ECON 2021 - Financial Economics I

Lecture 11 – Heterogeneous Beliefs & Speculative Bubbles

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MOTIVATION

- It is obvious that asset markets feature heterogeneous beliefs. However, their role is subtle.
- Heterogeneous beliefs arise for two basic reasons:
  1. Heterogeneous information
  2. Heterogeneous priors
- There are 2 difficulties with the heterogeneous information approach
  1. Often prices reveal the info of other traders (Grossman & Stiglitz (1980))
  2. No Trade Theorems: If people only trade to make a profit, so trade is a zero-sum game, then if someone wants to sell to you, why would you buy?
- Harrison & Kreps (1978) assume that people have heterogeneous prior beliefs. People simply interpret the same data differently. In this case, both sides can expect to profit from trade (even though only one will, ex post).
- However, there is a problem with the this approach - Learning. Eventually, agents priors should merge together. Harrison & Kreps ignore learning.
- Scheinkman & Xiong (2003) extend the model of Harrison & Kreps to develop of theory of speculative bubbles, based on resale option values.

Trading Volume

- Trading volume is enormous. Billions of shares change hands every day on the NYSE. Volume in the forex market is even larger.

- The number of shares traded is misleading, since it is not scale or unit invariant. A better measure is turnover rate, i.e., what percentage of the value of shares is traded over a given time period. Weekly turnover rates are in the range 1-2% on the NYSE.

- There are intriguing correlations between prices and trading volume. For example, volume is positively correlated with the magnitude of price changes (both positive & negative).

- There are 4 basic reasons why people might trade:
  1. Dynamic Spanning/Hedging (Portfolio Rebalancing)
  2. Asymmetric Info. (Requires background noise)
  3. Heterogeneous Beliefs (Differences in opinion)
  4. They're crazy (or they just like to trade).

- The only one we've discussed is dynamic spanning. However, this seems completely inadequate to explain observed volume and its high frequency volatility.

- Asymmetric info is likely important. But the fact that trading responds to public info suggests that it can't be whole story.

Simple Example of Harrison & Kreps

1. Single asset, in fixed supply, normalized to 1.
2. Asset yields a sequence of dividends at $t = 1, 2, 3, \cdots$
3. Dividends are nonstorable, and there is no goods market or rental market.
4. Two ∞-lived, risk-neutral, agent types: Mr. E and Mr. O
5. Agents have common discount factor β, and behave competitively.
6. No short-selling

6. **Beliefs**

E believes $d_t = \begin{cases} 1 & \text{if } t \text{ is even} \\ 0 & \text{if } t \text{ is odd} \end{cases}$

O believes $d_t = \begin{cases} 0 & \text{if } t \text{ is even} \\ 1 & \text{if } t \text{ is odd} \end{cases}$

7. No Learning. Beliefs are never revised.

8. No capital limits. Agents can always afford the asset.

9. Trading occurs at the beginning of each period, before dividends are announced.

10. Consider period 1. If agents must hold the asset,

$$V_O = 1 + \beta^2 + \beta^4 + \cdots = \frac{1}{1-\beta^2}$$
$$V_E = \beta + \beta^3 + \beta^5 + \cdots = \frac{\beta}{1-\beta^2}$$

11. Since $V_O > V_E$, Mr. O buys the asset. (Mr. E cannot short sell).

12. In period 2, roles are reversed. Mr. E buys from Mr. O.

13. It would appear the equilibrium asset price is $P = \frac{1}{1-\beta^2}$

14. Suppose $t = 1$, and Mr. O considers the following:

    Speculative Trading Strategy: *Buy asset now, collect the dividends I expect this period, then sell at $t = 2$ for $P = \frac{1}{1-\beta^2}$, when I expect no dividends*.

    $$\Rightarrow \qquad 1 + \beta\frac{1}{1-\beta^2} = \frac{1+\beta-\beta^2}{1-\beta^2} > \frac{1}{1-\beta^2}$$

15. Next period, the same reasoning applies to Mr. E. So now the equilibrium price appears to be $P = \frac{1+\beta-\beta^2}{1-\beta^2}$.

16. By induction we have

$$\bar{P} = \lim_{n\to\infty} \frac{1+\beta-\beta^n}{1-\beta^2} = \frac{1+\beta}{1-\beta^2} = \frac{1}{1-\beta} > \frac{1}{1-\beta^2}$$

17. At $\bar{P}$ there is no longer any advantage to the buy & sell strategy $1 + \beta\frac{1}{1-\beta} = \frac{1}{1-\beta}$. We have an equilibrium.

Comments

- It is not restrictive to limit the trading strategy to selling after just 1 period. (Doob's Optimal Stopping Theorem).
- There is still a sense in which price is the expected PDV of dividends. At each date we just use the expectation of the currently most optimistic agent:

$$\bar{P} = 1 + \beta + \beta^2 + \cdots = \frac{1}{1-\beta}$$

- The difference between the "buy & hold" price and the speculative trading price

$$\frac{1+\beta}{1-\beta^2} - \frac{1}{1-\beta^2} = \frac{\beta}{1-\beta^2}$$

can be interpreted as a resale option value or a bubble.

Scheinkman & Xiong (2003)

- Scheinkman & Xiong (2003) develop a continuous-time version of the Harrison/Kreps model in which belief heterogeneity is driven by filtering.
- Two risk-neutral (groups of) agents can trade an asset. The asset yields dividends

$$dD_t = f_t dt + \sigma_D dB_t$$

The drift, f_t is unobserved, but is known to follow the mean-reverting process

$$df_t = -\lambda(f_t - \bar{f})dt + \sigma_f dB_t^f$$

- The agents observe two unbiased signals

$$ds_t^i = f_t dt + \sigma dB_t^i \qquad i = A, B$$

Group A thinks dB^A is correlated with f_t, while group B thinks dB^B is correlated with f_t. In reality, neither is correlated with f_t. Hence, both agents are 'overconfident', but overreact to distinct signals. This drives belief heterogeneity.

- Let $g^i = \hat{f}^i - \hat{f}^{-i}$ be the belief differences between agent-i and the other agent. SX show that it follows

$$dg^i = -\rho g^i dt + \sigma_g dB^i$$

where $\rho > 0$ depends on the model's underlying parameters. Note that belief differences are mean-reverting.

- The buy-and-hold price of agent-i is

$$P_t^i = E_t^i \int_t^{\infty} e^{-r(s-t)} D_s ds = \frac{\bar{f}}{r} + \frac{\hat{f}^i - \bar{f}}{r+\lambda}$$

where r is a constant, exogenous, risk-free interest rate.

- However, if the agent can sell to the other agent, his valuation becomes

$$P_t^i = \max_{\tau \geq 0} E_t^i \left[\int_t^{t+\tau} e^{-r(s-t)} D_s ds + e^{-r\tau}(P_{t+\tau}^{-i} - c) \right]$$

where c is a transaction cost paid by the seller. Note that this is a perpetual American option pricing problem, with the added twist that the 'strike price' is endogenous, and depends itself on the option value.

- SX pursue a 'guess-and-verify' approach. They conjecture that the solution is of the form, $P_t^i = \frac{\bar{f}}{r} + \frac{\hat{f}^i - \bar{f}}{r+\lambda} + Q(g_t^i)$. Using Ito's lemma, they show Q satisfies the 2nd-order ODE

$$\frac{1}{2}\sigma_g^2 Q''(x) - \rho x Q'(x) - rQ(x) = 0$$

- The solution of this equation is given by the 'confluent hypergeometric function'

$$Q(x) = A \cdot M\left(\frac{r}{2\rho}; \frac{1}{2}; \frac{\rho x^2}{\sigma_g^2}\right)$$

where A is a constant of integration and M is given by the infinite series

$$M(a;b;z) = 1 + \frac{az}{b} + \frac{(a)_2 z^2}{(b)_2 2!} + \cdots$$

and $(a)_n = a(a+1)(a+2)\cdots(a+n-1)$.

- The integration constant A and the selling threshold, x^*, are determined by value-matching and a smooth-pasting conditions,

$$\begin{aligned} Q(x^*) &= \frac{x^*}{r+\lambda} + Q(-x^*) - c \qquad \text{Value-Matching} \\ Q'(x^*) &= \frac{1}{r+\lambda} - Q'(-x^*) \qquad \text{Smooth-Pasting} \end{aligned}$$

- SX interpret the resale option component of the price, Q, as a speculative bubble. It is state-dependent, with dynamics that resemble many observed bubble episodes. In contrast to 'rational bubble' theories, the SX model generates a positive correlation between bubbles and trading volume.

Chen & Kohn (2011)

- Chen & Kohn (2011) develop a simplified version of the SX model without filtering. Two risk-neutral agents have heterogeneous beliefs about mean reversion,

$$dD = \kappa_i(\theta - D)dt + \sigma dB \qquad \kappa_1 > \kappa_2$$

- Hence, Agent 1 is relatively optimistic when $D < \theta$, while Agent 2 is the relative optimist when $D > \theta$.

- The buy-and-hold price of agent-i is

$$P_t^i = E_t^i \int_t^\infty e^{-r(s-t)} D_s ds = \frac{\theta}{r} + \frac{D_t - \theta}{r + \kappa_i}$$

- Once again, the option to sell to a future relative optimist drives prices above this

$$P_t = \frac{\theta}{r} + \max_i \left[\frac{D_t - \theta}{r + \kappa_i} \right] + Q(D)$$

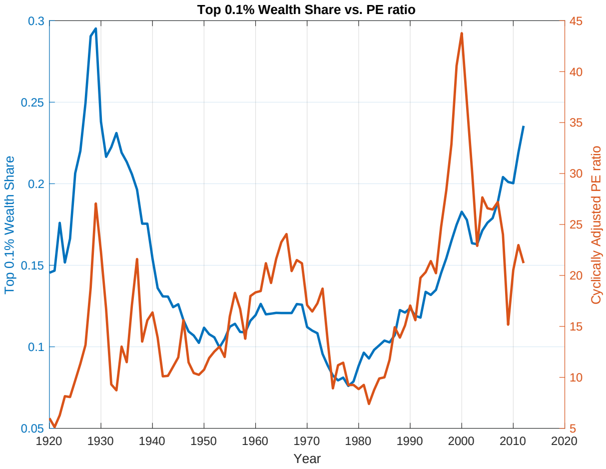
where again Q solves the hypergeometric ODE

$$\frac{1}{2}\sigma^2 Q'' + \max\{\kappa_1(\theta - D), \kappa_2(\theta - D)\}Q' - rQ = 0$$

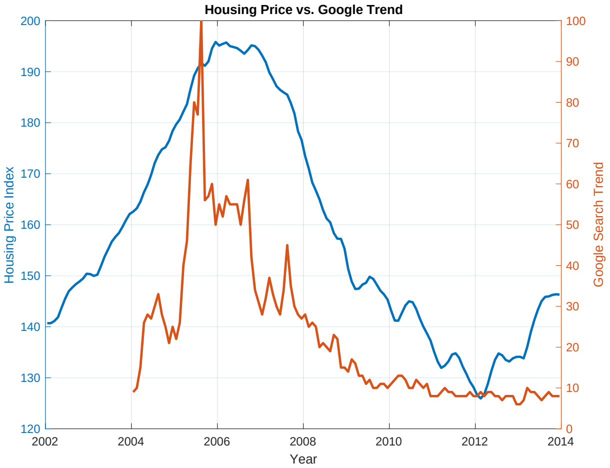
Doubts, Inequality, and Bubbles

- Cho & Kasa (2018) introduce ambiguity aversion into the Scheinkman/Xiong model.
- Doubts depend on wealth. Doubts increase with $\alpha W \cdot V'(W)$.
- Since wealth is endogenous, so is belief heterogeneity.
- Belief heterogeneity creates a resell option value.
  $\Rightarrow \quad V''(W) > 0$
  $\Rightarrow$ Doubts increase with wealth.
- Prediction: Bubbles arise when wealth inequality increases.

Bubbles and Inequality



Bubbles and Doubts (?)



Main Ingredients

- Two assets.
  1. Risk-free with constant interest rate $r = \rho$.
  2. Indivisible dividend-paying asset. Supply normalized to 1.

- No goods or asset rental markets. If you want to consume, you must own the dividend-paying asset.

- Two (sets of) agents. Agents are risk-neutral but ambiguity averse. Agents have 'doubts' about the dividend process.

- Doubts produce pessimistic belief distortions that increase with wealth.

- Relative wealth depends on asset ownership.

Technology & Preferences

Technology

$$dx = \alpha(b - x) \cdot dt + \sigma\sqrt{x} \cdot dB$$

Preferences

$$V(x_0) = \hat{E}_0 \int_0^\infty 2\rho x e^{-\rho t} dt$$

$$\begin{aligned} \hat{E}(x) &= \int x dq = E(zx) = \int zx dq^0 \\ z_t &= 1 + \int_0^t z_s h_s dB_s \end{aligned}$$

Two Robust Control Formulations

1. Distorted measure

$$V(x_0) = \min_h \hat{E}_0 \int_0^\infty \left[2\rho x_t + \frac{1}{2\varepsilon} h_t^2\right] e^{-\rho t} dt$$

subject to

$$dx = [\alpha(b - x) + \sigma\sqrt{x}h] \cdot dt + \sigma\sqrt{x} \cdot d\tilde{B}$$

2. Actual measure

$$V(z_0, x_0) = \min_h E_0 \int_0^\infty z_t \left[2\rho x_t + \frac{1}{2\varepsilon} h_t^2\right] e^{-\rho t} dt$$

subject to

$$\begin{aligned} dx &= \alpha(b - x) \cdot dt + \sigma\sqrt{x} \cdot dB \\ dz_t &= z_t h_t dB_t \qquad z_0 = 1 \end{aligned}$$

Single-Agent Competitive Equilibrium

HJB Equation

$$\rho V(z,x) = \min_h \left\{ z\left[2\rho x + \frac{1}{2\varepsilon}h^2\right] + \alpha(b-x)V_x + \frac{1}{2}\sigma^2 x V_{xx} + \frac{1}{2}(zh)^2 V_{zz} + \sigma\sqrt{x} z h V_{xz} \right\}$$

Proposition 3.1: *A first-order perturbation approximation of the buy-and-hold price is given by*

$$P(z,x) = \frac{2z}{\rho+\alpha}\left[(\alpha b + \rho x)\left(1 - \varepsilon\frac{\sigma^2\rho^2}{\rho(\rho+\alpha)^2}\right)\right] + O(\varepsilon^2)$$

Note: Without heterogeneity, robust prices are *lower*.

Planner's Problem

Optimal policy is characterized by a stopping time problem:

$$V(z_{1,t}, z_{2,t}, x_t, \iota) = \max_{\tau} \min_{h_1,h_2} E_t \left\{ \int_t^{t+\tau} [(\iota z_{1,s} + (1-\iota) z_{2,s}) x_s] + \frac{1}{2\varepsilon}(z_{1,s} h_{1,s}^2 + z_{2,s} h_{2,s}^2)] e^{-\rho s} \right.$$

$$\left. + \, \mathrm{e}^{-\rho(t+\tau)} [V(z_{1,t+\tau}, z_{2,t+\tau}, x_{t+\tau}, 1-\iota) - \varepsilon \cdot c] \right\}$$

subject to

$$\begin{aligned} dx &= \alpha(b-x)dt + \sigma\sqrt{x} \cdot dB \\ dz_i &= z_i h_i \cdot dB \qquad i = 1,2 \end{aligned}$$

Can show

$$V(z_1, z_2, x, \iota) = (z_1 + z_2)\hat{V}(\theta, x, \iota) \qquad \theta = \frac{z_1}{z_1 + z_2}$$

where

$$d\theta = \theta(1-\theta)(h_1 - h_2)\{-(\theta h_1 + (1-\theta)h_2)dt + dB\}$$

Value & Policy Functions

Value Function ($O(\varepsilon)$ approx.)

$$\hat{V}(\theta^o, x) = \theta^o(b+x)\Phi + \varepsilon B(\theta^o)e^{x/b}$$

where $\Phi = 1 - \varepsilon\frac{\sigma^2}{4\rho}$ and $B(\theta^o) = \frac{b\Phi}{2\varepsilon}\left[2\varepsilon e^{-A/2\varepsilon} + (1-2\theta^o)e^{-A/(1-2\theta^o)}\right]$

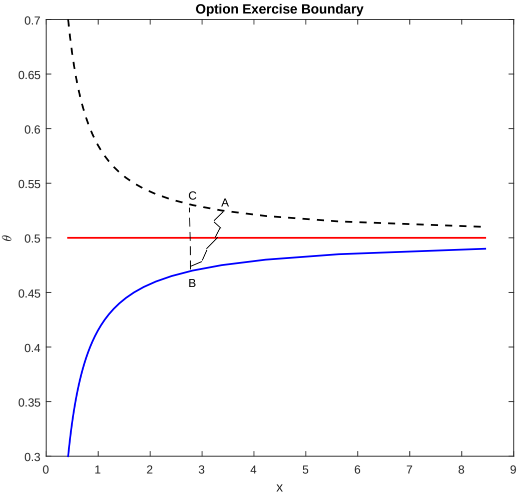
Policy Functions

$$\begin{aligned} h_1(\theta, x) &= -\varepsilon\sigma\sqrt{x}(\hat{V}_x + (1-\theta)\hat{V}_{\theta x}) \\ h_2(\theta, x) &= -\varepsilon\sigma\sqrt{x}(\hat{V}_x - \theta\hat{V}_{\theta x}) \end{aligned}$$

$$\begin{aligned} \hat{V}(\theta^o, x) &= \hat{V}(\theta^{no}, x) - \varepsilon \cdot c_z && \}\text{Value-Matching} \\ \hat{V}_x(\theta^o, x) &= \hat{V}_x(\theta^{no}, x) && \}\text{Smooth-Pasting} \\ \hat{V}_{\theta^o}(\theta^o, x) &= \hat{V}_{\theta^o}(\theta^{no}, x) && \}\text{Smooth-Pasting} \end{aligned}$$

Trading Dynamics

$$r = .02, \sigma^2 = .09, \varepsilon = .10, c = 2.0$$



DECENTRALIZATION

- These outcomes can be decentralized as a competitive equilibrium in many different ways, depending on the assumed asset market structure.
- These alternative decentralizations generate different trading volumes.
- Given our interests and observed data, it is natural to assume an equity claim and a riskless bond.
- Given the continuous-time/Gaussian information structure, this will deliver (dynamically) complete markets.

SIMULATIONS

$$r = .02,\ \sigma^2 = .09,\ \varepsilon = .10,\ c = 2.0$$