this case, we know now that an economy with two people, two goods (e.g., apples and oranges), a single firm, one round of decision making, and no uncertainty at all has exactly the same mathematical structure, takes no more time to describe, and has the same properties—such as the welfare theorems—as one with 2 billion people, 2 billion goods, 2 billion firms, and which will last for 2 billion years with all manner of uncertainty. That is, we see that with the right set of markets, Walrasian equilibria are efficient, meaning that linear prices can coordinate economic activity in incredibly rich settings.

But mathematical sameness clearly hides something vital: it doesn't immediately convey, for example, the fact that a literal Arrow-Debreu world is well-nigh unattainable. The presumption is that trading forums are costless to operate: there are no overhead costs, no costs for verifying claims, etc. While this was a bad assumption under conditions of certainty, it is far, far worse under uncertainty. The ADM model, under uncertainty, envisions the presence of a market (with a single, linear price) for quite literally every good in every discernible "state of the world." As a result, the ADM setting is wildly demanding in terms of the number of markets it imagines. Let's say that instead of two goods, we had $L = 1000$ different goods, and instead of just two states, we had $S = 500$ different contingencies (really, 500 entire *histories* of outcomes over the entire span of time for which households exist), we'd then need *half a million* markets at time zero under ADM trading, all of which would have to be of the fanciful "contingent" commodity variety! This is just too demanding to be realistic. But, strictly speaking, this is what complete Arrow-Debreu markets require. The cost of operating so many markets would rapidly exhaust all of society's resources; even the smallest department store or auction house takes space and some personnel to operate. It would be truly ironic, if not hilarious, if society fully squandered its resources in an ill-conceived attempt to create an efficient trading system.[4]

As for contingent commodities, there is no obvious contract available right now that I could purchase that would deliver me, for instance, a coconut and a ticket to the Caribbean if and only if "the weather in the preceding six months had an average heat index of less than 20 degrees Fahrenheit." But the Arrow-Debreu world presumes that there is such a thing. Of course, I *can* buy a coconut today and a plane ticket (also today) to the Caribbean on a flight leaving exactly six months from now. But these purchases are good for a coconut and a trip *irrespective* of whether the weather at home has been bad or good over the period in question. They are emphatically not contingent on all the uncertainty that may resolve itself between now and six months from now.

What is more, the Arrow-Debreu world would involve no *trade* after the first day of mankind's existence (or at least, after the first day in which mankind created a full set of Arrow-Debreu contingent claims). One would observe only *deliveries*! That is, if the Arrow-Debreu contingent-claims market actually were present among the ancients, *all* observed transactions today would simply be the fulfillment of the obligations created by those contracts, whereby the descendants of those alive at the "beginning of time" would simply be delivering on commitments agreed to by their most distant ancestors! To say that this seems not quite what occurs around us is a mild understatement.

A final nail in the coffin of "literal" complete Arrow-Debreu markets has to do with the incentives to manipulate prices that would arise under such a market structure. Notice that the requirement that there be a price for any Arrow-Debreu claim in which even two traders have any interest means that there will inevitably be many commodities for which one probably won't be able to establish price-taking behavior unless the parties were guileless enough to not exploit the market power they inevitably had. Moreover, recall that the Myerson-Satterthwaite theorem told us for sure that in such a setting, barring intimate knowledge of preferences, efficiency was impossible.

In light of all this negativity I've heaped on the ADM model, two questions immediately arise regarding how one views decentralized trade. First, a full set of Arrow-Debreu markets are, as I have repeatedly emphasized, a sufficient but perhaps not necessary condition for decentralized outcomes to be efficient.[5] And even within the class of purely "Walrasian," i.e., linear-price-mediated, trading arrangements, might there be arrangements that require fewer markets to be open at any one time, but which nonetheless, reproduce the Arrow-Debreu outcome? The answer is yes, and the most important example of such a setting is the so-called **Radner trading arrangement** (see the original Radner 1972), stemming from an earlier idea of Kenneth Arrow, and due to the eminent economist Roy Radner.