1. This question asks you to replicate some of the calculations in Hansen, Sargent, and Tallarini (1999). Consider the simplified Robust Permanent Income model discussed in Chapter 2 of the Robustness monograph (pages 44-50), which features a univariate dividend process and no habit persistence. With one exception, assume the same parameter values: $\beta=.9971, R=1 / \beta, \rho_{d}=.9992, b=32, \mu_{d}=13.6$. The one exception is $c_{d}$, the standard deviation of dividend innovations. Instead of $c_{d}=5.5819$ set $c_{d}=0.23$ (this is done to be consistent with the units used in the paper and in Chapt. 10).
(a) Use the Quantecon class LQ to solve the nonrobust Permanent Income model. Use the LQ method stationary_values to compute the optimal policy function. Verify that consumption follows a random walk. (Hints: Assume $b-c_{t}$ is the control, and augment the $R$ matrix to incorporate a small state cost to $k_{t}^{2}$ ).
(b) Following the discussion in class and in Chapter 13 of Robustness, compute the model's implied unconditional 'price of risk'. To do this, use the LQ method compute_sequence to generate a long sequence of the control process, which serves as the marginal utility of consumption. Set $\left(k_{0}, d_{0}\right)=\left(100, \mu_{d}\right)$ when generating the time path for $\left.b-c_{t}\right)$. How does your answer compare to data from the US stock market? (Remember, the units here are quarterly).
(c) Now use the Quantecon class RBLQ to solve the Robust Permanent Income model. Begin by verifying that for $\theta>10^{7}$, the robust policy approximately matches the nonrobust policy computed in part (a). (Note: the underlying python code implements the 'Simple Algorithm' outlined on pages 43-44 of the Robustness monograph. By default, it uses a doubling algorithm to accelerate the iterations).
(d) Verify that for $\theta<10^{7}$, the robust policy features a form of precautionary saving, How does the robust innovation variance of consumption compare to the nonrobust innovation variance? Explain.
(d) Using the monte carlo simulation strategy outlined in Chapter 9 of Robustness, compute the Detection Error Probabilities associated with values of $\theta<10^{7}$. For what value of $\theta$ is the detection error probability approximately equal to $10 \%$ ? Given this value of $\theta$, what is the implied market price of risk? Is the model now consistent with the data? What is the implied 'Market Price of Model Uncertainty'? [Note: When computing the robust price of risk, exploit the observational equivalence formula (10.3.18) on p. 231 of Robustness to adjust the value of $\beta$ so that the allocations in the robust economy remain the same as in the nonrobust economy].
(e) Finally, see if you can reproduce Figure 10.8 .1 on page 246 of Robustness, which illustrates the potential benefits from using a robust policy in the event the benchmark model is misspecified.
