NEW YORK UNIVERSITY
Department of Economics

Econ-GA 2021 — Prof. Kasa
Financial Economics I — Fall 2018

PROBLEM SET 5
(Due December 10)

Each of the following questions is worth 10 points.

1. **Heterogeneity & Long-Run Survival**. Consider a continuous-time, complete markets economy consisting of two agent types, one with log preferences ($\gamma = 1$), and one with constant relative risk aversion greater than one ($\gamma > 1$). Assume aggregate consumption follows a geometric brownian motion

$$\frac{dC}{C} = \mu dt + \sigma dB$$

(a) Write down the planner's problem that characterizes the optimal (and equilibrium) allocation of consumption to the two agent types.

(b) Prove that the share of the $\gamma > 1$ agent converges to zero.

(c) Briefly discuss how this outcome would be supported in a competitive equilibrium with asset trade. Explain intuitively why the log agent eventually dominates.

(d) Now suppose both agent types have log preferences, but one of the agent types fears model misspecification, and so computes 'robust' consumption/portfolio policies, as in Hansen & Sargent. Suppose the entropy penalty is scaled as in Maenhout (2004), as discussed in class. Re-do the previous analysis to show that the robust agent ultimately 'disappears' from the economy. Again, explain this result intuitively.

2. **Long-Run Risk in Continuous-Time**. Suppose agents have Stochastic Differential Utility with IES=1, so that the (normalized) aggregator takes the form

$$f(C_t, V_t) = \delta(1-\gamma)V_t \left[\log(C_t) - \frac{1}{1-\gamma}\log[(1-\gamma)V_t]\right]$$

Suppose aggregate consumption follows the process

$$d\log C_t = \mu_c dt + X_t dt + \sigma_c dB$$

where the predictable growth component, X_t, follows the mean-reverting process

$$dX_t = -\alpha X_t dt + \sigma_x dB_t^x$$

Assume $E(dB \cdot dB^x) = \rho$.

(a) Write down the HJB equation that characterizes $V(C, X)$.

(b) Guess-and-verify that its solution is of the form: $V = \frac{1}{1-\gamma}\exp[(1-\gamma)(\log(C) + f(X))]$.

(c) Derive the ODE that characterizes $f(X)$, and show that its solution is of the form: $f(X) = f_0 + f_1 X$.

(d) Given the solution for V, compute the SDF process, using the expression given in class. Show that it has a permanent/martingale component. Use the SDF process to compute the equilibrium risk premium on a claim to the consumption process.

(e) Briefly discuss how this economy could instead be interpreted as one in which agents have time-separable CRRA preferences, but fear model misspecification, as in Hansen & Sargent.