Topics for Today

1.) Labor Demand
- Production Functions
- Profit Maximization

2.) Labor Supply
- Utility Functions + Indifference Curves
- Income + Substitution Effects

3.) Labor Market Equilibrium
- Comparative Statics

Visualizing the Economy

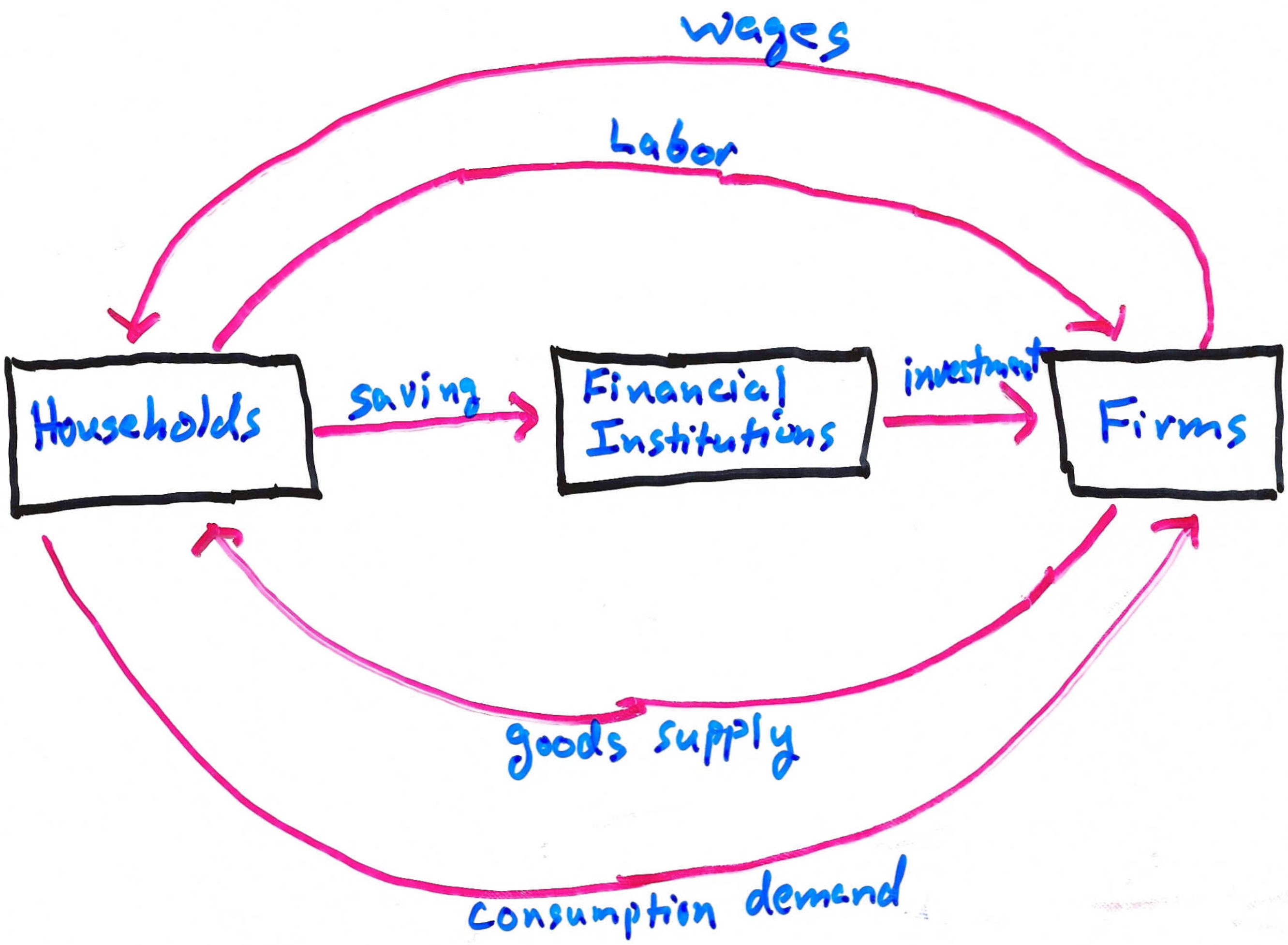
We are going to attempt to model 3 inter-related markets:

1.) Labor Market

2.) Goods Market

3.) Asset/Financial Market

Their relationship can be visualized as follows:



Macroeconomics is Based on Microeconomics

- Notice from this picture that:
  1.) Households supply labor and demand consumption goods
  2.) Firms demand labor and supply consumption goods

- Ultimately, we would like to be able to predict how the economy responds to various disturbances (e.g., macro policies). One way to do this is to assume that households and firms optimize something. We can then use the tools of comparative statics to predict how they will respond.

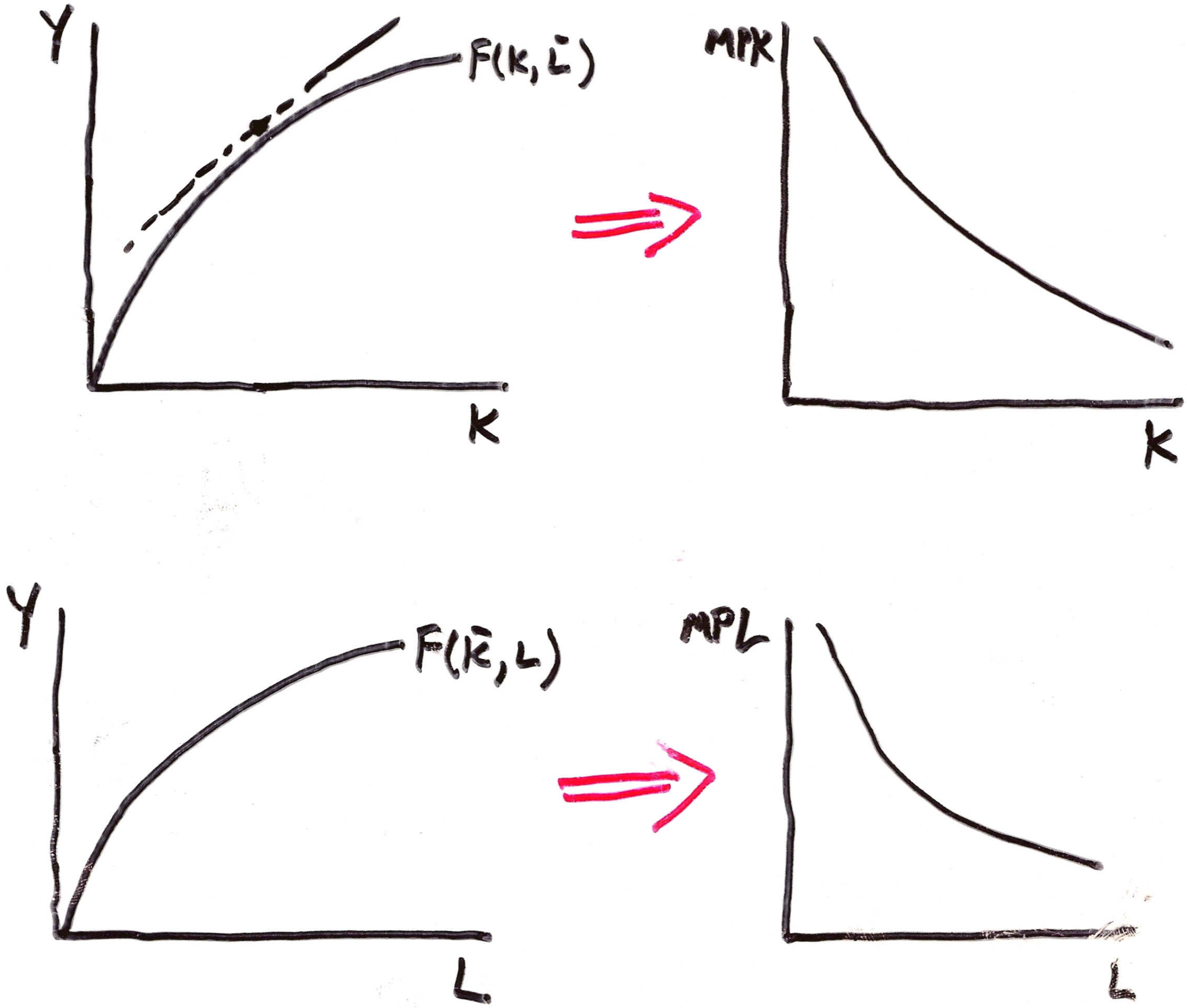
- As in micro, we will assume firms maximize profits subject to resource and technology constraints, and assume households maximize "utility" subject to budget constraints.

- The supply + demand decisions of households and firms are made consistent with each other via market-clearing adjustment in prices. In the goods market the relevant price is the interest rate. In the labor market it is the wage rate.

- We assume competitive markets, meaning that firms + households take wages + interest rates as given.

The Production Function

$$Y = F(K, L)$$



Factor Demand

We make 2 key assumptions:

1.) Profit Maximization

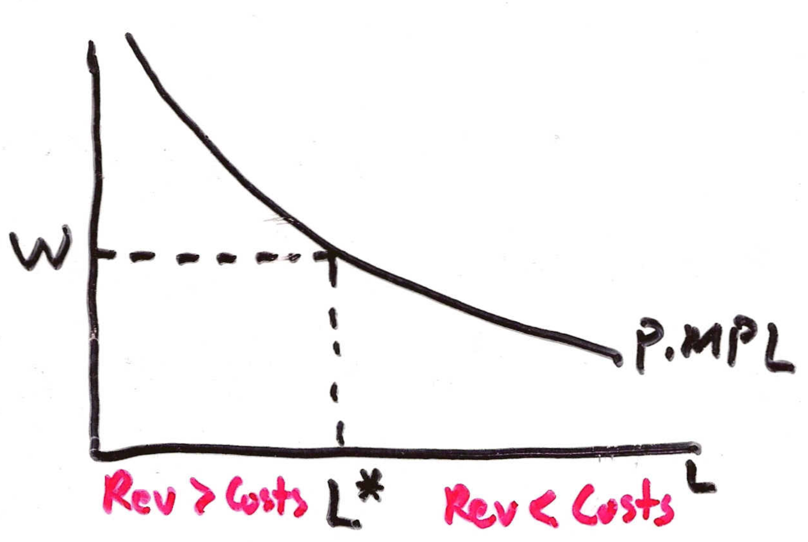
2.) Competitive Goods and Factor Markets

$$\text{Profits} = \text{Revenues} - \text{Costs}$$
$$= P \cdot F(K, L) - wL - RK$$

First-Order Conditions:

$$\left.\begin{aligned} P \cdot \frac{\partial F}{\partial L} - w &= 0 \\ P \cdot \frac{\partial F}{\partial K} - R &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} P \cdot MPL &= w \\ P \cdot MPK &= R \end{aligned}$$

$$\text{Marg. Benefit} = \text{Marg. Cost}$$



The Cobb-Douglas Production Function

Observed Aggregate Production Functions exhibit two properties:

1.) Constant Factor Shares

2.) Constant Returns to Scale

Question: What functional form has these two properties?

Answer: $\left. Y = A \cdot K^{\alpha} L^{1-\alpha} \right\}$ Cobb-Douglas

Verify

$$\begin{aligned} \text{Labor Income} &= MPL \cdot L \\ &= \left[(1-\alpha) \cdot A \cdot K^{\alpha} L^{-\alpha}\right] \cdot L \\ &= (1-\alpha) \underline{A K^{\alpha} L^{1-\alpha}} \\ &= (1-\alpha) Y \end{aligned}$$

$$\text{Labor Share} = \frac{\text{Labor Income}}{Y} = 1-\alpha$$

$\alpha \approx .3 \quad 1-\alpha = .7$

Returns to Scale: What happens to output if <u>all</u> factors are increased (or decreased) by x%

$$A(xK)^{\alpha}(xL)^{1-\alpha} = A x^{\alpha} x^{1-\alpha} K^{\alpha} L^{1-\alpha}$$
$$= x \cdot A K^{\alpha} L^{1-\alpha}$$
$$= xY \Longrightarrow \text{CRS}$$

Question: What if $Y = AK^{\alpha}L^{\beta}$?

$\alpha + \beta > 1 \Longrightarrow$ Increasing Returns

$\alpha + \beta < 1 \Longrightarrow$ Decreasing Returns

Calibrating a Cobb-Douglas Production Function

$$Y = AK^{\alpha}L^{1-\alpha}$$

Problem: Both A and K are difficult to measure

Solution: Use economic theory!

(1.) $$Y/L = A(K/L)^{\alpha}$$

(2.) $$MPK = r + \delta$$
$$= \alpha A K^{\alpha-1} L^{1-\alpha} = \alpha (Y/K)$$

From (2) $\Longrightarrow$ $$K/L = \frac{\alpha}{r+\delta} Y/L$$

<u>Observed</u>: $Y/L \approx 45{,}000$ $\quad \alpha = .3$

$\delta \approx .10$ $\quad r \approx .10$

$$\Longrightarrow K/L = 3/2\,(45{,}000) = 67{,}500$$

$$\Longrightarrow A = \frac{45{,}000}{(67{,}500)^{.3}} \approx 1600$$

Labor Supply

We assume households care about two broad categories of goods:

1.) Consumption Goods

2.) Leisure

Later, when we focus on the goods market, we will distinguish between current consumption and future consumption, but for now we just consider a static 1-period model.

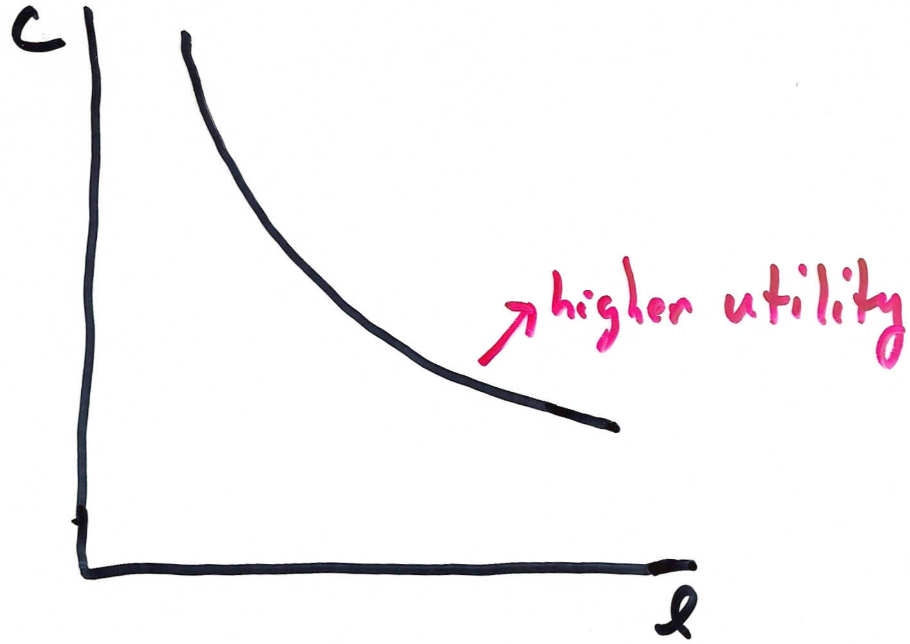
As usual, we assume the preferences of households can be described by an (ordinal) utility function, $U(c, \ell)$, where c = consumption and ℓ = leisure.

Naturally, households prefer to have more of both consumption and leisure, meaning $\frac{\partial U}{\partial c} > 0$ and $\frac{\partial U}{\partial \ell} > 0$. We also assume their desires are subject to diminishing marg. utility, $U_{cc} < 0$ $U_{\ell\ell} < 0$

These preferences can be visualized by means of Indifference Curves. An indifference curve describes combinations of C and ℓ that provide a constant, or fixed, amount of utility. [Of course, we can't say anything about how much utility, but for us, all we need to be able to do is compare (or rank) utility levels].

Clearly, since both C and ℓ are desirable, an Indifference curve must be negatively sloped. Moreover, diminishing Marginal utility is (almost) enough to ensure that they are "convex", meaning that households prefer to have a mixture of C + ℓ than to have a lot of one or the other.

Therefore we have,



Mathematically, the slope of an Indifference curve is given by:

$$dU = U_c\,dc + U_\ell\,d\ell = 0$$

$$\Longrightarrow \left.\boxed{\frac{dc}{d\ell} = -\frac{U_\ell}{U_c}}\right\}\ \text{MRS}$$

The ratio $-\frac{U_\ell}{U_c}$ is called the marginal rate of substitution of consumption for leisure. It tells us how much consumption someone is willing to give up in exchange for an additional unit of leisure. Notice this decreases as you acquire more & more leisure.

The household's goal is to attain the highest Indifference Curve subject to a budget constraint. The household confronts 2 constraints:

1.) A time constraint

$$\boxed{l + h = 1}$$

where h = time devoted to the market (labor supply)

and l = leisure time

[Note: We can just normalize the total time endowment to 1].

2.) An expenditure or market constraint

$$\boxed{p \cdot c = w \cdot h}$$

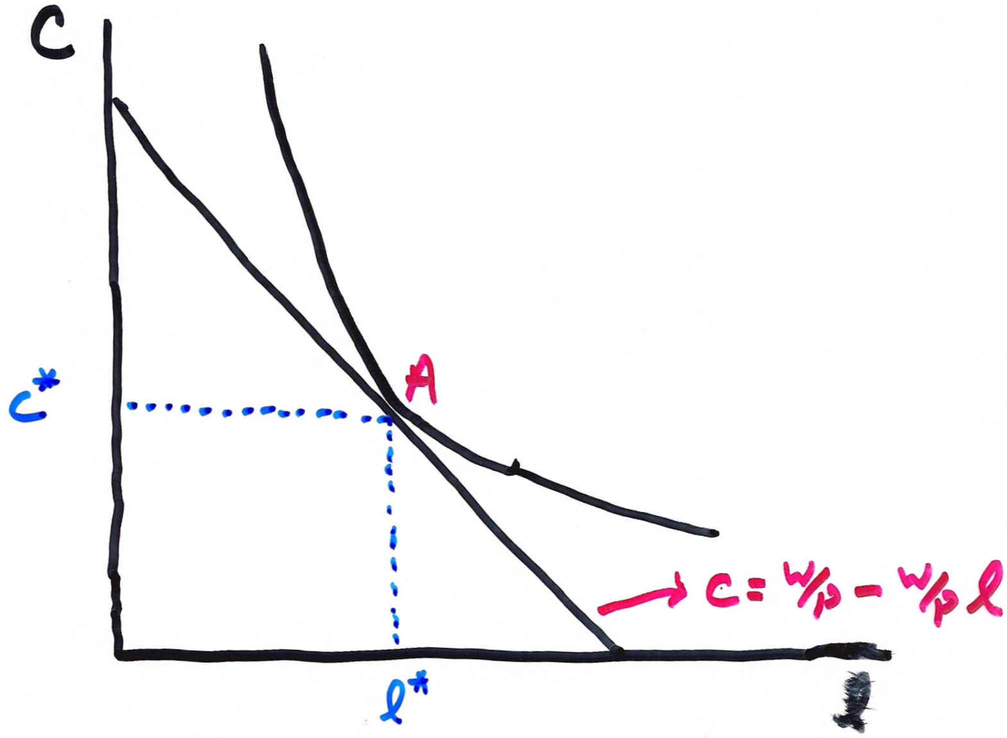
p = price of goods

w = wage rate

We can combine, or consolidate, these 2 constraints into 1 equation

$$\boxed{c + \frac{w}{p} l = \frac{w}{p}}$$

The household's optimal combination of (c, ℓ) is described by a tangency condition, where the slope of an Indifference Cure is equal to the slope of the budget constraint:



At pt. A the rate at which households are willing to substitute consumption and leisure is exactly equal to the rate at which they are able to substitute between them in the market. That is,

$$\boxed{\frac{w}{p} = -\frac{u_\ell}{u_c}}$$

Note that w/p can be interpreted as the "price of leisure".

Another way to derive this condition is to assume the household knows calculus!

$$\max_{c,\ell} \left\{ u(c,\ell) + \lambda \cdot \left[w/p - c - w/p\,\ell \right] \right\}$$

Lagrange Multiplier

First-Order Conditions

1.) ℓ : $\quad u_\ell - \lambda\, w/p = 0$

2.) c : $\quad u_c - \lambda = 0$

Divide (1.) by (2.)

$$\boxed{\frac{u_\ell}{u_c} = w/p} \quad \Big\} \text{ MRS = real wage}$$

Comparative Statics

Suppose $w/p \uparrow$. What happens to (c, ℓ)?

Intuition,

When $w/p \uparrow$ the household's real income goes up. Since c and ℓ are assumed to be "normal goods", when real income rises the household will want to consume more of both. Note, however, that when $w/p \uparrow$ the "price of leisure" goes up, and so the household will substitute away from leisure and toward consumption.

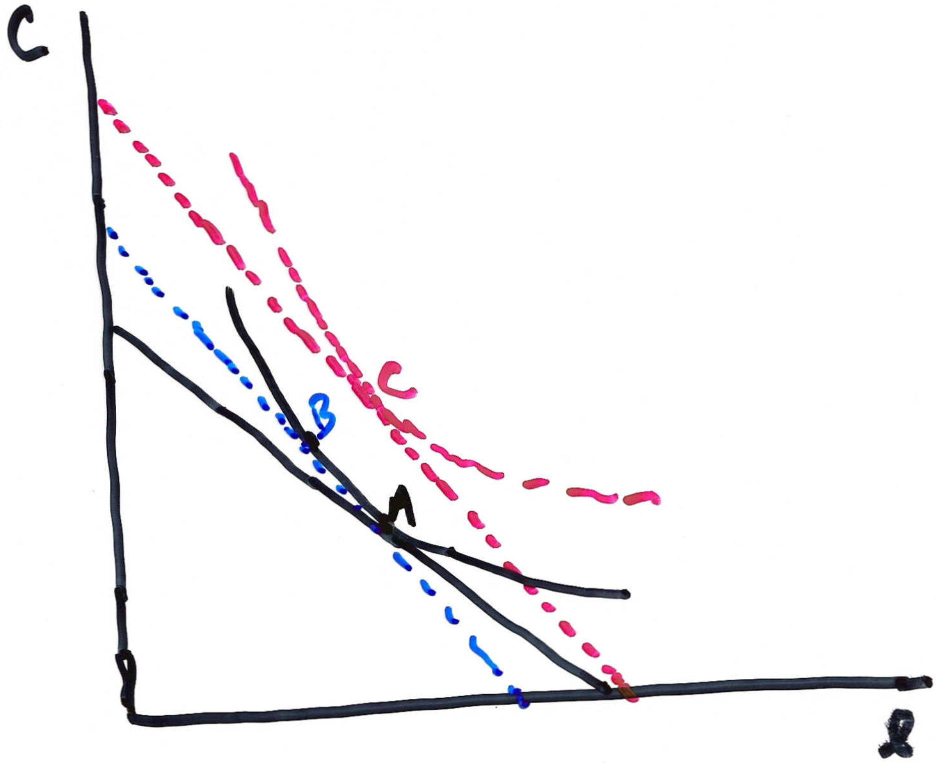
Notice that for c the income and substitution effects work in the same direction, leading to $c \uparrow$.

However, for ℓ the substitution effect $(\ell \downarrow)$ is offset by the income effect $(\ell \uparrow)$. Hence, the effect of w/p on ℓ is ambiguous.

Income effect dominates $\Longrightarrow \ell \uparrow$

Substitution effect dominates $\Longrightarrow \ell \downarrow$

Graphically,

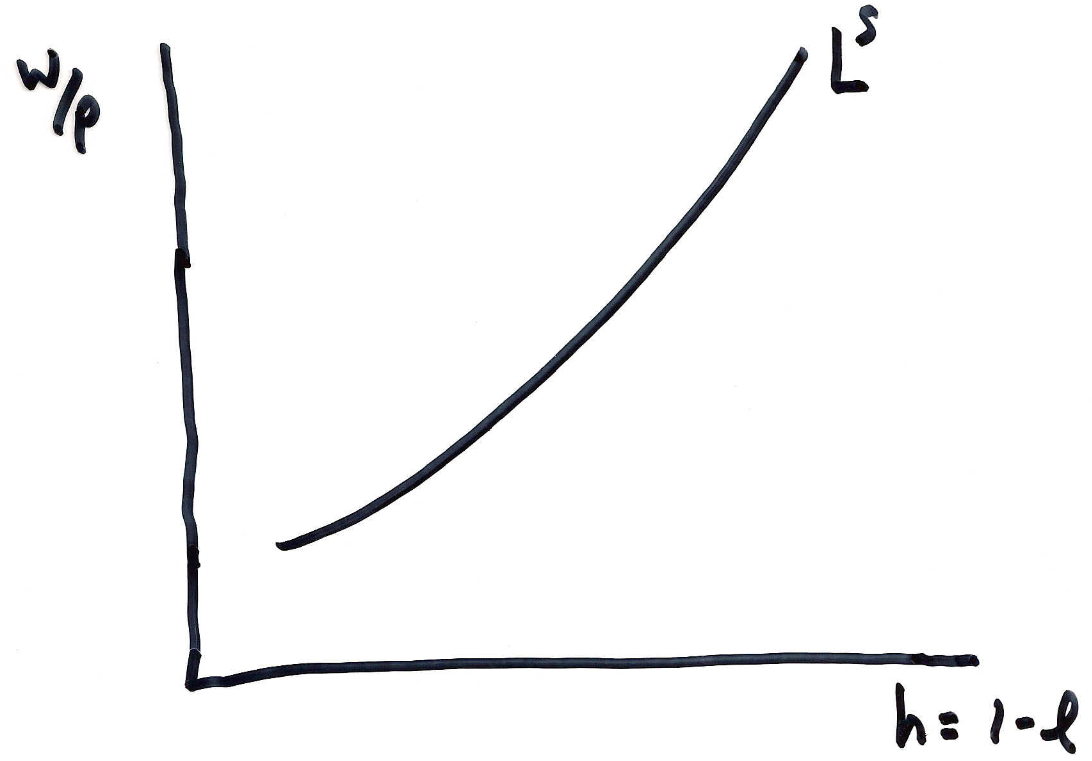


1.) A → B : Substitution Effect
Note $l\downarrow$ due to diminishing MRS (convexity of Indifference Curve)

2.) B → C : Income Effect
$l\uparrow$

3.) A → C : Total Effect
Ambiguous in general

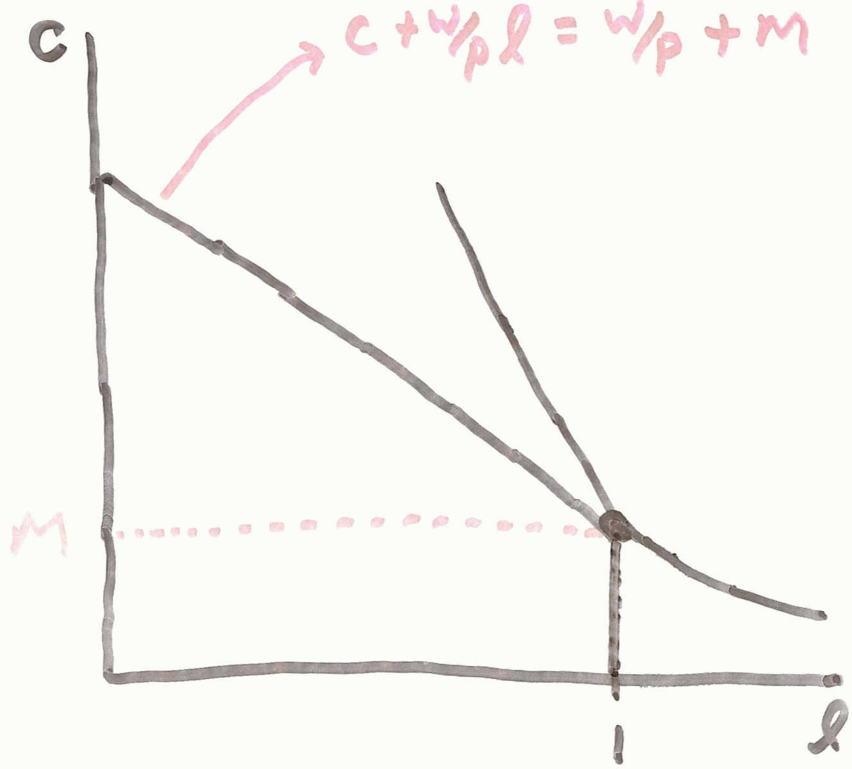
Normally we assume that the substitution effect dominates, so that we get an upward sloping labor supply curve,



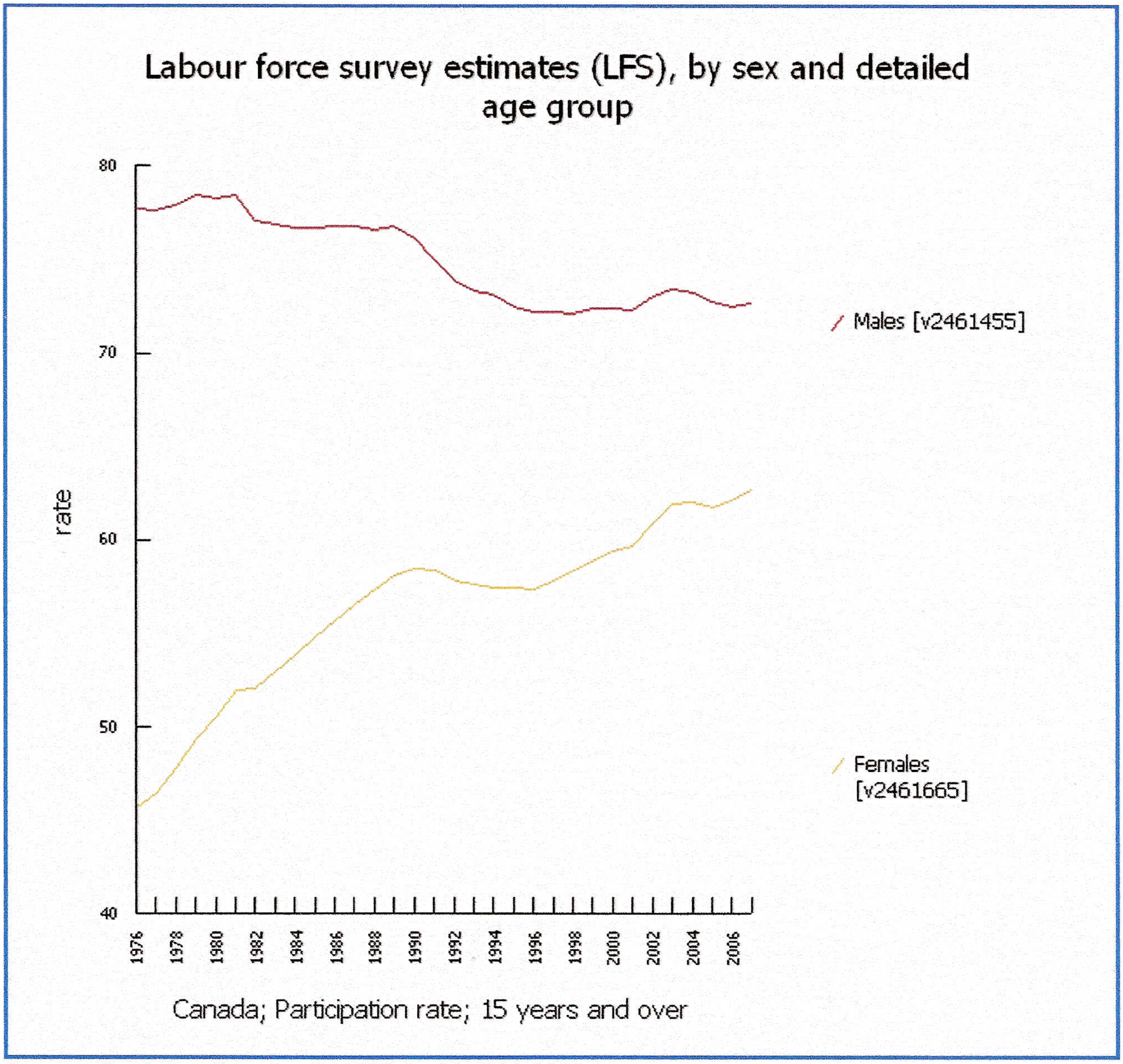
Note: This curve just traces out the points of tangency between an IC and budget constraint as w/p changes.

Labor Force Participation

We can make the model a little more realistic by assuming that households have some non-market income (or wealth).

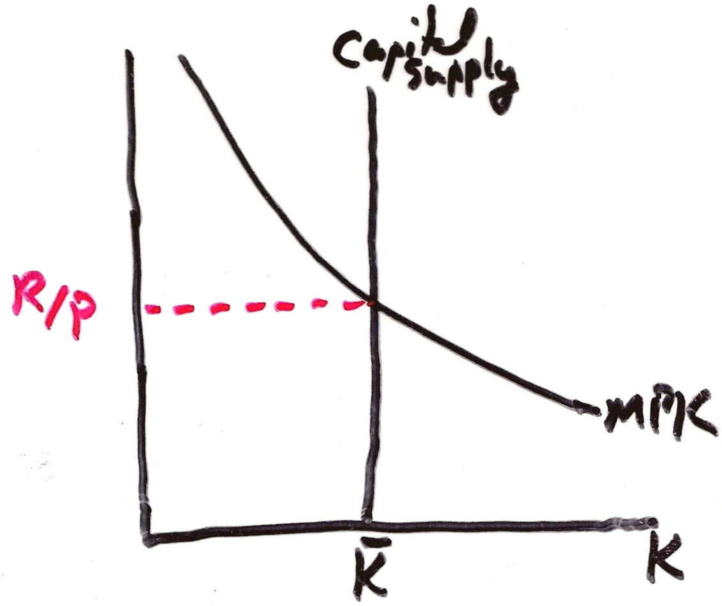


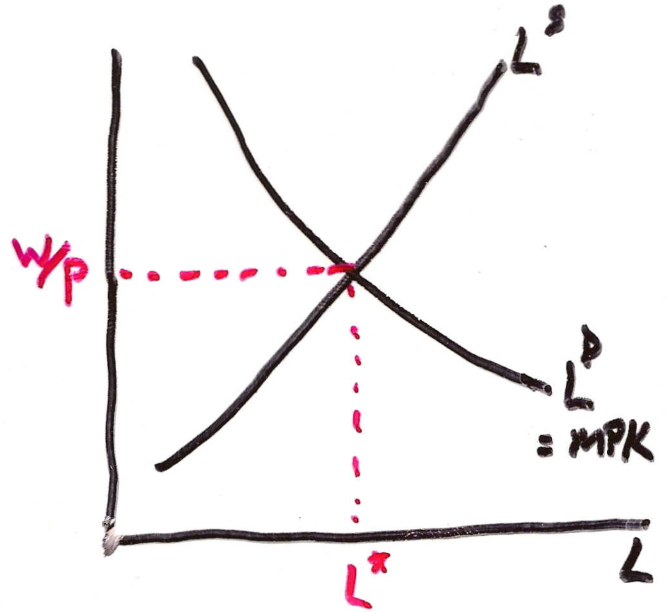
Notice that with non-market income, households might choose a corner solution (i.e., set $\ell = 1$ and supply no labor). In this case, "labor force participation" would be zero.



Source: Statistics Canada, CANSIM table 282-0002.

Factor Market Equilibrium





2 Things to Note

1.) Simultaneity. Each factor price depends on the supply of both factors. Markets are inter-dependent.

2.) Only real factor prices are determined. What determines P?

Distribution of Income

Distribution of Income = Factor Prices + Distribution of Factor Ownership

Unequal Distribution of Factor Ownership $\Rightarrow$ Unequal Distribution of Income.

Shifts in the Curves

Shifts in the Aggregate Demand for Capital

1.) Changes in Technology
2.) Changes in the Supply of Labor

Shifts in the Aggregate Supply of Capital

1.) Investment (gradual)
2.) Wars + Natural Disasters

Shifts in the Aggregate Demand for Labor

1.) Changes in Technology
2.) Changes in the Supply of Capital

Shifts in the Aggregate Supply of Labor

1.) Changes in Population
2.) Changes in labor force participation
3.) Changes in wealth
4.) Changes in expected future wages

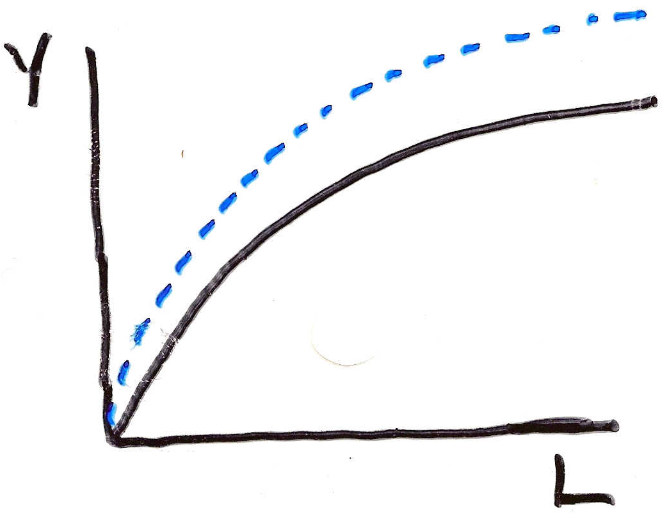
Supply Shocks

Examples: 1.) Changes in Technology, e.g., innovations

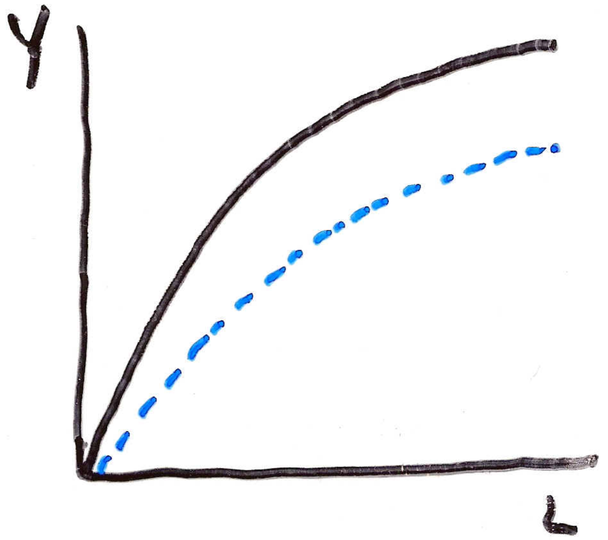
2.) Weather / Natural Disasters

3.) Laws / Taxes

3.) Changes in the Supply of omitted factors

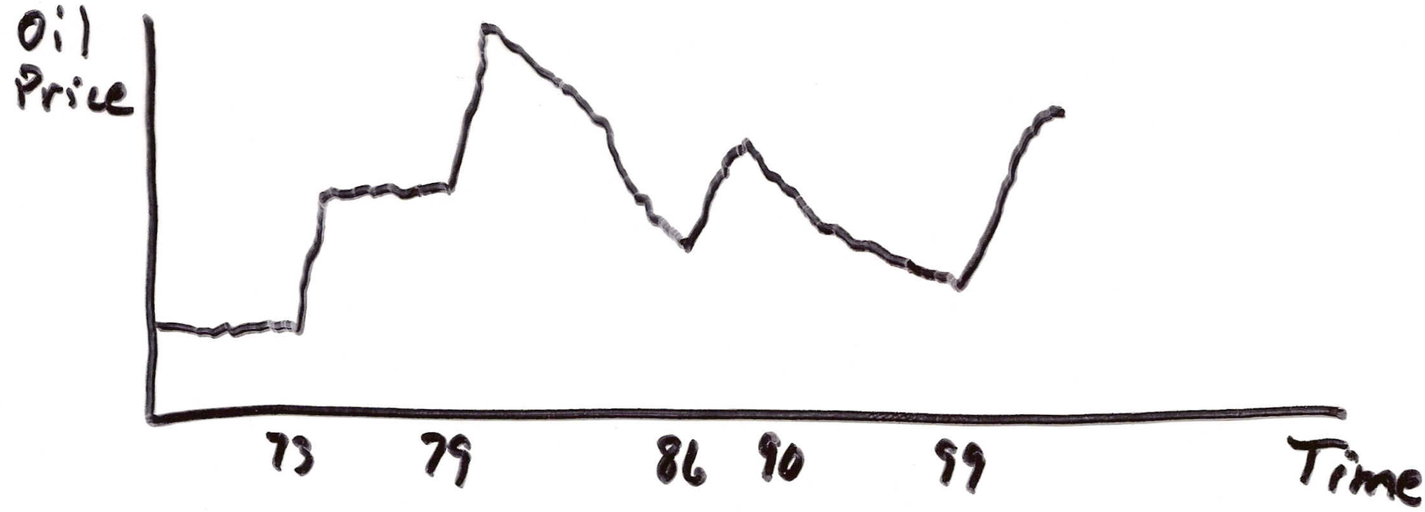


positive supply shock

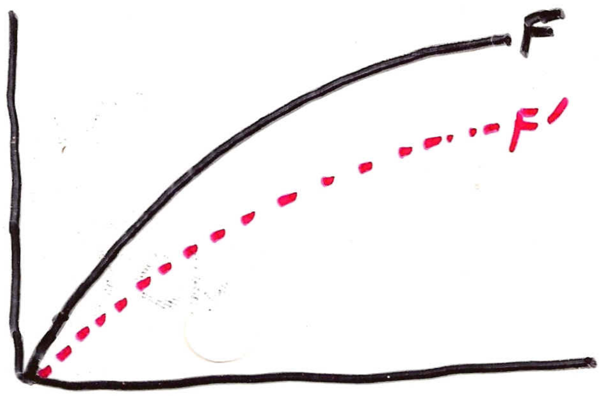


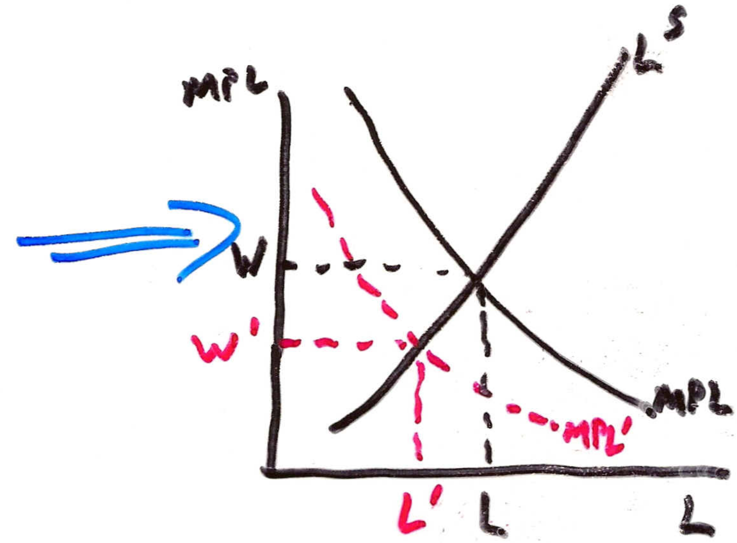
negative supply shock

Example 1 : Oil Prices



Oil Price Increase





Predicted Effects

1.) Real Wage Falls

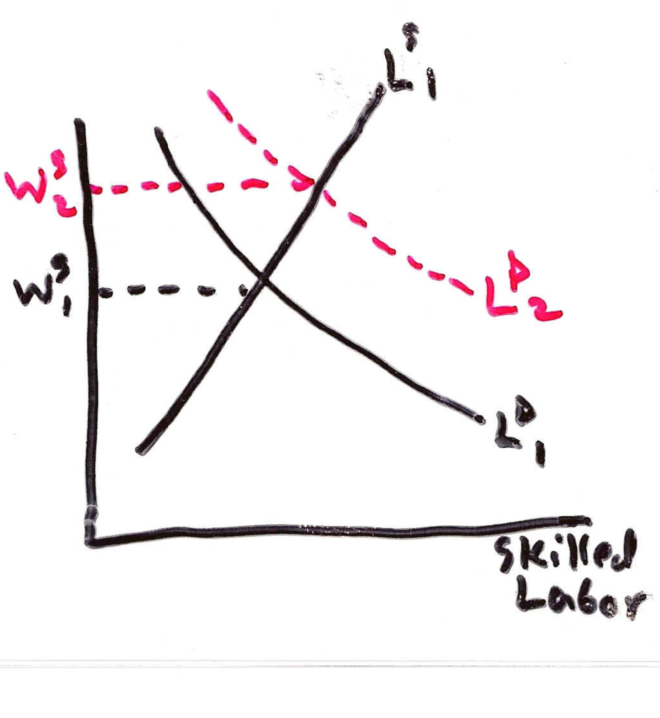
2.) Employment Falls

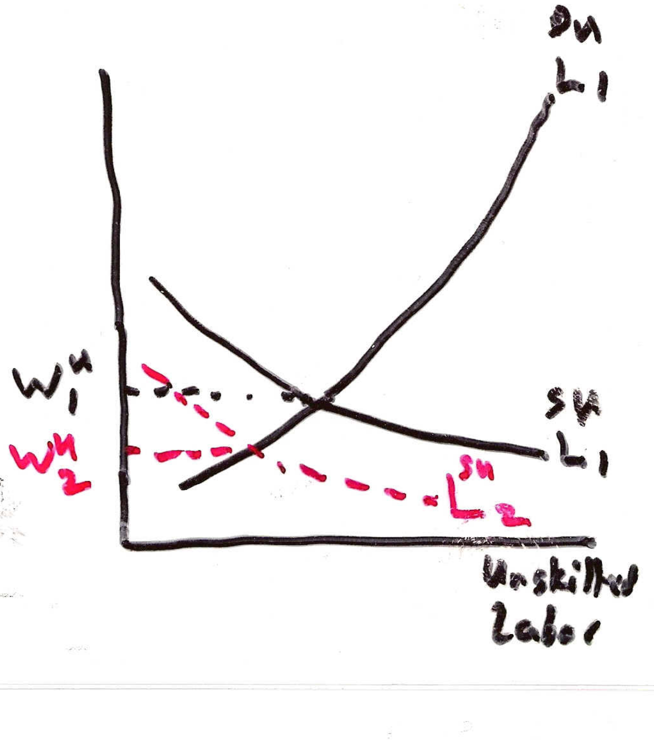
3.) Output Declines (for 2 reasons).

Example 2: Skill-biased Tech. Shock

The past 30 years has exhibited two broad labor market trends: (1.) Stagnent average real wage, and (2.) Growing inequality of labor income.

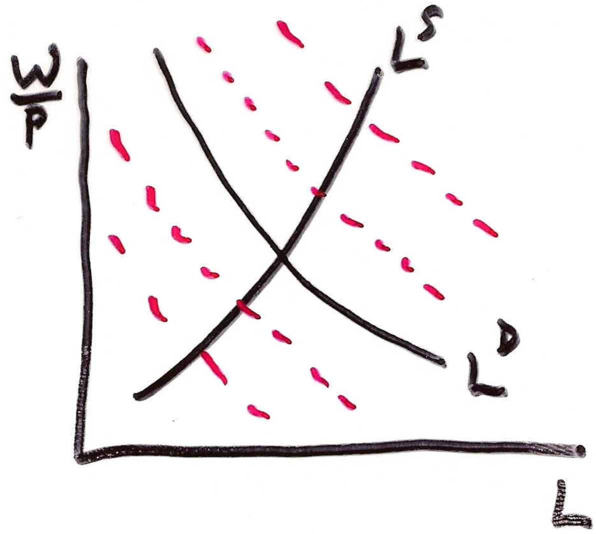
	1967	1975	1993
% of women < 20,000	86.6	73.1	61.6
% of men < 20,000	44.7	35.1	43.0
% of women > 40,000	0.6	2.6	7.7
% of men > 40,000	9.9	23.3	23.9
90-10 ratio women	45.8	47.9	42.7
90-10 ratio men	30.7	35.4	45.4





The Identification Problem

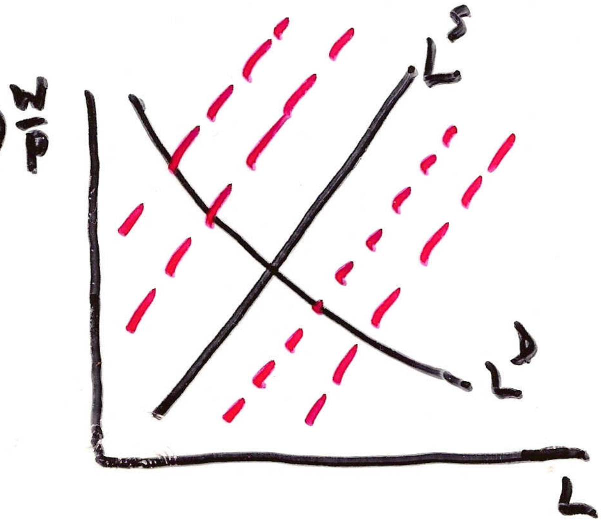
1.



only labor demand shifts

$\Longrightarrow$

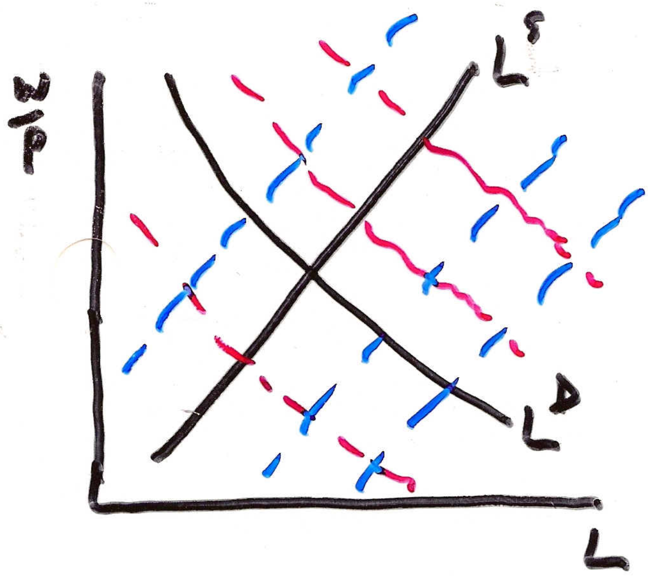
2.



only labor supply shifts

$\Longrightarrow$

3.



Both Shift

$\Longrightarrow$