Topics for Today

1.) Investment
   - Business Fixed Investment
   - Residential Investment
   - Inventory Investment

2.) The "User Cost of Capital"

3.) Adjustment Costs

4.) Taxes + Investment

5.) The Stock Market + "Tobin's Q"

6.) Financing Constraints + the "Financial Accelerator"

7.) Residential Investment

8.) Goods Market Equilibrium
   - Comparative Statics

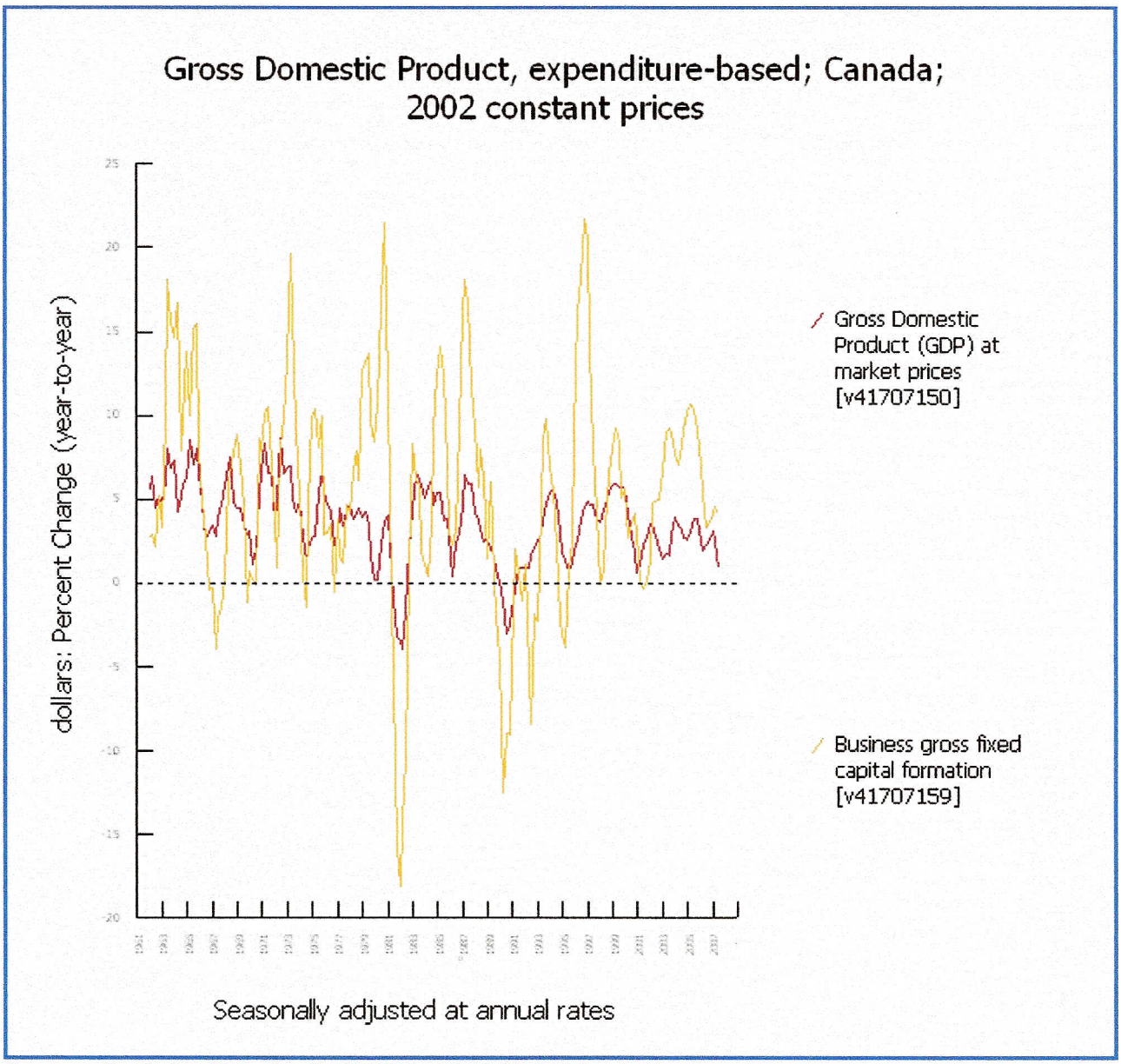
Investment

- The goods market is in equilibrium when the aggregate supply of goods equals the aggregate demand for goods, or equivalently, when Saving = Investment (in closed economies).

- The demand for goods comes from 2 main sources:
  1.) Consumption by households
  2.) Investment by firms

- Last time we studied consumption. Today we study investment.

- As with households, we assume investment represents the solution to an optimization problem, i.e., the attempt by firms to maximize (expected) profits.



Source: Statistics Canada, CANSIM table 380-0002.

Profit-Maximizing Investment

- Investment represents increments to the capital stock. That is,

$$\boxed{K_{t+1} = (1-\delta)K_t + I_t}$$

where δ is the depreciation rate, and I_t is gross investment (i.e., investment that includes replacement of worn out capital). That is, capital, K_t, is a stock variable, and investment, I_t, is the flow that changes it.

- Hence, to understand investment we need to understand the demand for capital.

- Firms use capital (like machinery + buildings) to produce output.

- A firm's profit maximizing choice of capital is based on the same principles as its choice of labor

$$\max_{K_t} \; p \cdot F(K, L) - R \cdot K$$

<u>FOC</u>

$$\underbrace{p \cdot MPK}_{\text{Marginal Benefit}} - \underbrace{R}_{\text{Marginal Cost}} = 0$$

- The variable R is sometimes called the "<u>user cost of capital</u>", or the "rental rate of capital". It consists of 4 main components

1.) The price of capital goods, P_K

2.) The interest rate, r

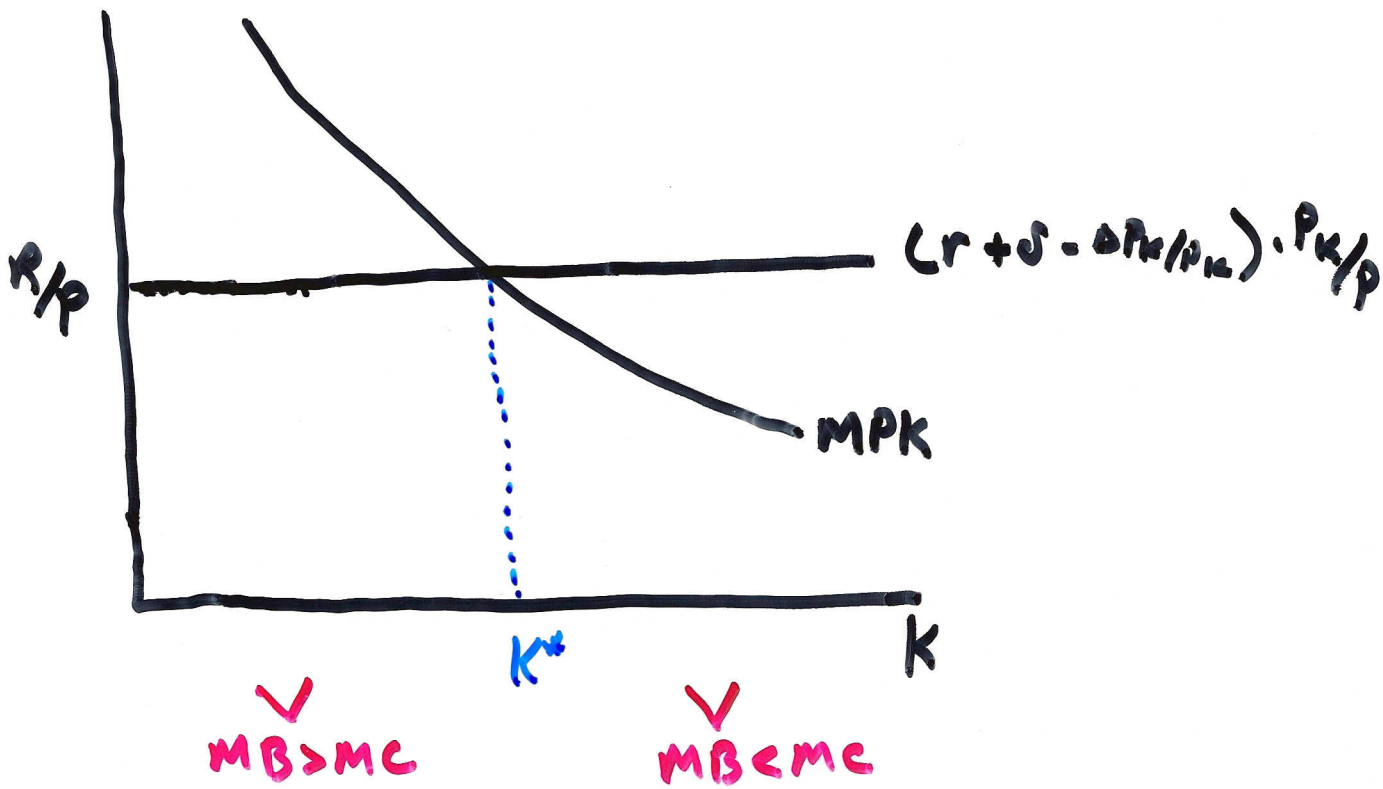
3.) The depreciation rate, δ

4.) Potential changes in the price of capital goods, ΔP_K

More precisely, we have

$$\boxed{\begin{aligned} R &= r P_K + \delta P_K - \Delta P_K \\ &= (r + \delta - \Delta P_K / P_K) \cdot P_K \end{aligned}}$$

Graphically then,

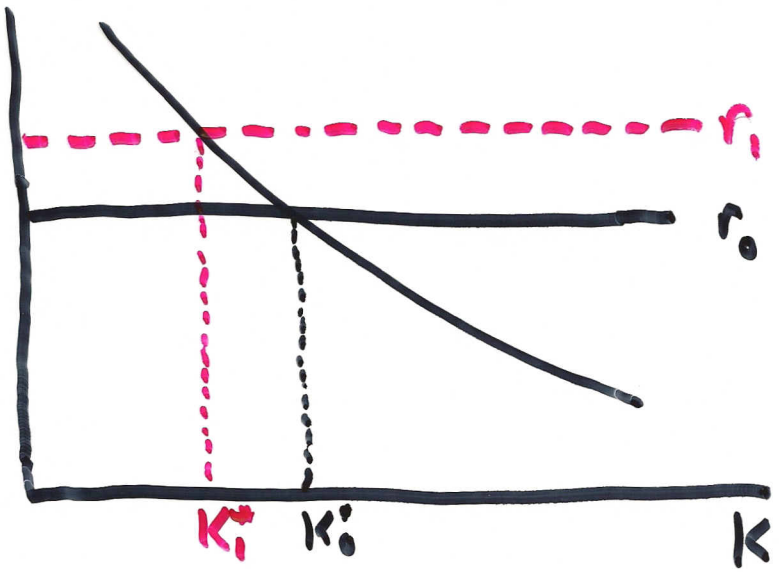


Therefore,

$$\boxed{I_t = (K^*_{t+1} - K_t) + \delta K_t}$$

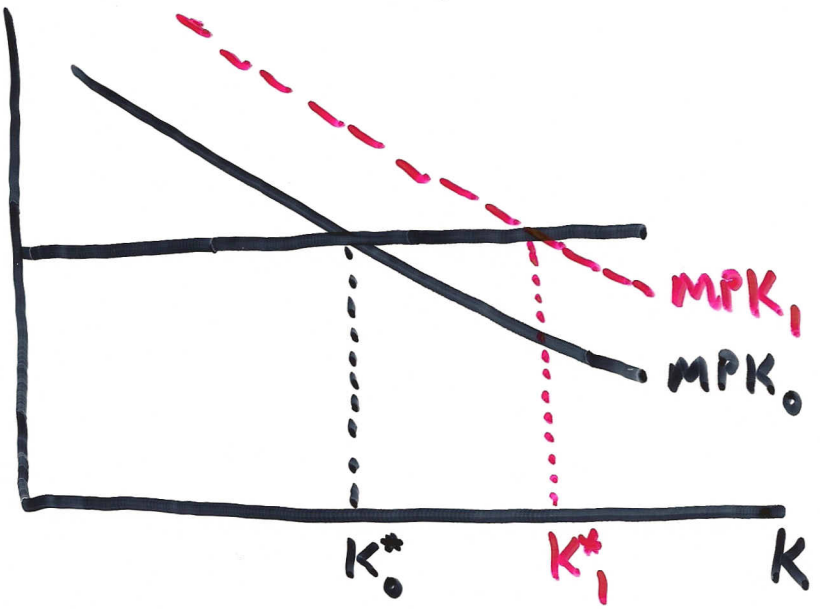
Comparative Statics

1. Suppose interest rate increases



$r\uparrow \Rightarrow R/p\uparrow \Rightarrow K^*\downarrow \Rightarrow I\downarrow$

2. Suppose productivity increases



$A\uparrow \Rightarrow MPK\uparrow \Rightarrow K^*\uparrow \Rightarrow I\uparrow$

Adjustment Costs

- In practice, it can be quite costly to rapidly change your capital stock, due to installation costs and other factors (think skyscrapers + Large Hadron Collider!).

- Therefore, firms only close part of the gap between the desired capital stock and the current capital stock. That is,

$$I_t = \lambda (K^*_{t+1} - K_t) + \delta K_t \qquad \lambda < 1$$

- Since net investment, $K_{t+1} - K_t$, is just gross investment less depreciation, we have

$$K_{t+1} - K_t = \lambda (K^*_{t+1} - K_t)$$

or

$$\boxed{K_{t+1} = \lambda K^*_{t+1} + (1-\lambda) K_t}$$

- That is, the actual current capital stock will be a convex combo of the current desired capital stock and last period's capital stock. History matters!

Taxes + Investment

- Government tax policy is often aimed at influencing investment. Sometimes it has unintended consequences on investment.

- We'll just consider a couple examples

1.) How does the corporate profits tax influence investment?

$$\text{Profits} = \text{Revenues} - \text{Costs}$$
$$= P \cdot F(K,L) - R \cdot K - w \cdot L$$

After-Tax Profits

$$(1-\tau) \cdot [P \cdot F(K,L) - R \cdot K - wL]$$

$\Rightarrow$ Same FOCs as before! A pure profits tax does not influence investment or labor demand.

(Of course, in a competitive industry there are no profits in equilibrium, so the tax wouldn't raise revenue either!)

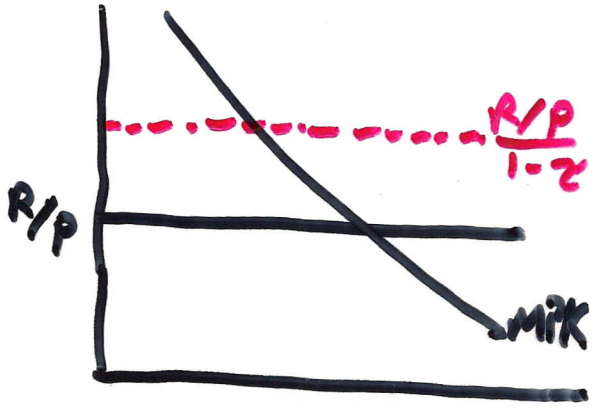
- In practice, profits are hard to measure, mainly because true economic costs are hard to measure.

- A leading example concerns depreciation. In practice, depreciation allowances are based on historical cost. When there is inflation, or when technological advance leads to rapid obsolescence, this practice tends to understate true depreciation, and thus overstate profits. In this case, the corporate profit tax could discourage investment.

- More generally, many tax policies have the effect of taxing revenues, not profits. In this case, we can define a tax-adjusted user cost of capital.

$$(1-\tau)\cdot PF(K,L) - RK$$

$$\Rightarrow \quad MPK = \frac{R/P}{1-\tau}$$



The Stock Market + Tobin's Q

Empirically, there is a close connection between stock prices and investment.

$$\boxed{\begin{array}{l} \text{Stock Prices} \uparrow \Longrightarrow \text{Investment} \uparrow \\ \text{Stock Prices} \downarrow \Longrightarrow \text{Investment} \downarrow \end{array}}$$

Why?

A share of equity entitles the holder to the PDV of all the future output (less labor costs) produced by the firm's capital stock

Assume for simplicity that the MPK and the price of the firm's output is constant.

Then the stock market value of the firm is

$$P \cdot MPK \cdot K \left[\frac{1}{1+r+\delta} + \left(\frac{1}{1+r+\delta}\right)^2 + \cdots \right]$$

$$= \frac{P \cdot MPK \cdot K}{r+\delta}$$

Note, with constant returns to scale $F(K,L) = MPK \cdot K + MPL \cdot L$ so that

$P \cdot F(\cdot) - P \cdot MPL \cdot L = P \cdot MPK \cdot K$

or

$P \cdot MPK \cdot K = P \cdot F(\cdot) - W \cdot L$

Next, the replacement cost of the firm's capital is just $P_K \cdot K$.

If we define

$$Q = \frac{\text{Market Value of Capital}}{\text{Replacement Cost of Capital}}$$

we get by substitution,

$$Q = \frac{MPK}{P_K/P\,(r+\delta)}$$

And our earlier investment theory can be stated,

$$\boxed{\begin{array}{lcl} Q > 1 & \Longrightarrow & \text{Investment} > 0 \\ Q < 1 & \Longrightarrow & \text{Investment} < 0 \end{array}}$$

Financing Constraints + the "Financial Accelerator"

- So far, we've assumed firms can borrow (and lend) as much as they want at the fixed market interest rate, r.

- In practice, this isn't true! Due to asymmetric info lenders face default risk (moral hazard). Hence, firms face financing constraints (e.g., credit rationing).

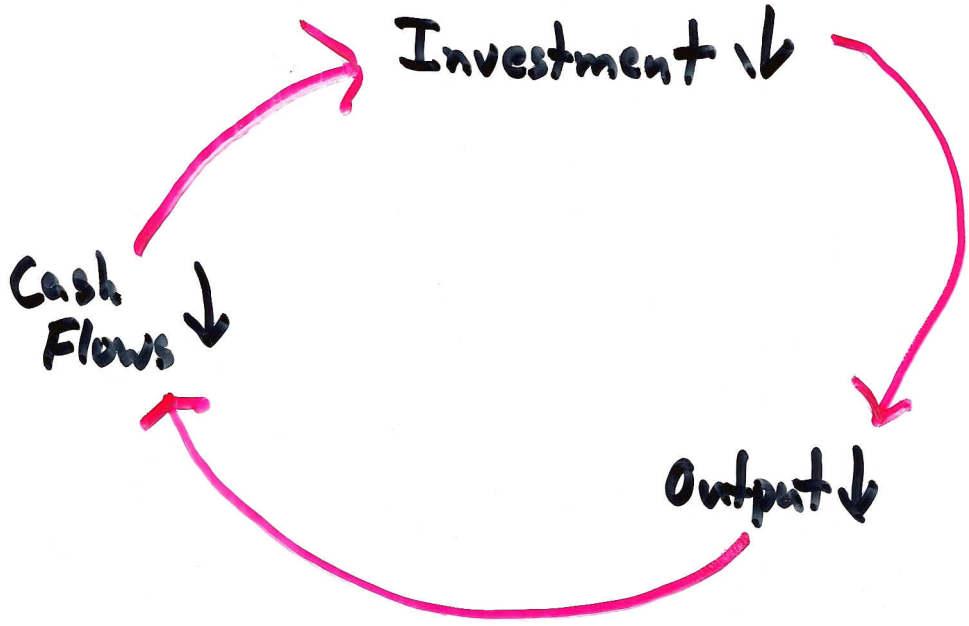
- Note, with limited liability just charging firms a higher interest rate to compensate for default risk might actually be counterproductive, as it attracts riskier borrowers (adverse selection).

- In practice, firms must often finance investment from their own cash flows and/or provide collateral to lenders.

- When investment is constrained by cash flows (or the value of collateral) a potentially nasty feedback occurs.

- This mutually reinforcing feedback is called the "Financial Accelerator".

Financial Accelerator



- This circular feedback can potentially explain why investment is so volatile.

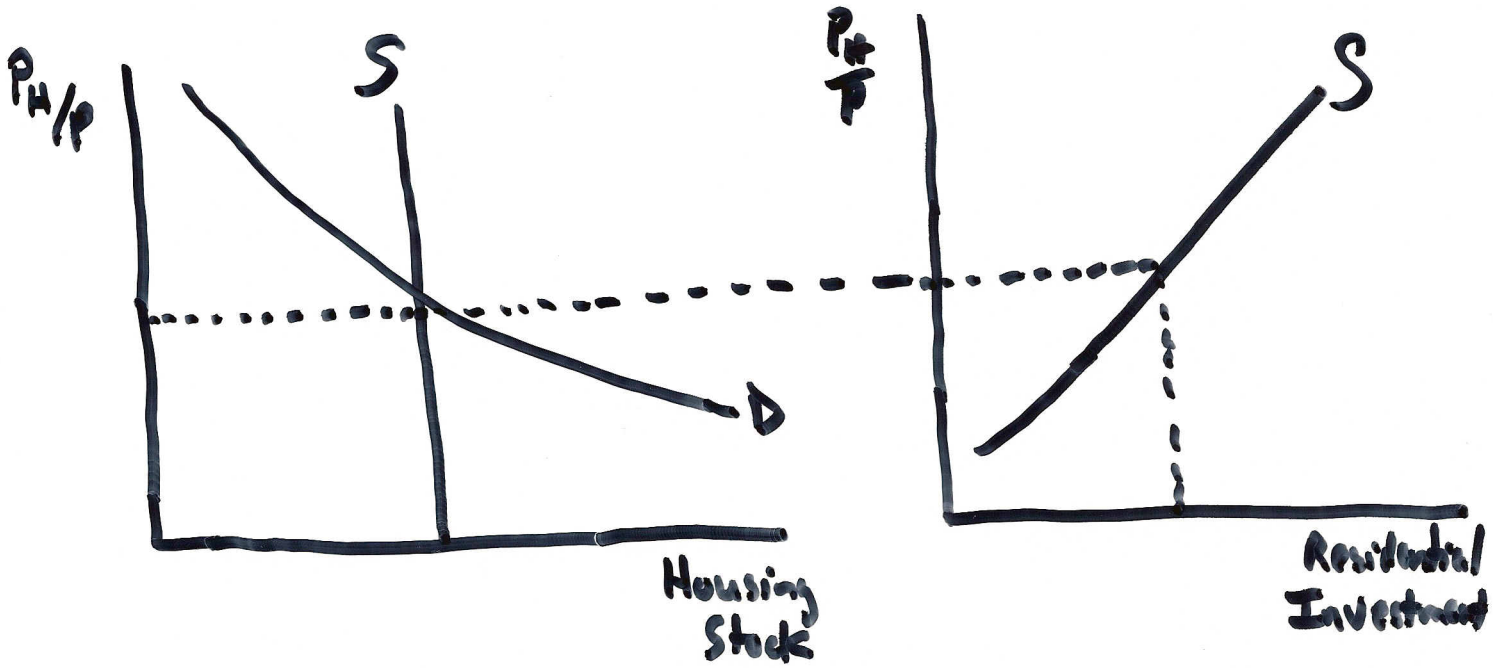
Residential Investment

- The housing market represents the interaction of 3 factors:

1.) At any given time, the supply of housing is fixed.

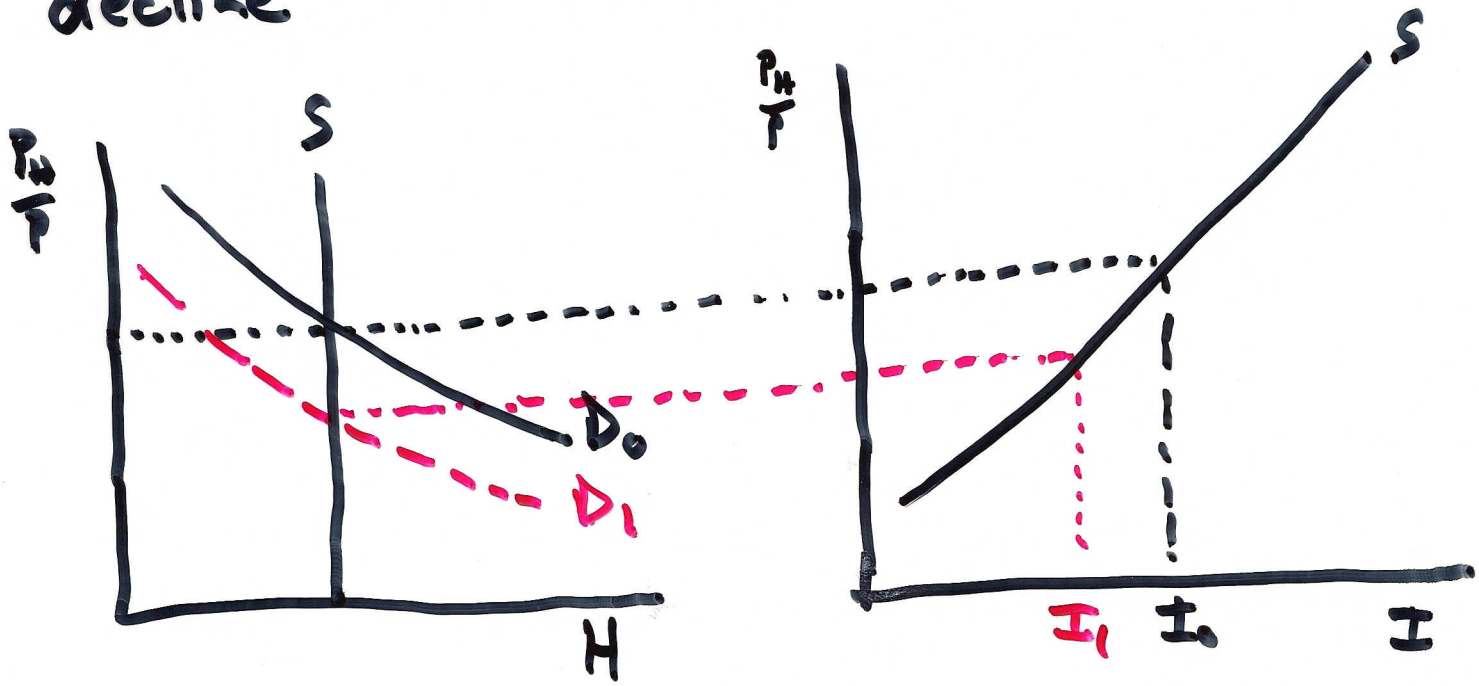
2.) The demand for this housing stock is a decreasing function of the relative price of housing, P_H/P.

3.) The supply of new housing is an increasing function of the relative price of housing.



Suppose P_H/P is expected to decline in the future.

Due to intertemporal substitution, this causes the demand for housing to decline



Since the supply is fixed, a drop in demand pushes prices down sharply.

This then reduces current residential investment.

Goods Market Equilibrium

From last time,

$$\text{consumption} = C(\overset{+}{Y}, \overset{-}{T}, \overset{+}{W}, \overset{?}{r})$$

$$\text{Investment} = I(\underset{-}{r}, \underset{-}{\delta}, \underset{+}{A})$$

Goods Mkt. Equil. : AS = AD

$$\boxed{\bar{Y} = C(\bar{Y}, T, W, r) + I(r, \delta, A) + G}$$

Given factor mkt. equil. $Y = \bar{Y}$

subtract $C + G$ from both sides,

$$\bar{Y} - C(\bar{Y}, T, W, r) - G = I(r, \delta, A)$$

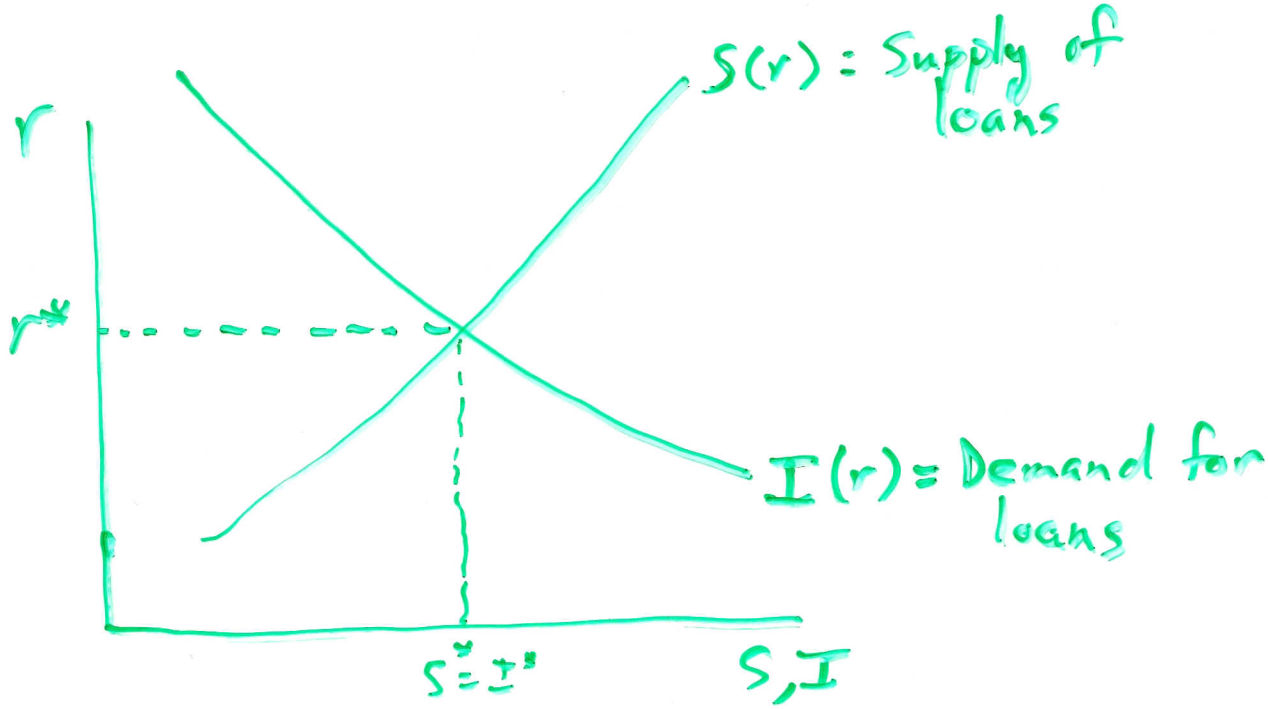
Note, National Saving = Priv. Saving + Govt. Saving

$$= (Y - T - C) + (T - G)$$

$$= Y - C - G$$

Equivalently,

$$S(\bar{Y}, T, W, \underset{+}{r}) = I(\underset{-}{r}, \delta, A)$$



Suppose, $r < r^*$

1.) Demand for loans > Supply of loans

2.) Interest Rate gets bid up

Alternatively
The Demand for Goods > Supply of Goods

A Numerical Example

Given,

$$Y = K^{1/2} L^{1/2}$$

$$\bar{K} = 62{,}500$$

$$\bar{L} = 400$$

$$G = 1{,}000$$

$$T = 1{,}000$$

$$C = 250 + .75(Y - T)$$

$$I = 1{,}000 - 50r$$

Find the equil. interest rate first using $AS = AD$, then using $S = I$

Step 1: Compute Equil. Output

$$Y^* = (12{,}500)^{1/2}(400)^{1/2}$$
$$= 5000$$

Step 2: Plug Y^* into Goods Mkt. Equil.

$$\bar{Y} = 5000 = C + I + G$$
$$= 250 + .75(5000 - 1000) + 1000 - 50r + 1000$$

$$5000 = 5250 - 50r$$
$$50r = 250 \Rightarrow \boxed{r^* = 5}$$

Now compute using $S = I$

$$S_p = Y - T - C$$

$$= 5000 - 1000 - [250 + .75(4000)]$$

$$= 750$$

$$S_g = T - G$$

$$= 1000 - 1000$$

$$= 0$$

$$S = S_p + S_g$$

$$= 750$$

$$750 = I = 1000 - 50r$$

$$50r = 250 \Rightarrow \boxed{r^* = 5}$$

Summary

1.) Given the production function and the Capital stock, find the wage rate where $L^S = L^D = L^*$

2.) Substitute L^* and $\bar{K}$ into the production function to get equilibrium output $Y^* = F(\bar{K}, L^*)$

3.) Substitute Y^* into the goods market equil. condition $AD = AS$ (or $S = I$) to get the market-clearing interest rate

That is,

1. Find equil. output, Y^*
2. Given Y^*, find the equil. interest.