2 Countries / Endogenous World Interest Rate

1.) Home :

$$u'(c_1) = \beta(1+r)u'(c_2)$$

$$c_1 + \frac{1}{1+r}c_2 = Y_1 + \frac{1}{1+r}Y_2$$

2.) Foreign :

$$u'(c_1^*) = \beta(1+r)u'(c_2^*)$$

$$c_1^* + \frac{1}{1+r}c_2^* = Y_1^* + \frac{1}{1+r}Y_2^*$$

3.) Market Clearing :

$$c_1^w = c_1 + c_1^* = Y_1 + Y_1^* = Y_1^w$$

$$c_2^w = c_2 + c_2^* = Y_2 + Y_2^* = Y_2^w$$

6 eqs. in $(c_1, c_2, c_1^*, c_2^*, r)$

[1 eq. is redundant (Walras' Law)]

Assume $u(\cdot) = \frac{1}{1-1/\sigma} c^{1-1/\sigma}$ (same for both countries)

From Euler eqs.,

$$\frac{1}{1+r} = \beta\left(\frac{c_2}{c_1}\right)^{-1/\sigma} = \beta\left(\frac{c_2^*}{c_1^*}\right)^{-1/\sigma} \Longrightarrow \boxed{\frac{c_2^*}{c_1^*} = \frac{c_2}{c_1}}$$

From Resource Constraints,

$$\frac{c_2 + c_2^*}{c_1 + c_1^*} = \frac{Y_2^w}{Y_1^w} = \frac{c_2(1 + c_2^*/c_2)}{c_1(1 + c_1^*/c_1)}$$

$$\Longrightarrow \boxed{\frac{c_2}{c_1} = \frac{Y_2^w}{Y_1^w}}$$

From either Euler eq.,

$$\frac{1}{1+r} = \beta\left(\frac{Y_2^w}{Y_1^w}\right)^{-1/\sigma} \Longrightarrow \boxed{1+r = \frac{1}{\beta}\left(\frac{Y_2^w}{Y_1^w}\right)^{1/\sigma}}$$

Current Account Implications,

$$C_1\left[1 + \frac{1}{1+r}\frac{C_2}{C_1}\right] = Y_1\left[1 + \frac{1}{1+r}\frac{Y_2}{Y_1}\right]$$

Sub in previous results,

$$C_1 = Y_1\left[\frac{1 + \beta\left(Y_2^w/Y_1^w\right)^{-1/\sigma} Y_2/Y_1}{1 + \beta\left(Y_2^w/Y_1^w\right)^{-1/\sigma} Y_2^w/Y_1^w}\right]$$

Therefore,

$$\boxed{\frac{CA_1}{Y_1} = 1 - \frac{C_1}{Y_1} = \frac{\left(Y_2^w/Y_1^w\right) - Y_2/Y_1}{1 + r + Y_2^w/Y_1^w}}$$

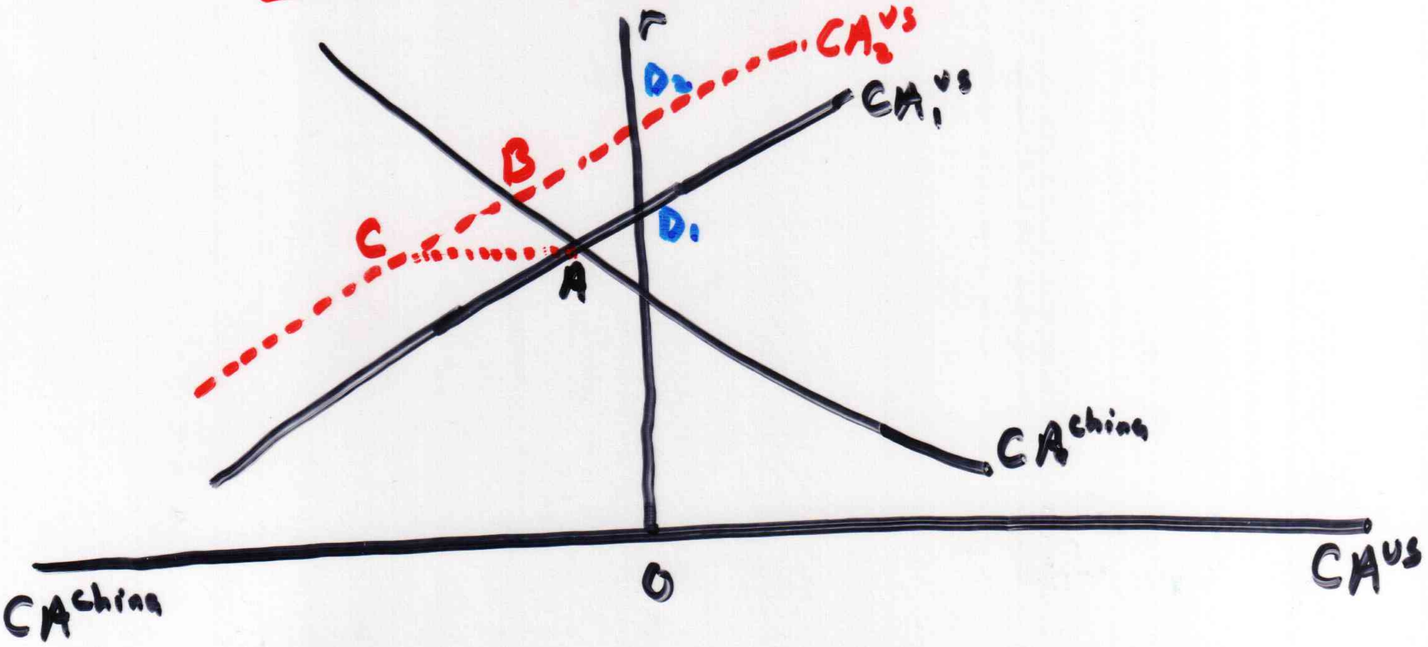
$$\frac{Y_2}{Y_1} > \frac{Y_2^w}{Y_1^w} \Longrightarrow CA_1 < 0$$

} Engel + Rogers (JME, 2006)

Large Countries

- The sum of all CAs must be zero. Let's suppose the world just consists of the U.S. + China.

$$\boxed{CA^{U.S.} + CA^{China} = 0}$$



1.) A : initial equil. (U.S. deficit, China Surplus)

A→B : Equil. after $A_2^{U.S.}\uparrow$ (Note: $r^*\uparrow$)

A→C : Response in "small" economy

$D_1 \rightarrow D_2$: Response in closed economy

Punchline : Response of large country lies between that of closed + small open economies.