Topics for Today

1.) Competitive Search Equilibrium ("Directed Search").

- Moen (1997)

Competitive Search Equilibrium

- Last time we saw that equilibria in the M-P model are generally inefficient, due to search externalities.

- Only when $\alpha = \phi$ (Hosios Condition) is the M-P equilibrium efficient.

- The basic reason for this is that wages play no allocative role in the M-P model. They are determined after a match is formed.

- Moen (1997) lets wages play an allocative role, by allowing firms to advertise wages. He shows that this "competitive search equilibrium" is always efficient. His model is an example of a "hedonic equilibrium", i.e., market price reflects the value of underlying attributes (in this case, search costs).

Assumptions

1.) Continuous time. A continuum of workers + firms. Measure of workers normalized to 1. Measure of firms/jobs is endogenous.

2.) Workers + firms are risk neutral.

3.) The labor market consists of a set of n "submarkets". Workers + firms are free to enter whichever market offers the highest expected utility/profits.

4.) Workers are identical. Jobs/Technology differ across submarkets.

5.) The matching function is the same across submarkets.

6.) Firms credibly announce wages + workers know the wage distribution. (workers cannot enter more than one submarket at a time).

7.) To open a vacancy, a firm must incur a sunk cost k. Productivity is then determined by a draw from a discrete prob. dist. $F(\cdot)$.

8.) Common exogenous job destruction rate s.

Basic Idea

Wages differ across submarkets despite the fact that workers are identical. Market tightness is an "equalizing difference" or "compensating differential". Tight markets have lower wages, since it is more likely that search is successful (for the worker). In equilibrium, the MRS between wages + search time are equalized across markets. High productivity firms offer higher wages. It is more costly for them to have unfilled vacancies, so they offer high wages to induce more applications. (kind of like "efficiency wages")

Moen's Paper Proceeds in 5 steps

1.) Compute value functions for workers + firms, given wages.

2.) Define a "no surplus" competitive equil., and use it to compute the wage dist.

3.) Solve a planner's problem (no wages / markets).

4.) Show that planner's FOCs match the equil. conditions $\Longrightarrow$ efficiency

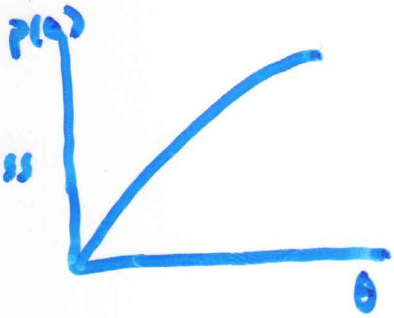
5.) Argue that firms do not have an incentive to deviate from the comp. equil. wage dist.

Matching

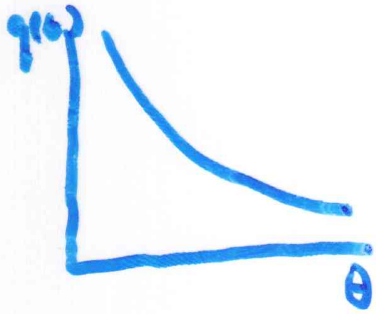
not probability

$$\frac{\chi(u,v)}{u} = \text{job finding rate} = p(\theta)$$

$\theta = v/u$ = tightness



$$\frac{\chi(u,v)}{v} = \text{vacancy filling rate} = q(\theta)$$



Workers Bellman Eqs.

(1) $rU_i = z + p(\theta_i)[E_i - U_i]$

(2) $rE_i = w_i - s(E_i - U_i)$

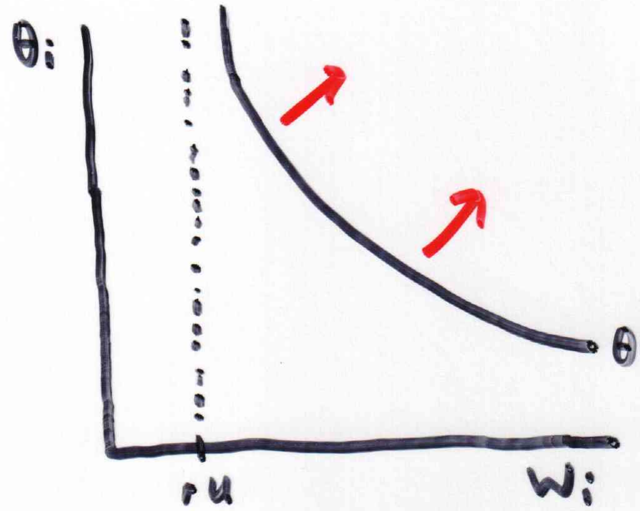
U_i : value of being unemployed in mkt. i.

Free to Choose + Homogeneity $\Longrightarrow U_i = U$

Solve (1) + (2) for U,

$$\boxed{rU = \frac{(r+s)z + w_i\, p(\theta_i)}{r+s+p(\theta_i)} \quad \text{or} \quad p(\theta_i) = \frac{rU - z}{w_i - rU}(r+s)}$$

Indifference Curve between θ and w



$w\uparrow \Rightarrow$ better off

$\theta\downarrow \Rightarrow$ worse off

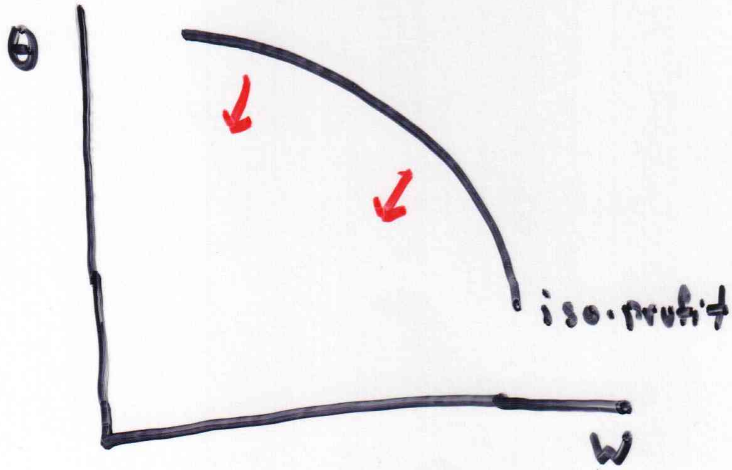
Firms Bellman Eqs.

$$rV(y_i, w, \theta_i) = -c + q(\theta_i)\left[J(y_i,w) - V(y_i,w,\theta_i)\right]$$

$$rJ(y_i,w) = y_i - w - sJ(y_i,w)$$

Solve for V

$$\Rightarrow \boxed{(r+q)V(y_i,w,\theta) = q(\theta)\frac{y_i - w}{r+s} - c}$$



$w\downarrow \Rightarrow$ better off

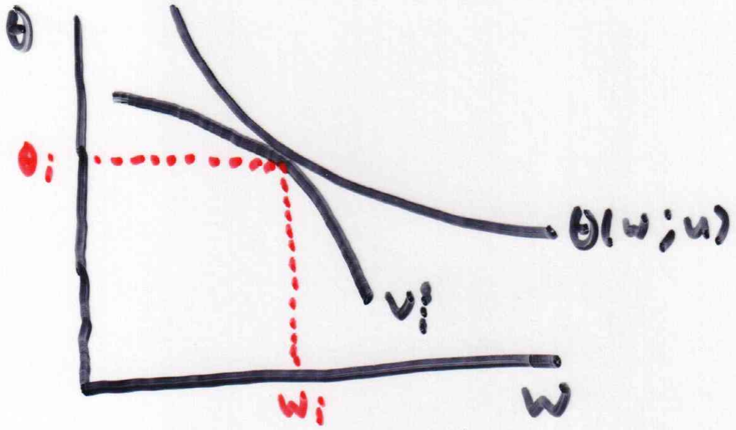
$\theta\uparrow \Rightarrow$ worse off

Equilibrium in Market i

Wages in mkt. i satisfy,

$$V_i^s = \sup_w V(y_i, w, \theta(w; u))$$

So that equil. in mkt. i can be visualized as,



Market Equilibrium

- To find the equilibrium, we must first determine the expected value from opening a vacancy

$$\boxed{\bar{V}(u) = \sum_{i=r}^{n} f_i \max_w V(y_i, w, \theta(w; u))}$$

threshold tech.
lowest vacancy type

Equilibrium Conditions

1.) $\bar{V}(\mathcal{U}) = k$ $\Longrightarrow \mathcal{U}$

2.) $w_i = \underset{w}{\arg\max}\; V(y_i, w, \theta(w; \mathcal{U}))$ $\Longrightarrow w_i$

3.) $r\mathcal{U} = \dfrac{(r+s)z + p(\theta_i)w_i}{r+s+p(\theta_i)}$ $\quad i \geq \hat{i}$ $\Longrightarrow \theta_i$

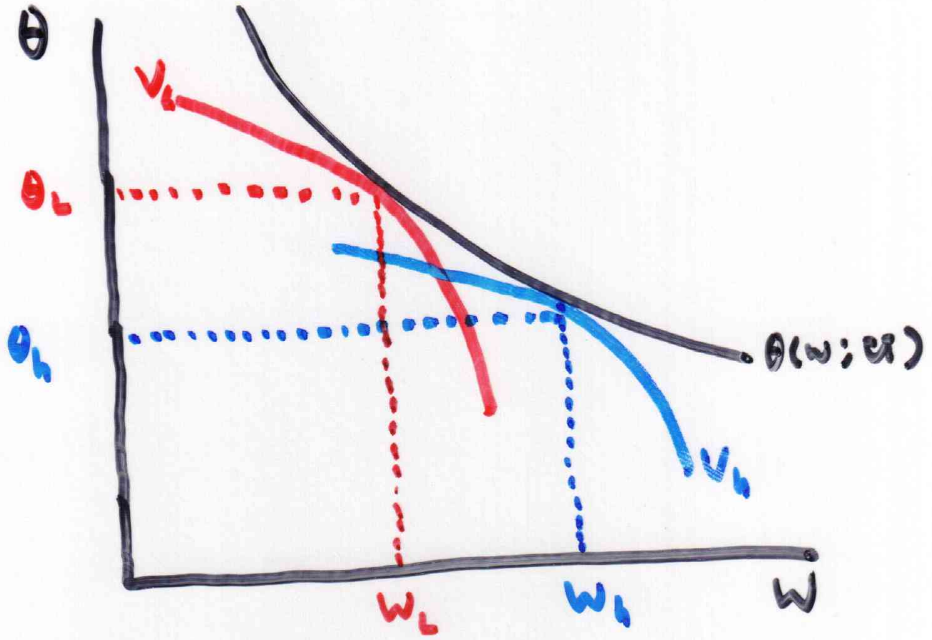
4.) $\underbrace{u_i\, p(\theta_i)}_{\text{job creation in } i} = \underbrace{\tilde{f}_i (1-u) s}_{\text{job destruction in } i}$ $\quad i \geq \hat{i}$ $\Longrightarrow u_i$ $\qquad \tilde{f}_i = \dfrac{f_i}{1-F_{\hat{i}-1}} = \dfrac{\text{prob } i}{\text{Prob}(i \geq \hat{i}-1)}$

5.) $\sum u_i = u$ $\Longrightarrow u$

Proposition: If $\dfrac{\sum_{i=1}^{n} f_i \max[y_i - z, 0]}{r+s} > k$

then an equilibrium exists.

- We can visualize the market as follows,



- There are just 2 firms here, a high productivity type (V_h), and a low productivity type (V_L). By allowing firms to advertise their wage, we achieve a separating equilibrium. High productivity firms have a high opportunity cost of an unfilled vacancy, so they offer higher wages to reduce wait times. In equilibrium, the expected profits in all submarkets are the same. Likewise, workers are indifferent as to where they apply for a job.