Topics for Today

1.) Dynamic Optimal Taxation
- The Ramsey Taxation Problem
- Chamley & Judd's Asymptotic Zero Tax on Capital Result
- Caveats to Chamley/Judd

Dynamic Optimal Taxation

- Last time we studied how an economy reacts to an exogenous sequence of flat-rate taxes. Now we make these tax rates endogenous, by letting the govt. choose them optimally over time.

Assumptions

1.) No Uncertainty / Infinite Horizon

2.) Govt. must finance an exogenous stream of expenditures using (time-varying) flat rate taxes on labor and capital income. [The govt. can issue debt, but this doesn't influence results about steady state taxation].

3.) The govt. seeks to maximize household welfare (Households can be heterogeneous).

4.) The govt. can commit to a tax policy. Hence, the govt. behaves as a 'Stackelberg leader' (i.e., it takes account of how the private sector will respond to taxes).

Comments

1.) This kind of problem was first studied by Frank Ramsey (EJ, 1927), so it's called a Ramsey taxation problem. (Ramsey studied commodity taxation in a static model).

2.) The tax structure (i.e., flat rate taxes) is imposed exogenously. What's endogenous are the tax rates.

Households (Identical for Now)

$$\max_{c_t,\, \ell_t} \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$$

s.t. 1.) $\ell_t + n_t = 1$

2.) $c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = (1-\tau_t^n) w_t n_t + (1-\tau_t^k) r_t k_t + (1-\delta) k_t + b_t$

Government

$$g_t = \tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{b_{t+1}}{R_t} - b_t$$

Firms

$$\max_{k_t,\, n_t} \pi(k_t, n_t) = F(k_t, n_t) - r_t k_t - w_t n_t$$

Aggregate Resource Constraint

$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1-\delta) k_t$$

FOCs (serve as constraints on govt tax policy)

Households

1.) c_t : $$U_c(t) = \lambda_t$$

2.) n_t : $$U_\ell(t) = \lambda_t(1-\tau_t^n)w_t$$

3.) k_{t+1} : $$\lambda_t = \beta\lambda_{t+1}\left[(1-\tau_{t+1}^k)r_{t+1} + (1-\delta)\right]$$

4.) b_{t+1} : $$\frac{\lambda_t}{R_t} = \beta\lambda_{t+1}$$

Firms

5.) k_t : $$F_k(t) = r_t$$

6.) n_t : $$F_n(t) = w_t$$

Sub (1.) into (2) and (3)

$$\boxed{\begin{aligned} U_\ell(t) &= U_c(t)(1-\tau_t^n)w_t \\ U_c(t) &= \beta U_c(t+1)\left[(1-\tau_{t+1}^k)r_{t+1} + (1-\delta)\right] \end{aligned}}$$

Combine (3) and (4)

$$\boxed{R_t = (1-\tau_{t+1}^k)r_{t+1} + 1 - \delta}$$

No Arbitrage
> Equal After-Tax returns on capital + bonds (Interest income not taxed).

- Formulate problem so that govt. chooses after-tax wages + rentals.

$$\tilde{r}_t = (1-\tau_t^k) r_t$$

$$\tilde{w}_t = (1-\tau_t^n) w_t$$

Then we have

$$\begin{aligned}
\tau_t^k r_t k_t + \tau_t^n w_t n_t &= (r_t - \tilde{r}_t) k_t + (w_t - \tilde{w}_t) n_t \\
&= r_t k_t + w_t n_t - \tilde{r}_t k_t - \tilde{w}_t n_t \\
&= F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t
\end{aligned}$$

Government's Problem

govt bud. constraint

$$\begin{aligned}
\mathcal{L} = \max_{\substack{c_t,\, k_t \\ n_t,\, \tilde{r}_t \\ \tilde{w}_t,\, b_t}} \sum_{t=0}^{\infty} \beta^t \Big\{ & U(c_t, 1-n_t) + \Psi_t \Big[F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t + \frac{b_{t+1}}{R_t} - b_t - g_t \Big] \\
& + \theta_t \big[F(k_t, n_t) + (1-\delta) k_t - c_t - g_t - k_{t+1} \big] \\
& + \mu_{1t} \big[U_\ell(t) - U_c(t) \tilde{w}_t \big] \\
& + \mu_{2t} \big[U_c(t) - \beta U_c(t+1) (\tilde{r}_{t+1} + 1 - \delta) \big] \Big\}
\end{aligned}$$

Note: Can omit household budget constraint due to Walras' Law

Comment

Though not explicit in the above Lagrangian, we might also have to restrict the tax rate on the initial capital stock (i.e., τ_0^k). This tax is nondistortionary (since K_0 is fixed), and so if possible, the govt. should finance all revenue requirements using τ_0^k. We assume this is not possible.

Definition: A feasible allocation is a set of sequences $\{k_t, c_t, \ell_t, g_t\}$ that satisfies the aggregate resource constraints.

Definition: A govt. policy is a set of sequences $\{g_t, \tau_t^k, \tau_t^n, b_t\}$

Definition: A price system is a set of non-negative bounded sequences $\{w_t, r_t, R_t\}$

Definition: A competitive equilibrium is a feasible allocation, a govt. policy, and a price system such that:

a.) Given prices + govt. policy, the allocation solves the household's + firm's problems.

b.) Given the allocation + price system, the govt. policy satisfies the govt's budget constraint.

Definition: Given k_0 & b_0, the Ramsey Problem is to choose a competitive equilibrium that maximizes household welfare.

- To obtain our main result, we only need to consider the govt's FOC w.r.t. k_{t+1}

$$\boxed{\theta_t = \beta\left\{\theta_{t+1}\left[F_k(t+1)+1-\delta\right] + \psi_{t+1}\left[F_k(t+1) - \tilde{r}_{t+1}\right]\right\}}$$

$F_k - (1-\tau)r$

$= F_k - (1-\tau)F_k = \tau r$

Note: The 2nd term on r.h.s. is the reduction in "excess burden" due to higher capital next period.

In a steady-state $[g_t = \bar{g}]$

$$\theta = \beta\left\{\psi[r-\tilde{r}] + \theta(r+1-\delta)\right\} \quad (*)$$

From consumer's s.s. Euler Eq., $1 = \beta(\tilde{r}+1-\delta)$

Sub this into $(*)$

$$\boxed{\beta(\theta+\psi)(r-\tilde{r}) = 0}$$

θ = marg. social value of goods

ψ = marg. social cost of raising revenue

= marg. social value of reducing taxes

$\Longrightarrow r = \tilde{r} \Longrightarrow \tau^k = 0$!

Proposition: If there exists a steady state Ramsey allocation, the associated asymptotic tax rate on capital is zero

Intuition

Go back to our results from last lecture. Suppose $\tau_{it} = 0 \ \forall t$ and $\tau_{Kt} = \bar{\tau}_K$. The household's after-tax Euler eq. becomes:

$$U_c(t) = \beta U_c(t+1)\left[\left(\frac{1+\tau_{ct}}{1+\tau_{ct+1}}\right)\left((1-\bar{\tau}_K)f'(k_{t+1}) + (1-\delta)\right)\right]$$

Notice that a constant tax rate on capital is equivalent to a rising tax rate on consumption. This kind of intertemporal distortion could never be optimal here, since there is no growth in the steady state.

Comment

- This result does not depend on the govt's ability (or inability) to issue debt. Note that we get the same result if $b_t = b_{t+1} = 0 \ \forall t$. [If the govt. can lend, then it would be optimal to accumulate claims on the private sector + to eventually use the interest to finance govt. expenditures. This would allow all taxes to be set to zero.

Heterogeneity

- One major role of tax policy is to redistribute resources among heterogeneous households. What if households differ then? Could capital taxation be part of an optimal redistribution program? Surprisingly, the answer is no (at least asymptotically)

- For simplicity, assume a balanced budget each period (we just saw this makes no difference to asymptotic results)

Pareto weight for household i

lump-sum transfer $s_t \geq 0$

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ & \sum_{i=1}^{N} \alpha^i u^i(c_t^i, 1-n_t^i) \\
& + \psi_t \left[F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t - g_t - s_t \right] \\
& + \theta_t \left[F(k_t, n_t) + (1-\delta) k_t - c_t - g_t - k_{t+1} \right] \\
& + \sum_{i=1}^{N} \epsilon_t^i \left[\tilde{w}_t n_t^i + \tilde{r}_t k_t^i + (1-\delta) k_t^i + s_t^i - c_t^i - k_{t+1}^i \right] \\
& + \sum_{i=1}^{N} \mu_{1t}^i \left[u_l^i(t) - u_c^i(t) \tilde{w}_t \right] \\
& + \sum_{i=1}^{N} \mu_{2t}^i \left[u_c^i(t) - \beta u_c^i(t+1)(\tilde{r}_{t+1} + 1 - \delta) \right] \Big\}
\end{aligned}
$$

Where $n_t = \sum_i n_t^i \quad k_t = \sum_i k_t^i$ etc.

FOC for k^i_{t+1}

$$\theta_t + \epsilon^i_t = \beta\Big\{\theta_{t+1}\left[F_k(t+1) + 1 - \delta\right] + \psi_{t+1}\left[F_k(t+1) - \tilde{r}_{t+1}\right] + \epsilon^i_{t+1}\left(\tilde{r}_{t+1} + 1 - \delta\right)\Big\}$$

In the steady state

$$\boxed{\theta + \epsilon^i\big[\underbrace{1 - \beta(\tilde{r} + 1 - \delta)}_{=0 \text{ from } i\text{'s FOC}}\big] = \beta\left[\psi(r - \tilde{r}) + \theta(r + 1 - \delta)\right]}$$

$\Longrightarrow$ Same as before $(\tau^k = 0)$

Judd's Special Case

Suppose 2 types: Type 1 = "Workers", Type 2 = "Capitalists"

Type 1 Budget Constraint: $c^1_t = \tilde{w}_t n^1_t + s^1_t$ > Work / Don't Save

Type 2 Budget Constraint: $c^2_t + k^2_{t+1} = \tilde{r}_t k^2_t + (1-\delta)k^2_t + s^2_t$ > Save / Don't Work

Note, this fits into above model, so even if $\alpha^1 > \alpha^2 = 0$ [i.e., you only care about "Workers"], you should still not tax capital, + instead tax labor!

Intuition: Wages depend on capital!

Caveats

1.) Assumes Commitment.

2.) Only applies to the steady state. [Ignores potential conflicts over the transition].

3.) Doesn't hold with finite horizons unless labor taxes can be made age-specific (Erosa + Gervais (JET, 2002)).

4.) Assumes no uncertainty. With uncertainty + incomplete mkts., households may overaccumulate capital + it becomes optimal to tax it. [Aiyagari (JPE, 1995)].

5.) Assumes all factors can be taxed. If some (complementary) factors cannot be taxed then it is optimal to tax capital [Correia (J. Pub. Econ, '96)].

6.) No asymmetric info / Incentive Problems.

Question: Does the result extend to models with endogenous growth? What if we incorporated human capital accumulation?