Topics for Today

1.) Fiscal Policy in a Growth Model
- competitive equilibria with distorting taxes
- the tax-adjusted "user cost of capital"

2.) Steady State Effects

3.) Computing Equilibrium Time Paths
- Shooting
- Linearization (Lag operators)

4.) Examples

Fiscal Policy in a Growth Model

Assumptions

1.) Exogenous distorting (flat rate) taxes

2.) No Uncertainty / Perfect Foresight

3.) Focus on taxation (govt. purchases separable in utility).

Main Issues

1.) Substantive (Dynamic effects of various taxes)

2.) Methodological (Solving dynamic models with distorted equilibria)

Preferences

$$\max_{c,\,n} \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$

Taxes

τ_{ct} = time-t consumption tax

τ_{nt} = time-t labor income tax

τ_{kt} = time-t capital income tax

τ_{it} = time-t investment tax credit

τ_{ht} = time-t lump-sum tax

Household Budget Constraint

$$\sum_{t=0}^{\infty} q_t(1+\tau_{ct})c_t + (1-\tau_{it})q_t[k_{t+1} - (1-\delta)k_t]$$
$$\leq \sum_{t=0}^{\infty} r_t(1-\tau_{kt})k_t + w_t(1-\tau_{nt})n_t - q_t\tau_{ht}$$

where

q_t = time-0 pre-tax price of consumption at t

r_t = time-0 pre-tax rental price of capital at t

w_t = time-0 pre-tax wage at t

(Note: Time-0 goods is numeraire).

Government Budget Constraint

$$\boxed{\sum_{t=0}^{\infty} q_t g_t \le \sum_{t=0}^{\infty} \Big\{ q_t \tau_{ct} c_t - q_t \tau_{it}\left[k_{t+1} - (1-\delta)k_t\right] + \tau_{kt} r_t k_t + \tau_{nt} w_t n_t + q_t \tau_{ht} \Big\}}$$

Note: Always optimal to use τ_{ht} + set all other taxes to 0. We will use τ_{ht} as a budget balance trick

User Cost of Capital (Tax-Adjusted)

Re-write household budget constraint, collecting terms in k_t

$$\sum_{t=0}^{\infty} q_t\left[(1+\tau_{ct})\right] c_t \le \sum_{t=0}^{\infty} w_t(1-\tau_{nt})n_t + \sum_{t=1}^{\infty}\Big[r_t(1-\tau_{kt}) + q_t(1-\tau_{it})(1-\delta) - q_{t-1}(1-\tau_{i,t-1})\Big] k_t$$

$$- \sum_{t=0}^{\infty} q_t \tau_{ht} + \left[r_0(1-\tau_{k0}) + (1-\tau_{i0})q_0(1-\delta)\right]k_0$$

$$- \lim_{T\to\infty} (1-\tau_{iT}) q_T k_{T+1}$$

- Since household doesn't care about k_t per se, to avoid arbitrage term multiplying k_t must always be 0:

$$q_t(1-\tau_{it}) = r_{t+1}(1-\tau_{k,t+1}) + q_{t+1}(1-\tau_{i,t+1})(1-\delta)$$

Buy capital at t

Rent it out + then sell undepreciated stock

This can be re-written as:

$$\boxed{r_{t+1} = \left(\frac{1}{1-\tau_{k,t+1}}\right)\left[q_t(1-\tau_{it}) - q_{t+1}(1-\tau_{i,t+1})(1-\delta)\right]}$$

- This is the tax-adjusted user cost of capital
- Note w/o taxes + in the steady state,

$$\frac{r_{t+1}}{q_{t+1}} = \frac{q_t}{q_{t+1}} - (1-\delta)$$

$$\frac{q_t}{q_{t+1}} = \frac{\text{time-0 goods}}{\text{time-}t\text{ goods}} \cdot \frac{\text{time-}(t+1)\text{ goods}}{\text{time-0 goods}} = 1+r$$

$$\Rightarrow \frac{r_{t+1}}{q_{t+1}} = r + \delta \quad \text{(usual formula)}$$

Household's FOCs

c_t : $\beta^t U_{1t} = \mu q_t (1+\tau_{ct})$

n_t : $\beta^t U_{2t} = \mu w_t (1-\tau_{nt})$

Lagrange multiplier

Firms

$$\max_{k,n} \sum_{t=0}^{\infty} q_t F(k_t, n_t) - w_t n_t - r_t k_t$$

$$\Longrightarrow \quad r_t = q_t F_{kt}$$
$$w_t = q_t F_{nt}$$

Definition : A feasible allocation is a sequence $\{c_t, x_t, n_t, k_t\}$ that satisfies the aggregate resource constraints :

1.) $c_t + x_t + g_t \leq F(k_t, n_t)$

2.) $k_{t+1} = (1-\delta) k_t + x_t$

Definition: A competitive equilibrium with distorting taxes is a budget feasible fiscal policy $\{g_t, \tau_{ct}, \tau_{nt}, \tau_{kt}, \tau_{it}, \tau_{ht}\}$, a feasible allocation and a price system $\{q_t, r_t, w_t\}$, such that the allocations solve the household's utility maximization problem and the firm's profit maximization problem. (where households + firms take prices as given).

Computing Equilibria

- For now, suppose labor supply is inelastic
- Sub the FOCs, $\beta^t u_{1t} = \mu q_t(1+\tau_{ct})$ and $r_t = q_t f_k$ into the expression for the user cost of capital,

$$\boxed{\begin{aligned} u'(c_t) &= \beta u'(c_{t+1})\left\{\left(\frac{1+\tau_{ct}}{1+\tau_{ct+1}}\right)\left[\frac{1-\tau_{i,t+1}}{1-\tau_{it}}(1-\delta) + \frac{1-\tau_{k,t+1}}{1-\tau_{it}} f'(k_{t+1})\right]\right\} \\ &= \beta u'(c_{t+1}) R_{t+1} \end{aligned}}$$

- Note that the tax-adjusted return to capital is higher when
  1.) Capital tax is low
  2.) Future consumption tax is relatively low
  3.) Current investment tax credit relatively high

- If we define the policy vector,

$$z_t = (g_t, \tau_{ct}, \tau_{it}, \tau_{kt})$$

and then sub in the aggregate resource constraint

$$c_t = f(k_t) + (1-\delta)k_t - g_t - k_{t+1}$$

into the after-tax Euler eq., we get a nonlinear 2^{nd}. Order difference equation that characterizes the equil. path of k_t

$$\boxed{H(k_t, k_{t+1}, k_{t+2}, z_t, z_{t+1}) = 0}$$

- The equil. time paths of the remaining endogenous variables can the be obtained as follows:

1.) $c_t = f(k_t) + (1-\delta)k_t - k_{t+1} - g_t$

2.) $q_t = \dfrac{\beta^t u'(c_t)}{1+\tau_{ct}}$

3.) $r_t = q_t f'(k_t)$

4.) $w_t = q_t [f(k_t) - k_t f'(k_t)]$

Steady State

- A steady state is the solution of the algebraic equation,

$$H(\bar{k}, \bar{k}, \bar{k}, \bar{z}, \bar{z}) = 0$$

From the tax-adjusted Euler eq., we have

$$1 = \beta\left[(1-\delta) + \left(\frac{1-\bar{\tau}_k}{1-\bar{\tau}_i}\right) f'(\bar{k})\right]$$

Defining $\beta = \frac{1}{1+\rho}$, we can write this as:

$$\boxed{f'(\bar{k}) = (\rho+\delta)\left(\frac{1-\bar{\tau}_i}{1-\bar{\tau}_k}\right)}$$

Comments

1.) Lump sum taxes yield a (constrained) Pareto Optimal allocation, with g_t simply deducted from output each period. [Note: τ_{ht} does not appear in the equil. conditions].

2.) With inelastic labor supply τ_n is non-distorting, and a constant τ_c policy is nondistorting.

3.) A time-varying τ_c policy is distorting

4.) Capital taxation is distorting [unless $\bar{\tau}_i = \bar{\tau}_k$].

2 Solution Strategies

① Shooting

1.) Solve $\bar{H}(\cdot)$ for $\bar{k}$

2.) Guess c_0

3.) Use k_0 + c_0 with resource constraint to get k_1

4.) Use Euler along with k_0, k_1, c_0 to get c_1

5.) Use resource constraint to get k_2

6.) Iterate 4+5 to get $\{k_t\}$ for $t = 1, 2, \ldots S$

7.) If $k_S > \bar{k}$, raise c_0 and start again

8.) If $k_S < \bar{k}$, lower c_0 " " "

- Note, if lump-sum taxes not available, then another loop will be necessary, to make sure the govt. budget constraint is satisfied.

Basic idea: 1.) Given taxes, compute equil. as above

2.) If Spending > Revenue raise taxes

If Spending < Revenue lower taxes

2.) Linearization

Linearize $H(\cdot)$ around the steady state,

$$H_{k_{t+2}}(k_{t+2}-\bar{k})+H_{k_{t+1}}(k_{t+1}-\bar{k})+H_{k_t}(k_t-\bar{k})$$
$$+H_{z_{t+1}}(z_{t+1}-\bar{z})+H_{z_t}(z_t-\bar{z})=0$$

where the partials are evaluated at the s.s.
Write this as:

$$\phi_0 k_{t+2}+\phi_1 k_{t+1}+\phi_2 k_t = A_0 + A_1 z_t + A_2 z_{t+1}$$

Or, in lag operator notation,

$$\boxed{\phi_0(1-\lambda_1 L)(1-\lambda_2 L)k_{t+2} = A_0 + A_1 z_t + A_2 z_{t+1}}$$

$-(\lambda_1+\lambda_2)=\frac{\phi_1}{\phi_0}$ $\qquad$ $\lambda_1\lambda_2=\frac{\phi_2}{\phi_0}$

<u>3 cases</u>

1.) $|\lambda_1|>1,\ |\lambda_2|>1 \implies$ No solution

2.) $|\lambda_1|<1,\ |\lambda_2|<1 \implies$ Infinite # of solutions

3.) $|\lambda_1|>1,\ |\lambda_2|<1 \implies$ Unique solution

- Let's assume a unique solution. (Since we know there is a saddle path when taxes $= 0$, this should be the case when taxes are not too big).

- To obtain a stationary solution we must "solve the unstable root forward". To see how this is done, note

$$(1-\lambda_1 L) = -\lambda_1 L(1-\lambda_1^{-1}L^{-1})$$

We now have

$$-\phi_0\lambda_1(1-\lambda_1^{-1}L^{-1})(1-\lambda_2 L)k_{t+1} = A_0 + A_1 z_t + A_2 z_{t+1}$$

Therefore,

$$k_{t+1} = \lambda_2 k_t - \frac{\lambda_2}{\phi_2}\frac{1}{1-\lambda_1^{-1}L^{-1}}\left\{A_0 + A_1 z_t + A_2 z_{t+1}\right\}$$

uses fact that $\lambda_1\lambda_2 = \frac{\phi_0}{\phi_2}$

$$\Rightarrow \boxed{k_{t+1} = \lambda_2 k_t - \frac{\lambda_2}{\phi_2}\sum_{j=0}^{\infty}\left(\frac{1}{\lambda_1}\right)^j\left\{A_0 + A_1 z_{t+j} + A_2 z_{t+1+j}\right\}}$$

transient. (movement toward the steady state)

anticipation of future policy

Comments

1.) In general, the adjustment speeds $(\lambda_2, 1/\lambda_1)$ are complicated functions of all the parameters. However, it is generally the case that

$\gamma\uparrow \implies \lambda_2 \uparrow$ [less intertemp. subst. $\implies$ slower adjustment to s.s.]

$\implies \lambda_1 \downarrow$ [future matters more with less intertemp. subst.]

2.) Long ago I noted that when evaluating "forecasts" of future variables it is convenient to define the state so as to obtain a 1st-order Markov specification

$$x_{t+1} = A_x x_t$$
$$z_t = G x_t$$

The particular specifications of x_t, A_x and G will depend on the details of the fiscal policies. With this specification we have:

$$\boxed{k_{t+1} = \lambda_2 k_t - \frac{\lambda_2}{\phi_2}\left\{(1-\lambda_1^{-1})^{-1}A_0 + A_1 G (I - 1/\lambda_1 A_x)^{-1} x_t + A_2 G (I - 1/\lambda_1 A_x)^{-1} A_x x_t\right\}}$$

Examples

Calibration: $f(k) = k^\alpha \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma}$

$$\alpha = .33 \quad \beta = .95 \quad \gamma = 2 \quad \delta = .2$$

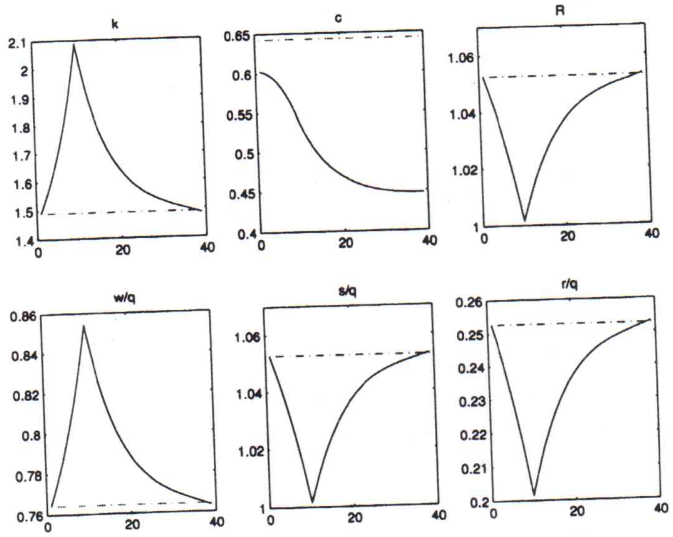

Figure 11.3.1: Response to foreseen once-and-for-all increase in g at $t = 10$. From left to right, top to bottom: $k, c, R, w/q, s/q, r/q$.

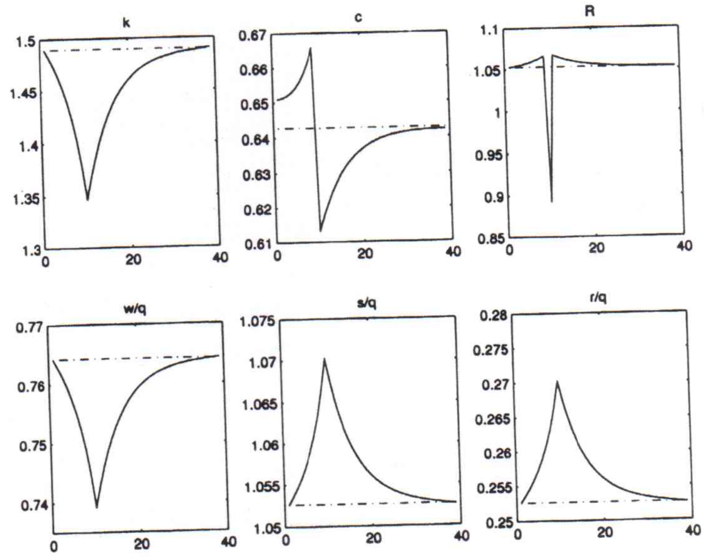

Figure 11.3.2: Response to foreseen once-and-for-all increase in τ_c at $t = 10$. From left to right, top to bottom: $k, c, R, w/q, s/q, r/q$.

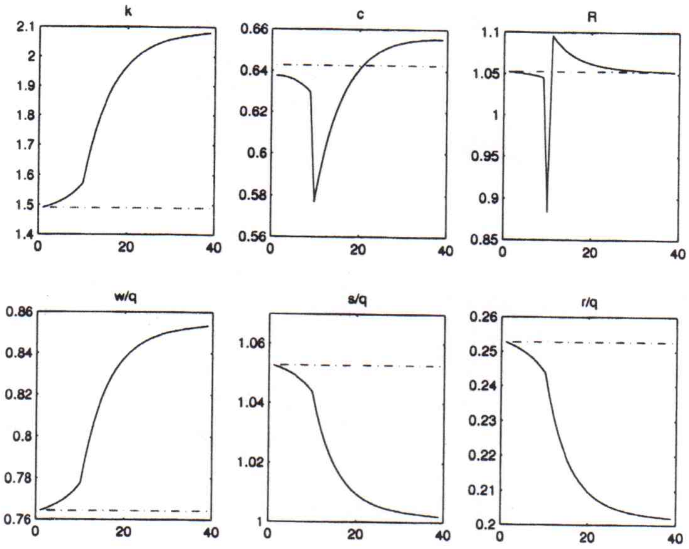

Figure 11.5.1: Response to foreseen once-and-for all increase in τ_i at $t = 10$. From left to right, top to bottom: $k, c, R, w/q, s/q, r/q$.

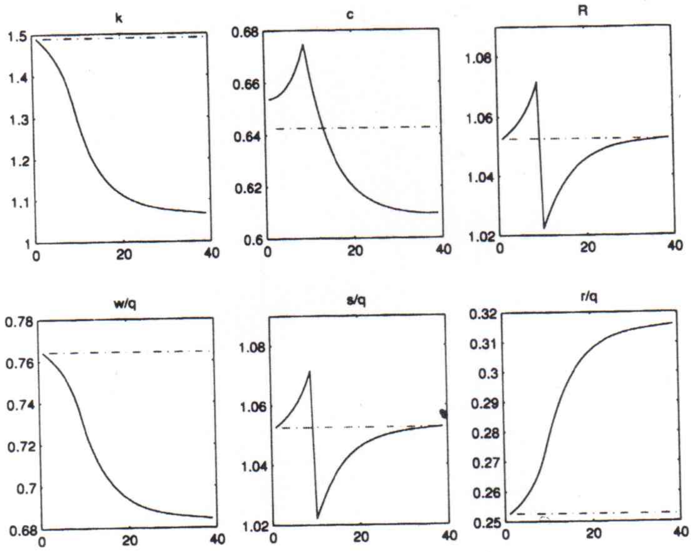

Figure 11.5.2: Response to foreseen increase in τ_k at $t = 10$. From left to right, top to bottom: $k, c, R, w/q, s/q, r/q$.

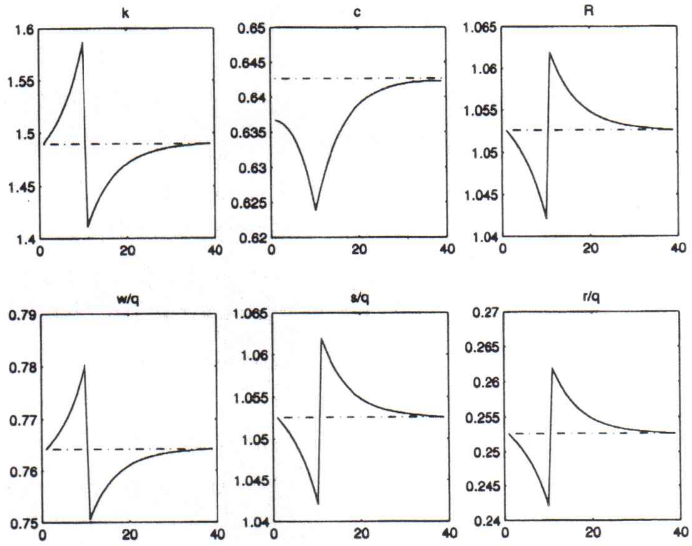

Figure 11.7.1: Response to foreseen one-time pulse increase in g at $t = 10$. From left to right, top to bottom: $k, c, R, w/q, s/q, r/q$.

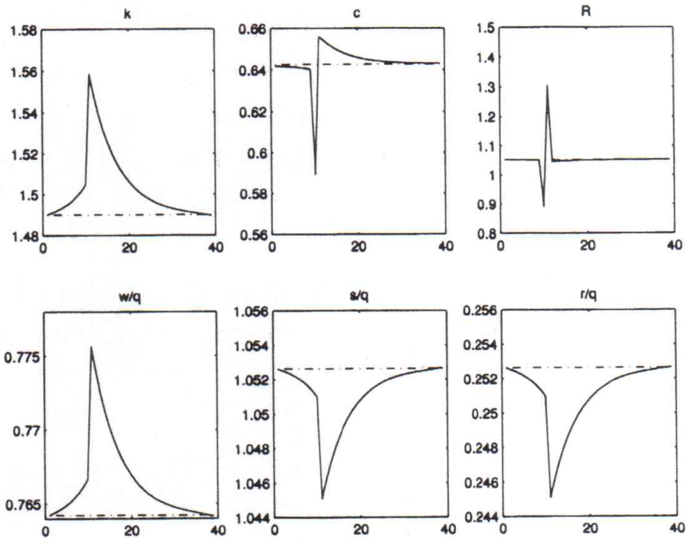

Figure 11.7.2: Response to foreseen one-time-pulse increase in τ_i at $t = 10$. From left to right, top to bottom: $k, c, R, w/q, s/q, r/q$.

Elastic Labor Supply

- Same as before, with 2 modifications:

1.) In the Euler eq., u_{1t} depends on n_t and u_{1t+1} depends on n_{t+1}, and f' depends on n_{t+1}

Thus, we have

$$H(k_{t+2}, k_{t+1}, k_t, n_{t+1}, n_t, z_{t+1}, z_t) = 0$$

2.) We must add the intratemporal optimality condition to the system:

$$\boxed{\frac{u_2(c_t, 1-n_t)}{u_1(c_t, 1-n_t)} = \frac{1-\tau_{nt}}{1+\tau_{ct}} F_2(k_t, n_t)}$$

- The solution algorithm now has 1 extra step

  - Solve $\bar{H}$ and G for the s.s. [write labor supply eq. as $G(k_{t+1}, k_t, n_t, z_t)$]
  - Linearize H around s.s.

$$H_{k_{t+2}}(k_{t+2}-\bar{k}) + H_{k_{t+1}}(k_{t+1}-\bar{k}) + H_{k_t}(k_t-\bar{k}) + H_{n_{t+1}}(n_{t+1}-\bar{n})$$
$$+ H_{n_t}(n_t-\bar{n}) + H_{z_{t+1}}(z_{t+1}-\bar{z}) + H_{z_t}(z_t-\bar{z}) = 0$$

  - Linearize G around s.s.

$$G_{k_{t+1}}(k_{t+1}-\bar{k}) + G_{k_t}(k_t-\bar{k}) + G_{n_t}(n_t-\bar{n}) + G_{z_t}(z_t-\bar{z}) = 0$$

→ solve for $n_t-\bar{n}$ + sub into H

Then proceed as before.