NEW YORK UNIVERSITY
Department of Economics

Econon-GA 2021 | Prof. Kasa

Financial Economics I | Fall 2018

PROBLEM SET 1
(Due September 24)

Each of the following questions is worth 10 points.

1. Download the dataset ProbSet1.xlsx from the class website. The final 2 columns contain (nominal) monthly returns on the 'market portfolio' (i.e., value-weighted portfolio of stocks listed on the NYSE, AMEX, and NASDAQ stock exchanges), and the 30-day Treasury Bill rate. (You can ignore the other 4 return series). The data run from July 1926 to June 2016.

   (a) Plot the market return and the T-Bill rate.

   (b) What were the annualized mean stock market and T-Bill returns? (Hint: Multiply the mean monthly returns by 12). What were their standard deviations? (Hint: Multiply the standard deviations of monthly returns by $\sqrt{12}$).

   (c) What was the mean equity premium? Is their any evidence that it has changed over time? Split the sample in half, and compute the mean equity premium in each half. Any difference?

   (d) What was the mean Sharpe ratio? Has it changed over time?

2. Assume there are two possible states of the world, s_1 and s_2. There are two assets: (1) a risk-free asset with an initial price of one, that pays R_f in each state, and (2) a risky asset with initial price one that pays R_d in state s_1 and R_u in state s_2. Assume without loss of generality that $R_u > R_d$.

   (a) What must be the relationship among (R_u, R_d, R_f) for there to be no arbitrage opportunities.

   (b) Assuming no arbitrage, compute the unique vector of state prices. Also compute the unique risk-neutral probabilities of states s_1 and s_2.

   (c) Now consider an option contract on the risky asset, which pays $\max[x - K, 0]$ for some constant K, where $x \in \{R_u, R_d\}$ is the unknown future price/payoff of the risky asset. Compute the no arbitrage price of this option.

3. Suppose you have a single return, R. By projecting onto the span of assets, we know $m_p = R/E(R^2)$ is one possible stochastic discount factor. (See, e.g., p. 64 in Back). What about R^{-1}? Clearly, $E(R^{-1}R) = 1$, so doesn't this violate the claim that m_p is unique? Do these stochastic discount factors rule out arbitrage?

4. Suppose a stock is currently worth \$20, and it is known that in 3 months it will be worth either \$22 or \$18. Consider on option on the stock with a strike price of \$21. This option will

either be worth \$1 (if the stock price increases) or worth nothing (if the stock price decreases). This question asks you to use no arbitrage reasoning to value this option contract. (Later we shall generalize this argument to much more complicated settings).

(a) Consider a portfolio consisting of a long position of Δ shares of the stock and a short position of one call option. Find a value of Δ that makes this a riskless portfolio (i.e., its payoff is the same, no matter what the future stock price turns out to be).

(b) Given this value of Δ, what will be the future value of the portfolio? Assuming the (annual) risk-free interest rate is 12%, what is the present value of this portfolio?

(c) What must therefore be the current no arbitrage price of the option? If the price deviated from this value, explain how you could make riskless profits.

(d) Why didn't we need to know the probability that the stock price would increase? Wouldn't this influence your valuation of a call option?

5. Suppose there are two states of the world, s_1 and s_2. Also suppose there are two assets: (1) A risky asset that pays 1 unit in s_1 and 3 units in s_2, and (2) A riskless asset that pays 1 unit in both states. Assume the riskless asset is in zero net supply. There are two agents: (1) A risk neutral agent with utility function $U = E(c)$, and (2) A risk averse agent with utility function $U = E\sqrt{c}$. Both agents assign equal probabilities to s_1 and s_2, and each is endowed with half the shares of the risky asset. Solve for the competitive equilibrium (relative) price of the risky asset, and compute the equilibrium allocation of the two securities. Explain your results intuitively.

6. This question explores the conditions that enable you to use observed asset prices to infer the (subjective) beliefs of market participants. Suppose there is a representative agent with time separable utility $U(\cdot)$ over consumption and time discount factor, δ. The investor's Euler equations price observed assets.

(a) First consider a stationary economy with no growth. Suppose there are two consumption states, C_H and C_L. Transitions between the two states follow a Markov process with transition probabilities f_{ij} for $i, j \in \{H, L\}$. Denote the four Arrow-Debreu security prices by p_{ij}. That is, p_{ij} is the price of a claim to one unit of consumption in state j tomorrow given state i today. Write down the four optimality (Euler) equations of the agent. Use them to solve for the ratio of marginal utilities, the time discount factor, and the transition probabilities as functions of the state prices, p_{ij}.

(b) Now introduce consumption growth. Assume that given the current level of consumption, consumption grows at either rate g_H or g_L. Consumption growth follows a Markov process with transition probabilities f_{ij} for $i, j \in \{H, L\}$. Finally, assume the representative agent has time-additive CRRA preferences, with coefficient of relative risk aversion, γ

(i) Show that the four Arrow-Debreu prices are independent of the current level of consumption. Solve for γ, δ, and f_{ij} as functions of the state prices and growth rates, g_H and g_L.

(ii) Now suppose consumption growth is i.i.d (i.e, $p_{LH} = p_{HH}$ and $p_{LL} = p_{HL}$). Show that recovery of beliefs from asset prices breaks down.

For more on this question, see Borovicka, Hansen, and Scheinkman (*Journal of Finance*, 2016) "Misspecified Recovery".