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Consumption and real exchange rates in dynamic economies with non-traded goods

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We examine the possibility that non-traded goods may account for several striking features of international macroeconomic data: large, persistent deviations from purchasing power parity, small correlations of aggregate consumption fluctuations across countries, and substantial international real interest rate differentials. A dynamic, exchange economy is used to show that non-traded goods in principle can account for each of these phenomena. In the theory there is a close relation between fluctuations in consumption ratios and those in bilateral real exchange rates, but we find little evidence for this relation in time-series data for eight OECD countries.

1. Introduction

Probably the most striking feature of international macroeconomic data is the regularity of large, persistent departures from purchasing power parity (PPP) apparent in national price indexes and foreign exchange rates. Isard (1977), Roll (1979), Frenkel (1981), Mussa (1986), and Huizinga (1987), among many others, have documented various aspects of this phenomenon, with the result that departures from PPP are one of the most clearly established empirical regularities in economics. As a consequence, deviations from PPP, and real exchange rate movements more generally, remain a central research topic in open economy macroeconomics.

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One way to account for these movements is to rely on non-traded goods in a competitive world economy. This approach has a long tradition in international macroeconomics, including papers by Cassel (1918), Samuelson (1948), Haberler (1961), Balassa (1964), Sanyal and Jones (1982), Jones and Purvis (1983), Stulz (1987), Neary (1988), and Stockman and Dellas (1989). The mechanism is fairly simple. Although the law of one price holds, in the sense that each good sells for a single price in all countries, PPP may not: price indexes combine prices of both traded and non-traded goods, and because the latter are sold in only one country their prices, and hence price indexes, may differ across countries. There is some evidence, like that reported by Isard (1977), Kravis and Lipsey (1977, 1978), Richardson (1978), Krugman (1987), and Lapham (1992), suggesting that the law of one price itself fails (both within and across countries), but this generally is based on comparisons of price indexes for disaggregated but nevertheless heterogeneous groups of goods. The evidence of Protopapadakis and Stoll (1983, 1986) implies that with homogeneous commodities, like metals and agricultural products, deviations from the law of one price are much smaller than departures from PPP. One therefore might follow the interpretation that all final goods contain a positive non-traded component. In that case even price comparisons between seemingly similar goods in different locations are affected by changes in relative prices of non-traded goods.

The application of non-traded goods to real exchange rates is direct. What makes this approach appealing is that it appears capable of explaining several other empirical regularities as well. One of these is the relatively small correlation between aggregate consumption fluctuations across countries. As noted by Leme (1984) and Scheinkman (1984), the most obvious implication of complete markets in a one-good world economy is that consumption by every individual and country should be deterministically related to aggregate, or world, consumption; with identical homothetic preferences, consumption in one country should be perfectly correlated with consumption in every other country. In fact the correlations are considerably less than one, and are similar to cross-country output correlations. There are a number of features of the theory that might be changed to account for this discrepancy between theory and evidence, including allowing for preferences which are non-separable between consumption and leisure [see Devereux et al. (1992)] and for incomplete markets [see Kollmann (1991)]. One of the most natural amendments is to abandon the assumption that countries consume the same basket of goods. With non-traded goods, countries 'share' the traded good but consume their own non-traded good. Movements in aggregate consumption, a function of consumption of the traded and non-traded components, may be imperfectly correlated if the quantities of non-traded goods are themselves imperfectly correlated.

A third empirical regularity, documented by Isard (1983), Cumby and

Obstfeld (1984), Mishkin (1984), Mark (1985), and Cumby and Mishkin (1986), is that real interest rates exhibit sizable differences across countries. One explanation is similar to that for deviations from PPP, but in first-differences: if nominal interest differentials reflect expected rates of depreciation, then real interest differentials, measured with price indexes, will reflect rates of change of deviations from PPP. Isard (1983), for example, interprets term structures of interest rates for Germany and the United States in the early 1980s as showing that such deviations last two to five years. Frankel (1986) also relates real interest rate differentials to imperfect goods market integration. Dornbusch (1983), Stulz (1987), Devereux (1988), and Stockman and Dellas (1989) provide examples of theoretical economies in which international interest-rate differentials reflect the role of non-traded goods. A further regularity is that many portfolios appear to be dominated by domestic assets. Eldor et al. (1988) and Stockman and Dellas (1989) use non-traded goods to account for imperfect international diversification. Engel and Kletzer (1989) and Tesar (1993) suggest, in addition, that non-traded goods may account for high correlations between savings and investment, country by country.

The strength of this approach is thus its potential for reconciling a wide range of international evidence by a single theoretical device. While non-traded goods have been suggested as an explanation for many features of international macroeconomic data, the focus of this paper is on general equilibrium restrictions. We derive several implications of non-traded goods simultaneously and point out the relations between them. The main theoretical finding is a monotonicity result, showing that, in theory, consumption ratios and real exchange rates are positively related for pairs of countries. This implies a similar relation for moments of these variables. For example, if fluctuations in the non-traded sector account for large variability in the ratio of two countries' consumptions, then their real exchange rate also should be relatively variable in competitive equilibrium. Moreover, relative consumptions and relative prices in the theory have similar dynamics and are positively correlated over time.

Section 2 describes a world exchange economy with one traded good, a non-traded good for each country, and an arbitrary number of countries. Consumers, whom we refer to as countries, are endowed with quantities of the traded good and their own non-traded good. Section 3 examines the benchmark case of a single traded good, with no non-traded goods. Section 4 describes the behavior of consumption and real exchange rates with non-traded goods and discusses ways of giving this behavior empirical content when non-traded goods are not directly observable. There we adopt a period utility function general enough to include many functions used by other researchers. This suggests some simple but general tests, which are applied to data for eight OECD countries in section 5.

The empirical work studies the general equilibrium interconnections between real exchange rates and cross-country consumption ratios and reports new evidence on the non-traded goods hypothesis. The main empirical finding is that there is little support for a central role for non-traded goods in accounting for the consumption and relative price evidence simultaneously. For example, the correlation between relative price movements and relative consumption movements is low (less than 0.17) for all of the pairs of countries studied. And pairs of countries with volatile real exchange rates do not tend to have volatile relative consumptions. We therefore conclude in section 6 by briefly considering some extensions and alternatives to the theory that might narrow the gap between predictions and evidence.

2. The theoretical economy

Our theoretical world is a stochastic exchange economy; the structure and notation extend Lucas (1984) to a multiagent setting. There are I countries, indexed by $i=1,2,\dots,I$, each represented by a single consumer who lives from date 0 to date T. At each date t, for $t=0,1,\dots,T$, an event z_t occurs that is observed by all agents. We assume, for mathematical simplicity, that T is finite and that each z_t is drawn from a finite set. A succession of events, known as a history, is denoted $z^t \equiv (z_1,\dots,z_t)$, an element of the finite set Z^t. The history, in addition to the initial event z_0, completely describes the state of the economy at date t. The probability of any state z^t, given z_0, is denoted $\pi(z^t)$.

In each state the economy has $I+1$ goods, a single traded good and a non-traded good associated with each country. We denote by w_i and x_i the endowments of the traded and non-traded goods in the ith country, and by $W=\sum_{i=1}^{I} w_i$ the world endowment of the traded good. The corresponding consumption quantities are, respectively, a_i and b_i. Date-t prices of these commodities will be denoted q_0 for the traded good and q_i for the ith non-traded good, while corresponding date-0 prices will be denoted Q_0 and Q_i. Both quantities and prices are functions of the current state.

The representative consumer in country i maximizes the expected utility function

$$U_i=\sum_{t=0}^{T} \beta^t \sum_{z^t \in Z^t} \pi(z^t)u[a_i(z^t),b_i(z^t)], \tag{2.1}$$

where $\beta \in (0,1)$ is a common discount factor. The function u is common to all countries, increasing in both arguments, concave, and homothetic. A similar condition is traditional in static trade theory, where it serves to eliminate distribution effects on relative prices.

We shall study the implications of competitive equilibrium for price and quantity indexes, defined as follows. The price index is a linear homogeneous scalar function $p_i(q_0, q_i)$. The quantity index is a linear homogeneous function $c_i(a_i, b_i)$ such that state utility can be expressed as $u[a_i, b_i] = v[c_i(a_i, b_i)]$ for all a_i and b_i for some monotone increasing function v. Homotheticity of the direct utility function is sufficient for there to exist price and quantity indexes p_i and c_i such that $q_0 a_i + q_i b_i = c_i(a_i, b_i) p_i(q_0, q_i)$ at utility-maximizing quantities. We study utility functions with this property, as Persson and Svensson (1985) did for the same reason. As the notation indicates, we assume strong (additive) separability by state and time, which greatly simplifies the solution. However, the state or period utility function generally will not be separable between traded and non-traded goods.

Given price indexes for countries i and j, we define the real exchange rate between them as the ratio of their price indexes:

$$e_{ij} \equiv p_j(q_0, q_j)/p_i(q_0, q_i). \tag{2.2}$$

We define real interest rates from prices of risk-free bonds, defined to pay one unit of the aggregate consumption bundle for each event z_{t+1} following z^t. The price of such a bond in country i in state z^t is found by summing the prices of Arrow securities:

$$s_i(z^t) = \sum_{z_{t+1}} P_i[Q_0(z^{t+1}), Q_i(z^{t+1})]/P_i[Q_0(z^t), Q_i(z^t)]. \tag{2.3}$$

The real interest rate is defined as

$$r_i(z^t) = s_i(z^t)^{-1} - 1, \tag{2.4}$$

which for small returns can be approximated by the continuously compounded return

$$r_i(z^t) \approx -\log[s_i(z^t)]. \tag{2.5}$$

Both bond prices and real interest rates may vary across countries if price indexes do.

Consumers face date-0, or present-value, budget constraints. The date-0 values of consumers' expenditures are bounded by the values of their endowments:

$$\sum_{t=0}^{T} \sum_{z^t} [Q_0(z^t) a_i(z^t) + Q_i(z^t) b_i(z^t)]$$

$$= Q_0(z_0) n_i(z_0) + \sum_{t=0}^{T} \sum_{z^t} [Q_0(z^t) w_i(z^t) + Q_i(z^t) x_i(z^t)], \tag{2.6}$$

where $n_i(z_0)$ is the net foreign asset position of country i at the start of period 0, measured in units of the traded good. Clearly $\sum_{i=1}^{I} n_i(z_0)=0$. If we let x, for example, denote a complete sequence $\{x(z^t)\}$, with one element for each state z^t, then consumer i's problem is to choose a_i and b_i to maximize utility (2.1) subject to the budget constraint (2.6).

An equilibrium in this economy consists of prices and quantities satisfying two conditions. First, quantities consumed in each country maximize utility given prices and budget constraints. Second, markets for goods clear in all states z^t:

$$\sum_{i=1}^{I} a_i(z^t)=\sum_{i=1}^{I} w_i(z^t)\equiv W(z^t),$$

$$b_i(z^t)=x_i(z^t), \quad \forall i, z^t.$$

3. Equilibrium without non-traded goods

We begin by describing an equilibrium when there are no non-traded goods. This serves as a basis of comparison with later developments and clarifies the contribution of non-traded goods. The properties of this model also apply to the widely-used multiple-traded-good model of Lucas (1982) if preferences are homothetic. The notation simplifies because $c_i=a_i$ and $p_i=q_0$. The representative consumer of country i has utility function

$$U_i=\sum_{t=0}^{T} \beta^t \sum_{z^t} \pi(z^t)u[c_i(z^t)]. \tag{3.1}$$

We follow Negishi (1960) and Mantel (1971) in computing a competitive equilibrium as the solution to a social planning problem. The planner chooses quantities c_i to maximize the welfare function

$$\sum_{i=1}^{I} \lambda_i U_i=\sum_{i=1}^{I} \lambda_i \sum_{t=0}^{T} \beta^t \sum_{z^t} \pi(z^t)u[c_i(z^t)], \tag{3.2}$$

subject to the resource constraints

$$\sum_{i=1}^{I} c_i(z^t)\leq \sum_{i=1}^{I} w_i(z^t)\equiv W(z^t), \quad \forall z^t. \tag{3.3}$$

Each choice of welfare weights $\{\lambda_i\}$ produces a different Pareto-optimal allocation, corresponding to a different distribution of initial wealth. The appropriate weights for a particular distribution of wealth can be determined by imposing the individual budget constraints [see Mantel (1971)]. However, many of the properties of a competitive equilibrium hold for all distributions of wealth (because preferences are identical and homothetic), so we shall not need this additional step.

It is well known that, with stationary, additively-separable preferences that are homothetic and identical across countries, cross-country consumption correlations are unity [see, for example, Scheinkman (1984)]. To illustrate that result simply we shall let $u(c)=c^{1-\gamma}/(1-\gamma)$, with $\gamma>0$; that is, consumers have identical, concave, isoelastic utility functions. This functional form makes calculating equilibrium prices and quantities simple.

Proposition 1. An equilibrium in the economy with isoelastic utility and without non-traded goods is characterized by prices p and quantities c given by

$$p(z^t)=W(z^t)^{-\gamma}, \tag{3.4}$$

$$c_i(z^t)=\lambda_i' W(z^t), \tag{3.5}$$

where $\lambda_i' \equiv \lambda_i^{1/\gamma}/\sum_{j=1}^{I}\lambda_j^{1/\gamma}$, *for some choice of welfare weights,* $\{\lambda_i\}$.

Proof. We label the Lagrange multipliers on the resource constraints $\beta^t\pi(z^t)p(z^t)$. Then the planning problem decomposes into a number of identical problems, one for each state, of the form: choose $\{c_i\}$ to maximize $\sum_{i=1}^{I}\lambda_i c_i^{1-\gamma}/(1-\gamma)$ subject to $\sum_{i=1}^{I} c_i \leqq W$. The first-order conditions imply $p(z^t)=[\sum_{j=1}^{I}\lambda_j^{1/\gamma}]^{\gamma}/[W(z^t)]^{\gamma}$ and $c_i(z^t)=\lambda_i' W(z^t)$. Date-0 prices are simply the multipliers on the resource constraints, so $P(z^t)=\beta^t\pi(z^t)p(z^t)=\beta^t\pi(z^t)[\sum_{j=1}^{I}\lambda_j^{1/\gamma}]^{\gamma}/[W(z^t)]^{\gamma}$. Because prices are determined, in equilibrium, only up to a multiplicative constant, the values given in eq. (3.4) constitute an equilibrium. □

The implications for real exchange rates and relative consumptions are immediate consequences of Proposition 1. First, it is clear that PPP holds exactly, because the price of the traded good is the same everywhere. Second, consumption in every country is monotonically related to consumption in every other: from (3.5) we see that in every state consumption is a linear function of the aggregate endowment. Ratios of consumption quantities, c_i/c_j, equal ratios of normalized welfare weights, λ_i'/λ_j', which are constant along any equilibrium path. Thus consumption in country i is perfectly correlated with consumption in every other country j, if correlations are defined. Third, real interest rates are equalized across countries. From eq. (3.4) date-t prices $p(z^t)$ are simply marginal utilities. Using eqs. (2.3) and (3.4), the price of a one-period risk-free bond can be expressed as

$$s_i(z^t)=\sum_{z_{t+1}} P_i(z^{t+1})/P_i(z^t)$$

$$= \sum_{z_{t+1}} \beta\pi(z_{t+1} \mid z^t)[p(z^{t+1})/p(z^t)]$$

$$= \beta \mathrm{E}_t[W(z^t)/W(z^{t+1})]^\gamma, \tag{3.6}$$

where $\pi(z_{t+1} \mid z^t) \equiv \pi(z^t, z_{t+1})/\pi(z^t)$ is the probability of z_{t+1} conditional on z^t and E_t is the expectation based on this density. The rate of return on the bond is the same for all countries because there is only one good: the real interest rate differential between any two countries is zero.

4. Equilibrium with non-traded goods

We now examine the influence of non-traded goods on equilibrium prices and quantities. We derive properties of the theoretical economy, and point out implications for aggregate time series for real exchange rates, consumption, and real interest rates. The main finding is a monotonicity result, showing that consumption ratios and real exchange rates are positively related in a cross-section of pairs of countries.

We compute equilibria by the Mantel–Negishi algorithm: the social planner chooses quantities $\{a_i, b_i\}$ to maximize

$$\sum_{i=1}^{I} \lambda_i U_i = \sum_{i=1}^{I} \lambda_i \sum_{t=0}^{T} \beta^t \sum_{z^t} \pi(z^t) u[a_i(z^t), b_i(z^t)], \tag{4.1}$$

subject to the resource constraints

$$\begin{aligned} &\sum_{i=1}^{I} a_i(z^t) \leqq W(z^t), \quad \forall z^t, \\ &b_i(z^t) \leqq x_i(z^t), \quad \forall i, z^t. \end{aligned} \tag{4.2}$$

The Lagrange multipliers will be denoted $\beta^t\pi(z^t)q_0(z^t)$ for the traded goods constraints and $\beta^t\pi(z^t)q_i(z^t)$ for the non-traded goods. Again the first-order conditions are

$$\begin{aligned} &\lambda_i\, \partial u[a_i, b_i]/\partial a_i = q_0, \\ &\lambda_i\, \partial u[a_i, b_i]/\partial b_i = q_i, \end{aligned} \tag{4.3}$$

$\forall i, z^t$.

The first-order conditions illustrate some of the features of an equilibrium. In general PPP does not hold, because endowments, and hence the prices of non-traded goods, differ across countries. Likewise consumption indexes are not perfectly correlated, in general, because endowments of non-traded goods, x, will not be so and because marginal utilities of consumption of the traded good will not be equalized if the state utility function is not additively separable between traded and non-traded goods.

As in section 3, we work with isoelastic period utility functions; now there also is a constant elasticity of substitution between traded and non-traded goods. The period utility function is

$$u(a,b)=[[\alpha a^{\rho}+(1-\alpha)b^{\rho}]^{1/\rho}]^{1-\gamma}/(1-\gamma), \tag{4.4}$$

with $\rho \leqq 1$, $\gamma \geqq 0$, and $\alpha \in [0,1]$. Goods are perfect substitutes if $\rho=1$; $\rho=0$ is the Cobb–Douglas case. The elasticity of substitution between traded and non-traded goods is $(1-\rho)^{-1}$. This functional form is general enough to include most of the forms used in applied studies. For example, (4.4) is used by Tesar (1993) and by Stockman and Tesar (1990), who adopt $\rho=-1.27$, a value suggested by some empirical evidence. Other researchers have used special cases, discussed below, with unit elasticity or additive separability.

Index numbers can be found by Euler's Theorem and the first-order conditions in (4.3). In this case,

$$c(a,b)=[\alpha a^{\rho}+(1-\alpha)b^{\rho}]^{1/\rho}, \tag{4.5}$$

which is the homogeneous image of (4.4). Combining the first-order condition with the requirement that $a_i q_0 + b_i q_i = p_i c_i$ gives the linear homogeneous price index:

$$p(q_0,q_i)=[\alpha^{1/(1-\rho)}q_0^{\rho/(\rho-1)}+(1-\alpha)^{1/(1-\rho)}q_i^{\rho/(\rho-1)}]^{(\rho-1)/\rho}, \tag{4.6}$$

where index weights are budget shares.

Proposition 2. Let the period utility function be isoelastic as in (4.4). Then along any equilibrium path there is a monotone relation between the bilateral real exchange rate, e_{ij}, and the consumption ratio, c_i/c_j: if the real exchange rate is higher in one state than in another, then so is the consumption ratio.

Proof. The first-order conditions are

$$q_0=\lambda_i c_i^{1-\rho-\gamma}\alpha a^{\rho-1}$$

and

$$q_i=\lambda_i c_i^{1-\rho-\gamma}(1-\alpha)b^{\rho-1}.$$

Substituting these conditions into the definition of $p(q_0,q_i)$ in (4.6) gives

$$\lambda_i c_i^{-\gamma}=p_i.$$

Thus

$$(\lambda_j/\lambda_i)(c_i/c_j)^\gamma = p_j/p_i = e_{ij}, \tag{4.7}$$

so that the consumption ratio and real exchange rate are monotonically related. □

The functional form of the period utility function in eq. (4.4), which is sufficient for this monotonicity result between consumption ratios and real exchange rates, is of interest because many applications use period utility with this form. Some other studies use additive separability (i.e. $\partial^2 u/\partial a\, \partial b = 0$ or $\rho + \gamma = 1$) which is stronger but also will give the result in the proposition more generally even without the isoelastic utility function (4.4) [e.g. as in Stockman and Dellas (1989)].

Proposition 2 applies state by state and hence applies to moments, when they exist. The isoelastic form suggests that we study moments of growth rates: eq. (4.7) implies that

$$\gamma \Delta \log(c_i/c_j) = \Delta \log(e_{ij}). \tag{4.8}$$

The proposition gives sufficient conditions for two types of implications. First, a pair of countries for which the growth rate in the ratio of aggregate consumptions has a relatively large mean or standard deviation will have a real exchange rate with similar properties. The means and standard deviations of the two growth rates will not be equal unless $\gamma = 1$, but the monotonicity implication can be tested using cross-section correlations of the moments, which are robust to the value of γ, or cross-section rank correlations of the moments, which also are robust to some measurement error. The autocorrelations of the two growth rates are equal in the theory, and this moment also can be studied using rank orderings. The second implication is that the time-series cross-correlation between the growth rate of relative consumptions and the growth rate of relative prices should be unity for all pairs of countries.

Although the endowments of non-traded goods, x_i, are unobservable, Proposition 2 thus suggests several simple tests of the theory, for any values of ρ, γ, and α. We next illustrate the implications with two examples. Then in section 5 we test the monotonicity property and the cross-correlation property for eight OECD countries.

Example 1. Suppose that $\rho = 0$ and $\gamma = 1$. Thus there is additive separability and the state utility function is

$$u(a, b) = \alpha \log a + (1 - \alpha) \log b. \tag{4.9}$$

This form was used by Stulz (1987), for example. Price and quantity indexes are

$$p_i(q_0, q_i) = q_0^{\alpha} q_i^{1-\alpha} / [\alpha^{\alpha}(1-\alpha)^{1-\alpha}] \tag{4.10}$$

and

$$c_i(\alpha_i, b_i) = a_i^{\alpha} b_i^{1-\alpha}. \tag{4.11}$$

Using the normalization $\sum_{j=1}^{I} \lambda_j = 1$, the solution to the planning problem is, for every state,

$$q_i = (1-\alpha)\lambda_i / x_i,$$

$$q_0 = \alpha / W,$$

$$a_i = \lambda_i W,$$

$$b_i = x_i,$$

with aggregate indexes

$$p_i = (\lambda_i / x_i)^{1-\alpha} / W^{\alpha},$$

$$c_i = \lambda_i^{\alpha} x_i^{1-\alpha} W^{\alpha}.$$

Consider the properties of the real exchange rate, real interest rate, and consumption ratios in this economy. The real exchange rate between countries i and j is

$$e_{ij} \equiv p_j / p_i = (\lambda_j / \lambda_i)^{1-\alpha} (x_i / x_j)^{1-\alpha}. \tag{4.12}$$

It varies over time as the relative endowments of non-traded goods in the two countries vary. This example has the property that endowments of other countries do not affect the bivariate relation between prices. The aggregate consumption ratio has the same property:

$$c_i / c_j = (\lambda_i / \lambda_j)^{\alpha} (x_i / x_j)^{1-\alpha}. \tag{4.13}$$

Except for the multiplicative constant, fluctuations in e_{ij} and c_i/c_j are identical. Their growth rates are perfectly correlated and have the same probability density function. Note that if $\alpha = 1$, then there are no non-traded goods and, as in section 3, relative prices and quantities do not fluctuate.

The price of a risk-free claim to the domestic consumption bundle is

$$s_i(z^t) = \beta E_t[p_i(z^{t+1}) / p_i(z^t)]$$

$$=\beta \mathrm{E}_t[x_i(z^t)/x_i(z^{t+1})]^{1-\alpha}[W(z^t)/W(z^{t+1})]^{\alpha}.$$

This price depends on the growth rates of both the traded and non-traded goods and on their covariance. Clearly it varies across countries as long as the behavior of non-traded goods endowments differs. The bond price differential between countries i and j is

$$s_i(z^t)-s_j(z^t)=\beta \mathrm{E}_t[[x_i(z^t)/x_i(z^{t+1})]^{1-\alpha} - [x_j(z^t)/x_j(z^{t+1})]^{1-\alpha}][W(z^t)/W(z^{t+1})]^{\alpha}. \tag{4.14}$$

As an illustration, suppose that z^{t+1} is predictable one step ahead. In that case the (continuously compounded) real interest differential is

$$r_i(z^t)-r_j(z^t)=(1-\alpha)[\Delta \log x_i(z^{t+1})-\Delta \log x_j(z^{t+1})]. \tag{4.15}$$

In this illustration the real interest differential (or the ratio of risk-free claim prices) depends only on the non-traded goods endowments.

The separable period utility function in Example 1 gives rise to strong predictions about the real exchange rate, consumption, and real interest rates: the growth rate of the real exchange rate, $\Delta \log(e_{ij})$, the growth rate of the consumption ratio, $\Delta \log(c_i/c_j)$, and (approximately) the real interest rate differential have identical moments and dynamics. Thus introducing non-traded goods into the theoretical model may yield realistic relative price movements and consumption correlations but also has strong implications for the relation between the two. The next example suggests that the strong form of the connection between variables depends on the functional form, but from Proposition 2 some weaker restrictions remain even with more general preferences.

Example 2. Suppose that $\rho=0$. The period utility function is

$$u(a,b)=[a^{\alpha}b^{1-\alpha}]^{1-\gamma}/(1-\gamma). \tag{4.16}$$

The special case of $\gamma=0$ gives the Cobb–Douglas function used by Dornbusch (1983), for example. Aggregate price and consumption indexes are unaffected by the power function applied to the CES aggregator and so are the same as in Example. 1. Equilibrium prices and quantities are

$$q_0=\alpha(D/W)^{\delta},$$

$$q_i=(1-\alpha)(\lambda_i x_i^{-\gamma})^{1/\delta}(W/D)^{1-\delta},$$

$$a_i=[(\lambda_i x_i^{\delta-\gamma})/D]\,W,$$

where $\delta \equiv 1-\alpha(1-\gamma)>0$ and $D \equiv \sum_{j=1}^{I} \lambda_j^{1/\delta} x_j^{(\delta-\gamma)/\delta}$. The indexes, at these values, are

$$c_i = \lambda_i^{\alpha/\delta} x_i^{(1-\alpha)/\delta} (W/D)^{\alpha}$$

and

$$p_i = (\lambda_i x_i^{-\gamma})^{(1-\alpha)/\delta} (W/D)^{\alpha\delta}.$$

The real exchange rate is

$$e_{ij} = (\lambda_j/\lambda_i)^{(1-\alpha)/\delta} (x_i/x_j)^{\gamma(1-\alpha)/\delta}. \tag{4.17}$$

The consumption ratio is

$$c_i/c_j = (\lambda_i/\lambda_j)^{\alpha/\delta} (x_i/x_j)^{(1-\alpha)/\delta}. \tag{4.18}$$

As illustrated more generally in eq. (4.8), the consumption ratio is smoother and less sensitive to differences in growth rates of the endowments of non-traded goods than is the real exchange rate if $\gamma > 1$, and the reverse if $\gamma < 1$.

5. OECD consumption ratios and real exchange rates, 1971–1990

The tests in this section use data from eight OECD countries: Australia (A), Canada (C), France (F), West Germany (G), Japan (J), Sweden (S), the United Kingdom (K), and the United States (U) for 1971–1990. Data sources are given at the end of the paper. Consumption series are measured as quarterly, seasonally adjusted, real, total private consumption expenditures. Their deflators are used with quarterly average nominal exchange rates to construct real exchange rates.

We first study the monotonicity property. The theory implies a positive relation between consumption ratios and real exchange rates state by state and hence also in moments. We use the growth-rate transformation suggested by isoelastic utility and illustrated in Examples 1 and 2 of section 4. Figs. 1–3 graph moments of the growth rates of real exchange rates [$\Delta \log(e_{ij})$] on the horizontal axes versus moments of the growth rates of consumption ratios [$\Delta \log(c_i/c_j)$] on the vertical axes. Data points for pairs of countries are labelled with the letters given in brackets in the previous paragraph, with country i listed first. For example, the point labelled CU (Canada/United States) has as its abscissa a moment of $\Delta \log(e_{\mathrm{CU}}) = \Delta \log(p_{\mathrm{U}}/p_{\mathrm{C}})$ and as its ordinate the same moment of $\Delta \log(c_{\mathrm{C}}/c_{\mathrm{U}})$. Fig. 1 gives quarterly standard deviations $\times$ 100, fig. 2 gives first-order autocorrelations, and fig. 3 gives means $\times$ 100.

In section 1 we referred to the extent and persistence of deviations from PPP. The horizontal axis of fig. 1 shows the variability in real exchange rate

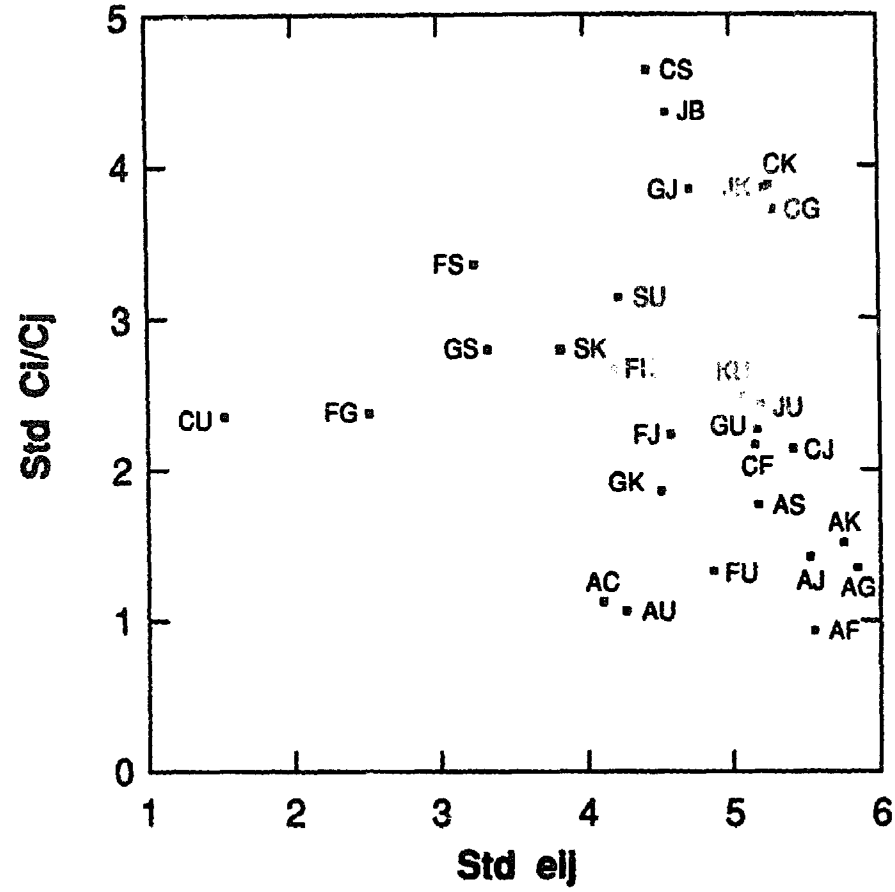

Fig. 1. Standard deviations (growth rates).

changes. The horizontal axis of fig. 2 shows the persistence in growth rates of relative prices, while that of fig. 3 shows their small means. This evidence can be compared with that of Mussa (1986), for example. Similar properties for cross-country consumption ratios can be read from the vertical axes.

From Proposition 2 and Example 2, the means and standard deviations of the growth rates of the price ratios exceed those of the quantity ratios if $\gamma > 1$, that is if there is less intertemporal substitution than that characterized by the logarithmic period utility function. From figs. 1 and 3 we observe that real exchange rates tend to be more variable and have larger means (in absolute value) than do consumption ratios, which facts are consistent with $\gamma > 1$.

Proposition 2 shows that, in the theoretical economy with standard preferences and non-traded goods, scatterplots of these moments lie around upward-sloping lines. For example, a pair of countries with a relatively variable consumption ratio also has a relatively variable real exchange rate. More specifically, the theoretical points in figs. 1 and 3 lie on a line through the origin with slope γ^{-1}, while those in fig. 2 lie on a 45° line through the origin. Some specific evidence seems consistent with these predictions. Fig. 1

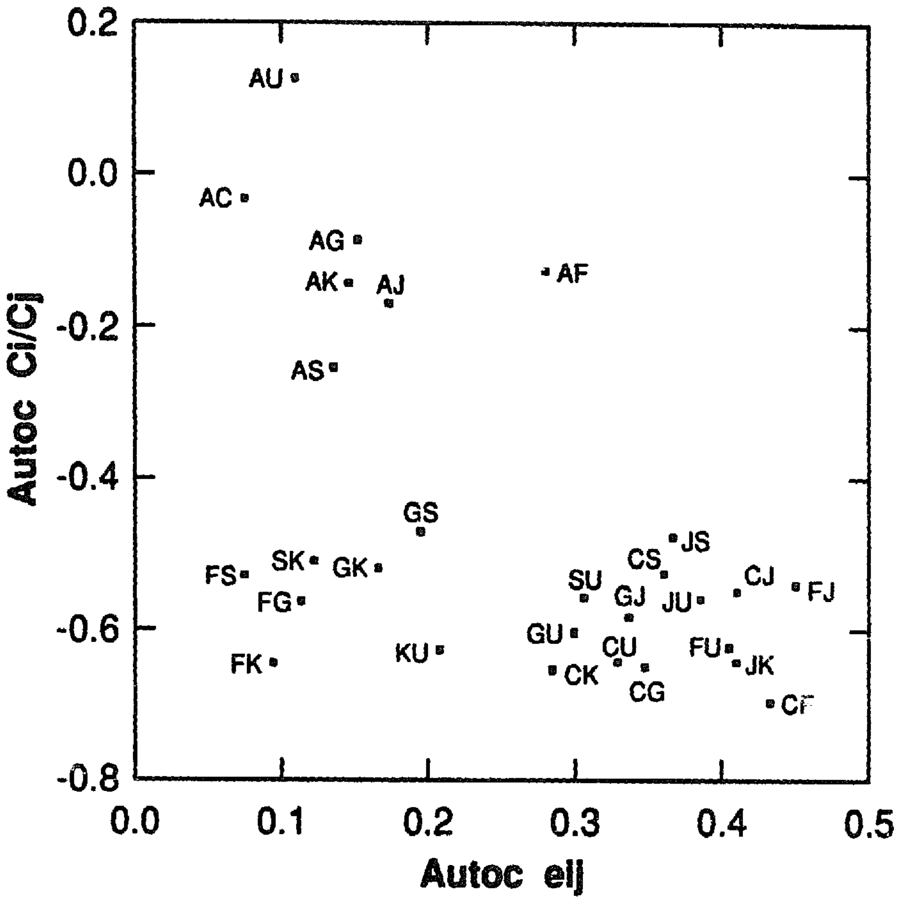

Fig. 2. Autocorrelations (growth rates).

shows that Canada–U.S. real exchange rate growth has the smallest variance among values for these pairs of countries and that the growth rate of the Canada–U.S. consumption ratio also is among the least variable in the set. In fig. 3 there is a positive relationship between mean growth rates if Japan is excluded.

In general, though, the cloud-like patterns found in the figures provide little support for the theoretical model. The rank correlations in the three figures are -0.263, -0.466, and 0.074. The null hypothesis that there is no coincidence in rankings can be studied using the normal approximation to the sampling distribution; when $n=28$ this gives a standard error of $(n-1)^{-0.5}=0.192$. With this standard error only the negative rank correlation in fig. 2 (autocorrelations) is significant at conventional significance levels. Thus there certainly is no evidence of positive rank correlation, and figs. 1–3 show no positive relationships.

Some care is required in interpreting the rank correlation in fig. 3 (means). This diagram contains only 7 observations, because the rest follow from transitivity: $\mathrm{E}(\Delta\log(c_i/c_j))+\mathrm{E}(\Delta\log(c_j/c_k))\equiv\mathrm{E}(\Delta\log(c_i/c_k))$ and similarly for relative prices. The effect of this can be seen by noting that in theory

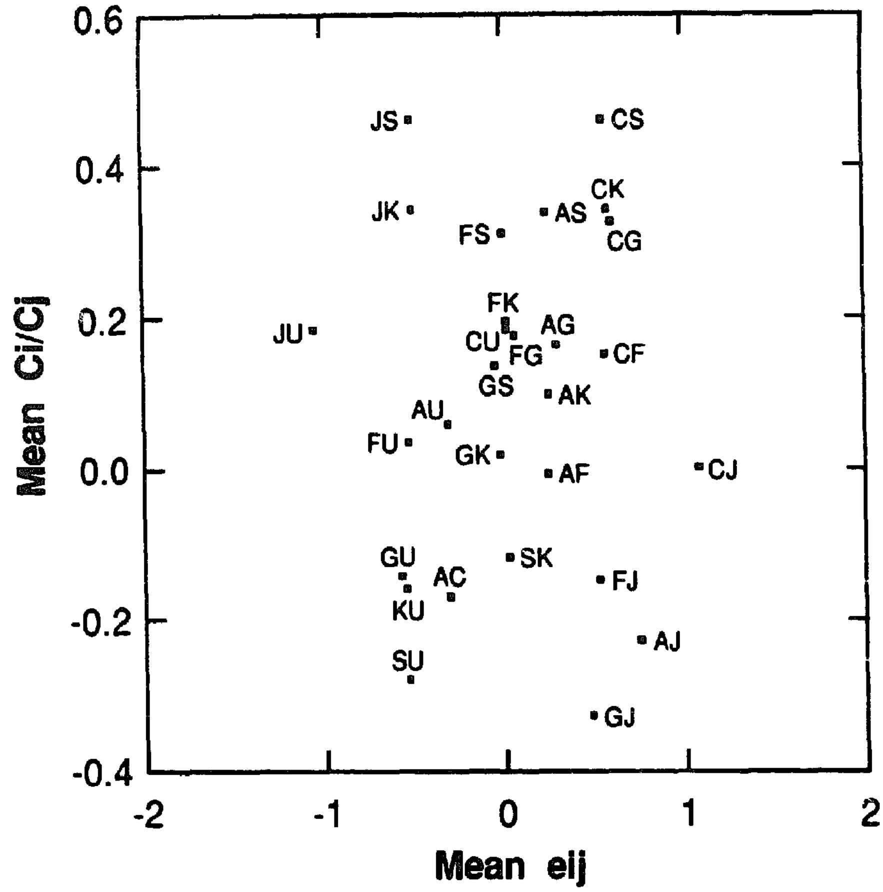

Fig. 3. Means (growth rates).

$E(\Delta \log(c_i/c_j))$ and $E(\Delta \log(e_{ij})$ have the same sign so that the graph lies in the first and third quadrants, in addition to sloping up. If consumption grows more rapidly in country i than in country j on average, then country i's real exchange rate with country j should depreciate on average. When this implication holds, points ij, jk, and ik lie on a straight line, sloping up, by transitivity. Here the standard error quoted above would be too small, because in fact $n=7$. (For points in the second and fourth quadrants this straight-line relation does not hold, despite transitivity, because of reflection in one axis.) Thus, including all 28 points biases the case in favor of the theory; even so, no significant positive relation can be detected.

Further implications of the theory are that the growth rates of consumption ratios and of real exchange rates should have identical dynamics and be perfectly correlated. Fig. 2 shows that the growth rates of all 28 bilateral real exchange rates are positively autocorrelated, while 27 of the growth rates of consumption ratios are negatively autocorrelated. In addition, the cross-correlation between the growth rate of the consumption ratio and the growth rate of the real exchange rate, averaged across countries, is 0.045, with a range of $[-0.08, 0.17]$. Thus there is little evidence in favor of either of these implications of the theory. A related prediction from the first-order conditions in (4.7) is that $\log(c_i)$, $\log(c_j)$, and $\log(e_{ij})$ are cointegrated, if integrated. Kollmann (1991) finds that this implication can be statistically rejected in several data sets, including the OECD data studied here.

The conclusions do not change when we measure consumption in per capita terms. In annual, per capita data the rank correlations in the counterparts to figs 1–3 are -0.114, -0.045, and 0.170. The average correlation between relative price growth and real consumption growth is -0.056, with a range of $[-0.63, 0.21]$. The figures do change with per capita data (because Australian population grew much faster in these two decades than did German population, for example) but the findings do not. We also find no support for the theory when we measure consumption only of non-durables and services. We have found similar results for other transformations and also for moments of interest rates. Further evidence could be collected for a longer span of annual data or for additional countries.

The results of inspecting the figures and of statistical tests of the monotonicity and cross-correlation implications are striking because they are based on weak assumptions. They do not require us to restrict parameter values, to make auxiliary assumptions about detrending, to specify laws of motion for the endowment processes, or to identify specific categories of consumption goods as traded or non-traded (although implicit separability is required for aggregation into two groups).

6. Conclusions

We have examined non-traded goods as a device to account for two features of international time series: deviations from purchasing power parity and imperfect correlations of consumption fluctuations across countries. Although changes in endowments of non-traded goods may be unobservable, their effect on several observable properties of prices and quantities can be used to evaluate this approach. Empirically, growth rates of relative consumption tend to be negatively autocorrelated, whereas growth rates of real exchange rates tend to be positively autocorrelated. And there is no systematic cross-correlation between the two growth rates. Moreover, pairs of OECD countries with relatively stable consumption ratios do not have relatively stable real exchange rates. Yet such parallels should be found if fluctuations in the non-traded good sector account for international fluctuations, under standard models of preferences.

What features of a more general model might reproduce the patterns evident in figs. 1–3? One possibility would be to admit demand-side shocks (such as taste shocks), in addition to the endowment shocks studied here. Taste shocks lead to a negative correlation between changes in relative consumption and in the real exchange rate in contrast to the positive correlation arising from endowment shocks. The relative importance of the two types of shocks might be identified from the empirical correlations given in section 5, which are near zero [see also Stockman and Tesar (1990)]. But the theory still predicts that pairs of countries with volatile relative consumptions will have volatile relative prices, unless there is considerable heterogeneity across countries either in preferences or in the relative importance of the two types of shocks. Fig. 1 provides little evidence of this property.

Other possibilities include (i) wealth effects (i.e. departures from homotheticity, which could be studied by simulation), (ii) measurement error in these data (e.g. from the use of fixed-weight indexes), and (iii) incomplete markets. A further possibility is that the empirical evidence can be accounted for by a model with non-competitive features and spatial separation or segmentation (pricing to market). Perhaps future work, as well as evidence for other countries and time periods, will help us distinguish among these alternatives.

Appendix: Data sources and definitions

(a) Consumption and deflators. Quarterly, seasonally adjusted, total private consumption expenditure and deflator, in constant prices of 1985, for 1971:1–1990:4. The first observation is lost in calculating growth rates. Volume and price indices come from the *OECD Quarterly National Accounts*; comparative tables.

(b) Nominal exchange rates. Quarterly average nominal exchange rates, 1971:1–1990:4, from CITIBASE. These are expressed in units of foreign currency per U.S.$, with the exception of rates for Australia and the United Kingdom which are the inverse.

(c) Population. Annual 1971–1990, from IFS.

(d) Consumption of non-durables and services. Quarterly, seasonally adjusted, consumption expenditures on non-durables and services in current prices and in constant prices, for 1971:1–1990:4. These series come from tables 6A and 6B of the *OECD Quarterly National Accounts.* They are given only for Canada, France, and the United States. The same source lists disaggregated consumption expenditures for Germany, Japan, and the United Kingdom, but these are not seasonally adjusted.

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