

## Topics for Today

1.) The Mirrlees Approach to Dynamic Optimal Taxation

- Ramsey vs. Mirrlees
- Insurance vs. Incentives
- The Inverse Euler Equation Theorem

2.) Review for Final

## Dynamic Mirrlees Taxation

- Last time we studied dynamic optimal taxation when the tax structure was exogenous (i.e., flat-rate taxes on labor + capital). Now we endogenize the tax structure itself.
- When doing this we assume the govt. not only needs to raise revenue to fund expenditures, but it also needs to provide incentives, since it does not observe an individual's skill level. In addition, when individual skills are random, the govt. also wants to provide insurance against 'skill shocks' (e.g., disability).
- Hence, the key issue here is resolving an inherent trade-off between insurance and incentives. We solve this problem using a mechanism design approach. The govt. computes optimal allocations subject to Incentive Compatibility constraints, and then (implicit) taxes are inferred from the resulting wedges between Marginal Rates of Substitution + Marginal Rates of Transformation.

## Assumptions

- 1.) Workers' skills are heterogeneous & random.
  - 2.) The govt. does not observe individual skills, but it knows the distribution.
  - 3.) No a priori restrictions on govt's tax policy.  
(Lump-sum taxes possible).
  - 4.) Govt. can commit (The nature of the time inconsistency problem is different from Ramsey).
  - 5.) Preferences are separable between consumption & leisure. [Our soln. strategy requires the govt. be able to observe the MU of consumption].
  - 6.) No Aggregate Uncertainty. (Can be relaxed).
- Note, with separable preferences & without aggregate uncertainty, the insurance/incentive trade-off becomes particularly clear. Without aggregate uncertainty, perfect consumption insurance (everybody gets equal consumption) is feasible, & would be optimal absent informational frictions. However, if the govt. cannot observe skills, then the highly skilled will pretend to be disabled. (They get the same consumption, but would get more leisure).

## A 2-Period Example

### Preferences

☰ a continuum of workers who live 2-periods.

$$\max E\{u(c_1) + v(n_1) + \beta[u(c_2) + v(n_2)]\}$$

### Skills / Production

$$y = \theta \cdot n$$

$\downarrow$        $\downarrow$   
output      skills      effort/labor  
 $(observed)$

$\theta(i)$  is observed (only) by agent  $i$  (at beginning of period)

$\pi_{1,i}(i)$  = period-1 dist. of skills

$\pi_{2,j|i}$  = conditional dist. of period-2 skills

### Resource Constraint

$$\sum_i \left\{ [c_{1,i} + \frac{1}{R} \sum_j c_{2,i,j} \pi_{2,j|i}] \pi_{1,i}(i) \right\} + G_1 + \frac{1}{R} G_2 \\ \leq \sum_i \left[ y_{1,i} + \frac{1}{R} \sum_j y_{2,i,j} \pi_{2,j|i} \right] \pi_{1,i}(i) + R K,$$

## Govt's Problem

$$\max_{\substack{c_{1(i)}, c_{2(i)} \\ y_{1(i)}, y_{2(i)}}} \sum_i \left\{ u(c_{1(i)}) + v\left(\frac{y_{1(i)}}{\theta_{1(i)}}\right) + \beta \sum_j \left[ u(c_{2(i,j)}) + v\left(\frac{y_{2(i,j)}}{\theta_{2(i,j)}}\right) \right] \pi_a(w|i) \right\} \pi_i(i)$$

s.t. 1.) Resource Constraint

2.) Incentive Compatibility Constraints

Revelation Principle : Govt. just asks what your skill is, and allocates consumption + labor contingent on your answer.

$i_r$  = first-period skills report (depends on realized  $i^*$ )

$j_r$  = 2nd-period skills report (depends on realized  $j$ )

## Incentive Compatibility Constraints

$$u(c_{1(i)}) + v\left(\frac{y_{1(i)}}{\theta_{1(i)}}\right) + \beta \sum_j \left[ u(c_{2(i,j)}) + v\left(\frac{y_{2(i,j)}}{\theta_{2(i,j)}}\right) \right] \pi_a(w|i)$$

$$\geq u(c_{1(i_r)}) + v\left(\frac{y_{1(i_r)}}{\theta_{1(i_r)}}\right) + \beta \sum_j \left[ u(c_{2(i_r,j_r)}) + v\left(\frac{y_{2(i_r,j_r)}}{\theta_{2(i_r,j_r)}}\right) \right] \pi_a(w|i)$$

(Just one constraint since general reporting strategy allowed).

## Characterization of Optimum

Consider the following simple variational argument

1.) Fix a 1<sup>st</sup> period realization  $i$  and a hypothetical optimum  $c_1^*(i)$ ,  $c_2^*(i)$ .

2.) Increase 2<sup>nd</sup> period utility uniformly across 2<sup>nd</sup> period realizations,

$$u(\tilde{c}_2(i,j;\Delta)) \equiv u(c_2^*(i,j)) + \Delta$$

3.) Hold total utility constant by decreasing 1<sup>st</sup> period utility by  $\beta\Delta$

$$u(\tilde{c}_1(i;\Delta)) = u(c_1^*(i)) - \beta\Delta$$

4.) Note, this variation does not affect IC constraint. Only the resource constraint is potentially affected.

5.) Therefore, for  $c_1^*(i)$  and  $c_2^*(i)$  to be optimal,  $\Delta = 0$  must minimize resources expended on the allocation.

Note, can express the resource cost of the perturbed allocation as follows:

$$\tilde{c}_1(i; \Delta) + R^{-1} \sum_j \tilde{c}_2(i, j; \Delta) \pi(j|i) = u'(u(c_1(i)) - \beta \Delta) \\ + R^{-1} \sum_j (u'(u(c_2(i, j)) + \Delta) \pi(j|i))$$

FOC evaluated at  $\Delta = 0$

$$(Note \frac{d}{dx} f^{-1}(x) = \frac{1}{f'(x)})$$

$$\frac{1}{u'(c_1(i))} = \frac{1}{\beta R} \sum_j \frac{1}{u'(c_2(i, j))} \pi_j(i)$$

} Inverse Euler Eq.

### 3 Cases

1.) Skills Observable  $\Rightarrow u'(c_1) = \beta R u'(c_2)$

2.) Skills Unobservable but not random (constant overtime)  $\Rightarrow u'(c_1) = \beta R u'(c_2)$

3.) Skills Unobservable + Random

$$\frac{1}{u'(c_1)} = \frac{1}{\beta R} E\left[\frac{1}{u'(c_2)}\right] > \frac{1}{\beta R} \frac{1}{E u'(c_2)} > \frac{1}{\beta R} \frac{1}{u'(c_2)} \neq$$

$$\Rightarrow u'(c_1) < \beta R E[u'(c_2(i, j))]$$

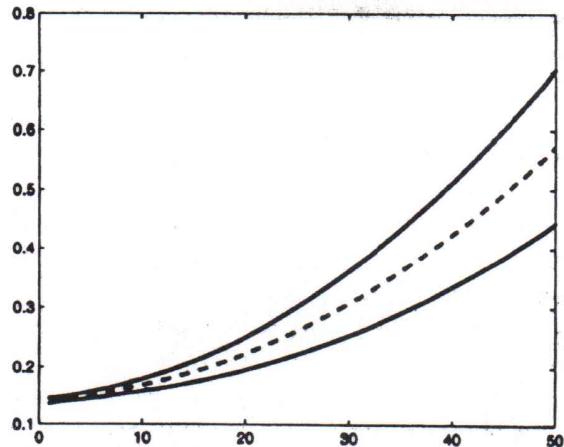
$$\Rightarrow \gamma_K > 0$$

## Intuition

Saving affects the incentive to work. Need to discourage saving to prevent the following deviation by the high-skilled: 1.) Save more today  
2.) Work less tomorrow.

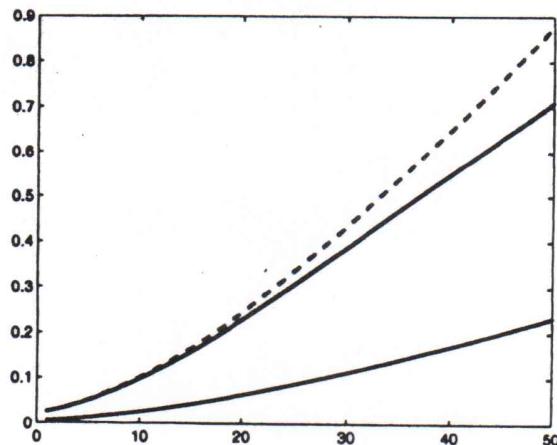
## Other Features of Optimal Policy

- 1.) On average (across individuals) wealth/capital taxes are zero ex ante.
  - 2.) However, they depend on future labor income
    - If labor income is below average, then your capital tax is positive
    - If labor income is above average, then your capital tax is negative
- $\Rightarrow$  "Regressive" (for incentive reasons)
- The fact that the capital tax varies in this regressive way makes investment risky + creates a positive risk premium. This explains how it is possible to have a positive intertemporal wedge/ $t_{\text{ex}}$ , even though taxes are zero ex ante.



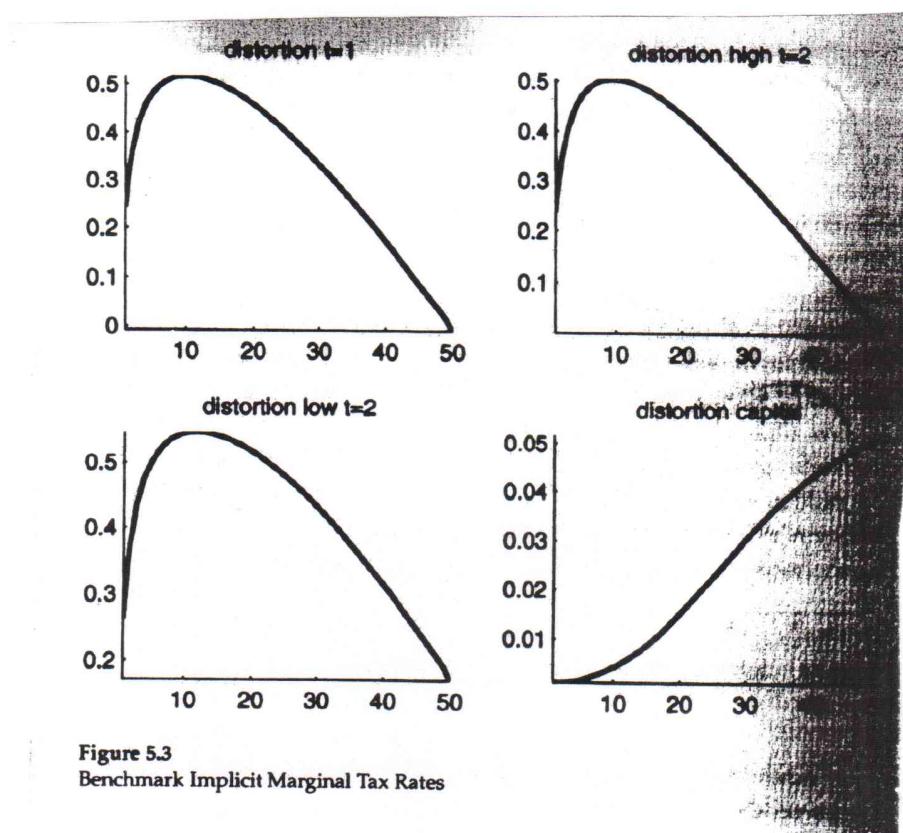
**Figure 5.1**

Consumption Allocation. Middle Dotted Line Shows First Period Consumption; Outer Solid Lines Are Second Period Consumption

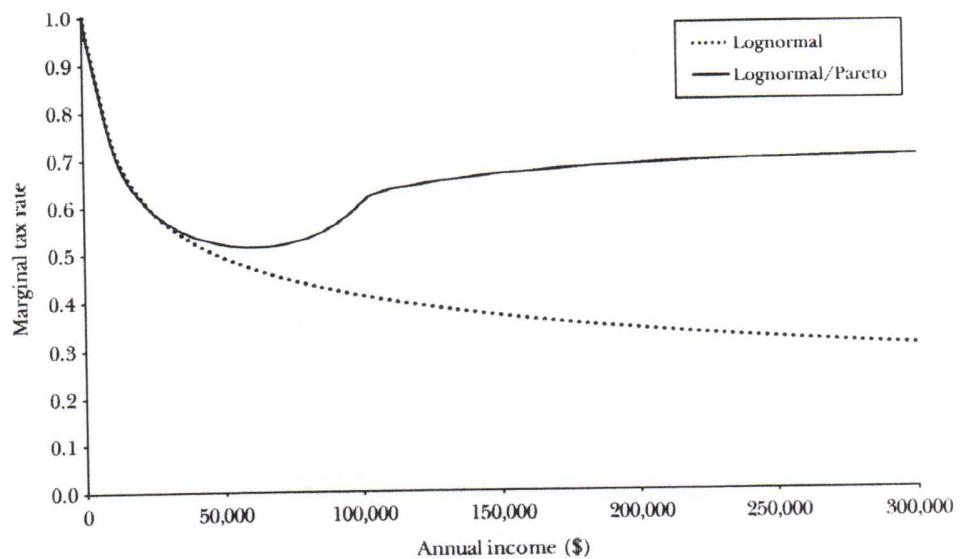


**Figure 5.2**

Effective Labor Allocation. Dashed Line Is for First Period. Solid Lines Are for Second Period, Top Is High Shock, Bottom Low Shock



*Figure 3*  
**Optimal Marginal Tax Simulations, with Different Ability Distributions**



*Note:* The figure shows optimal marginal tax rates given two different ability distributions: one lognormal; and one lognormal until approximately \$43 per hour and Pareto thereafter.

# Main Topics for Final

## 1.) Growth

- Basic Facts
- Problems with Solow
- Cass-Koopmans (Saddlepaths, transition rates)
- Diamond (Dynamic Inefficiency)
- Endog. Growth (AK, Romer '86, Lucas '88, Romer '90)

## 2.) Asset Pricing

- Lucas Model (Consumption-CAPM)
- Equity Premium Puzzle
- Hansen Jagannathan Bounds
- Strategies for Resolving the EP Puzzle

## 3.) Ricardian Equivalence

## 4.) Dynamic Effects of Taxation

- Lag Operators + Linearization

## 5.) Dynamic Ramsey Taxation

- Chamley / Judd Result

## 6.) Dynamic Mirrlees Taxation

- Inverse Euler Eq.

## Main Readings

- 1.) Text - Chpts. 10-15 (except 12)
- 2.) Lucas (JME, 1988)
  - "The Mechanics of Economic Development"
- 3.) Kocherlakota (JEL, 1996)
  - "The Equity Premium : It's Still A Puzzle"
- 4.) Atkeson, Chari + Keboe (Minn. Fed, 1999)
  - "Taxing Capital : A Bad Idea"
- 5.) Golosov, Tsyvinski + Werning (NBER, 2006)
  - "New Dynamic Public Finance : A User's Guide"