Topics for Today

1.) Long-Run Growth
   - Basic Growth Facts
   - Sources of Economic Growth
     ("Growth Accounting")

2.) The Solow Growth Model
   - The Dynamics of Capital Accumulation
   - The "Golden Rule"
   - Population Growth
Small Differences in Growth Rates Make a Big Difference

<table>
<thead>
<tr>
<th></th>
<th>1870</th>
<th>1998</th>
<th>Annual Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>$3,645</td>
<td>$20,390</td>
<td>1.4%</td>
</tr>
<tr>
<td>Japan</td>
<td>737</td>
<td>20,084</td>
<td>2.6%</td>
</tr>
</tbody>
</table>

(per capita GDPs in 1990 U.S. $)

Understanding why growth differs across countries and over time is the fundamental question in all of economics!
<table>
<thead>
<tr>
<th>Country</th>
<th>Per Capita Income (2000 Intl. $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>$39,535</td>
</tr>
<tr>
<td>Canada</td>
<td>31,600</td>
</tr>
<tr>
<td>Japan</td>
<td>26,658</td>
</tr>
<tr>
<td>Mexico</td>
<td>8,883</td>
</tr>
<tr>
<td>Brazil</td>
<td>7,800</td>
</tr>
<tr>
<td>China</td>
<td>5,772</td>
</tr>
<tr>
<td>India</td>
<td>3,212</td>
</tr>
<tr>
<td>Sudan</td>
<td>1,254</td>
</tr>
<tr>
<td>Zambia</td>
<td>1,065</td>
</tr>
<tr>
<td>Niger</td>
<td>863</td>
</tr>
<tr>
<td>Congo (Dem. Rep.)</td>
<td>417</td>
</tr>
</tbody>
</table>
**Figure 1.6**
The Distribution of Growth Rates, 1960–2000

<table>
<thead>
<tr>
<th>Average annual growth rate</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0%–7.5%</td>
<td>Singapore</td>
</tr>
<tr>
<td>6.5%–7.0%</td>
<td>Taiwan</td>
</tr>
<tr>
<td>6.0%–6.5%</td>
<td>South Korea</td>
</tr>
<tr>
<td>5.5%–6.0%</td>
<td>Hong Kong</td>
</tr>
<tr>
<td>5.0%–5.5%</td>
<td>Botswana</td>
</tr>
<tr>
<td>4.5%–5.0%</td>
<td>Thailand</td>
</tr>
<tr>
<td>4.0%–4.5%</td>
<td>Japan, China, Ireland</td>
</tr>
<tr>
<td>3.5%–4.0%</td>
<td>Romania, Portugal, Malaysia</td>
</tr>
<tr>
<td>3.0%–3.5%</td>
<td>Norway, Greece, Spain</td>
</tr>
<tr>
<td>2.5%–3.0%</td>
<td>United States, France, India, Israel, Brazil, Italy</td>
</tr>
<tr>
<td>2.0%–2.5%</td>
<td>United Kingdom, Sweden, Canada, Turkey, Australia</td>
</tr>
<tr>
<td>1.5%–2.0%</td>
<td>Algeria, Nepal, Zimbabwe, Colombia, Mexico</td>
</tr>
<tr>
<td>1.0%–1.5%</td>
<td>Philippines, Argentina, Kenya, Ecuador, Jordan</td>
</tr>
<tr>
<td>0.5%–1.0%</td>
<td>Tanzania, Burkina Faso, Jamaica, Peru</td>
</tr>
<tr>
<td>0.0%–0.5%</td>
<td>Benin, Bolivia, Ethiopia, Cameroon</td>
</tr>
<tr>
<td>-0.5%–0.0%</td>
<td>Venezuela, Senegal, Rwanda, Burundi, Mali</td>
</tr>
<tr>
<td>-1.0%–0.5%</td>
<td>Nigeria, Chad, Madagascar, Zambia</td>
</tr>
<tr>
<td>-1.5%–1.0%</td>
<td>Nicaragua, Mozambique</td>
</tr>
<tr>
<td>-2.0%–1.5%</td>
<td>Niger</td>
</tr>
<tr>
<td>-2.5%–2.0%</td>
<td>Central African Republic</td>
</tr>
</tbody>
</table>

*Source: Heston, Summers, and Aten (2002).*
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>2.9</td>
<td>1.8</td>
<td>2.7</td>
</tr>
<tr>
<td>France</td>
<td>4.3</td>
<td>1.6</td>
<td>2.2</td>
</tr>
<tr>
<td>Germany</td>
<td>5.7</td>
<td>2.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Italy</td>
<td>4.9</td>
<td>2.3</td>
<td>4.7</td>
</tr>
<tr>
<td>Japan</td>
<td>8.2</td>
<td>2.6</td>
<td>1.1</td>
</tr>
<tr>
<td>U.K.</td>
<td>2.4</td>
<td>1.8</td>
<td>2.5</td>
</tr>
<tr>
<td>U.S.</td>
<td>2.2</td>
<td>1.5</td>
<td>2.9</td>
</tr>
</tbody>
</table>
Figure 1.7
GDP per Capita by Country Group, 1820–1998

Country group and population in 1998 (in millions)

- Western Europe: 388
- China: 1,243
- Eastern Europe: 121
- India: 975
- Former USSR: 291
- Other Asia: 1,172
- Western Offshoots: 323
- Africa: 760
- Latin America: 508
- World: 5,908
- Japan: 126


Figure 1.8
World Inequality and Its Components, 1820–1992

Inequality

* Bourguignon and Morrison (2002).
† Mathematical Note: The mean logarithmic deviation is defined as

\[
\frac{1}{n} \sum_{i=1}^{n} \ln \left( \frac{x_i}{\bar{x}} \right)
\]

where \( \bar{x} \) is the mean value of \( x \) and \( n \) is the number of observations.

Source: Bourguignon and Morrison (2002).
Figure 4. Average Annual Growth (1980–2000) on Initial Level of Real GDP per Capita.

Note: The data are values for real GDP in U.S. dollars per equivalent adult.
Source: Penn World Tables, version 6.1 (available online: http://pwt.econ.upenn.edu/).

Figure 5. Average Annual Growth (1980–2000) on Initial Level of Real GDP per Capita (as in Fig. 4, but with Area Proportional to Population in 1980)

Note: The data are values for real GDP in U.S. dollars per equivalent adult.
Source: Penn World Tables, version 6.1 (available online: http://pwt.econ.upenn.edu/).
FIGURE 2.3
Relationship Between Latitude and Income per Capita

GDP per capita, 2000 (ratio scale)

FIGURE 2.4
Relationship Between Income per Capita and Population Growth

Population growth rate, 1960–2000 (% per year)
FIGURE 3.1
GDP and Capital per Worker, 2000

Capital per worker (2000 dollars)

United States
Japan
Mexico
China
Nigeria
India

Source: Calculations based on Heston et al. (2002).
Empirically, countries with higher investment rates have higher capital to output ratios:

![Graph showing the relationship between investment rate and capital-output ratio for various countries.](image)

**FIGURE 5.3 Explaining Capital in the Solow Model**

**CHAPTER 5 The Solow Growth Model**
Income per person in 1992 (logarithmic scale)

Investment as percentage of output (average 1960–1992)
FIGURE 3.3
Capital's Share of Income in a Cross-Section of Countries

Capital's share of national income

Ecuador
Botswana
South Korea
Spain
Canada
United States

Average = 0.35

GDP per capita in 2000

Source: Bernanke and Gürkaynak (2002), table 10 and note 18.
Summary of Basic Growth Facts

1.) Historically, sustained growth in per capita income is a recent phenomenon. Before 1800, increases in productivity + income were offset by population growth (i.e., the so-called "Malthusian Trap").

2.) Since the Industrial Revolution, per capita output has grown over time, and its growth rate does not tend to diminish. If anything, growth has accelerated during the past 100 years.

3.) Growth rates and living standards differ substantially across countries.

4.) If anything, inequality across countries in per capita incomes has increased over time. However, if countries are weighted by population, there is some recent evidence of convergence.
5.) Per capita incomes are positively correlated with latitude, and negatively correlated with population growth.

6.) The shares of Labor & Capital in National Income are nearly constant over time, and nearly equal across countries.

7.) The Capital/Labor ratio grows over time.

8.) Per capita income is positively correlated with capital/labor ratio, both over time and across countries.

9.) The rate of return on capital is nearly constant.
Growth Accounting

Decompose growth into 3 basic sources:
1.) Increases in Capital
2.) Increases in Labor
3.) Advances in Technology

\[ Y = F(K, L) \]

\[ dY = MPK \cdot dK + MPL \cdot dL \]

\[ = MPK \cdot K \left( \frac{dK}{K} \right) + MPL \cdot L \left( \frac{dL}{L} \right) \]

\[ \Rightarrow \frac{dY}{Y} = \frac{MPK \cdot K}{Y} \cdot \frac{dK}{K} + \frac{MPL \cdot L}{Y} \cdot \frac{dL}{L} \]
If constant returns,

\[
\frac{dY}{Y} = \alpha \frac{dK}{K} + (1-\alpha) \frac{dL}{L}
\]

output growth = (capital's share) \times (capital growth) + (labor's share) \times (labor growth)

With Technology,

\[ Y = A \cdot F(K, L) \]

\[
\frac{dY}{Y} = \frac{dA}{A} + \alpha \frac{dK}{K} + (1-\alpha) \frac{dL}{L}
\]
Growth Accounting in Canada

1900 - 2000

\[
\frac{\Delta Y}{Y} \approx 3\%
\]

\[
\frac{\Delta L}{L} \approx 1\%
\]

\[
\frac{\Delta K}{K} \approx 3\%
\]

\[\alpha \approx \sqrt{3}\]

Contribution of Labor = \(\frac{2}{3} \times 1 = \frac{2}{3}\%\)

Contribution of Capital = \(\frac{1}{3} \times 3 = 1\%\)

Contribution of Tech = 3 - 1 - 0.67 = 1.33%

or in percentage of total

Labor 22%  Capital 33%  Tech. 45%
The Solow Growth Model

• So far, we've assumed the capital stock is given. This is not a bad assumption in the short to medium-run. However, over the long-run, saving & investment produce changes in the capital stock. How do these changes feedback to influence the economy? Can capital accumulation explain growth over time? These are the questions the Solow model addresses.

Sources of Growth in the Solow Model

1.) Saving & Investment (Capital Accumulation)

2.) Labor Force Growth (assumed exogenous)

3.) Technological Progress (assumed exogenous)
There are 2 Main Ingredients in the Solow Model

1.) The Production Function: \( Y = A \cdot F(K, L) \) 

2.) A Consumption/Saving Function: \( C = (1-s)Y \)
   \( s \) = (exogenous) marginal propensity to save

There are 3 Simplifying Assumptions

1.) Closed Economy (No int'l. capital flows)

2.) No Government

3.) Exogenous labor force/population
   (we will make labor supply endogenous when we discuss Real Business Cycle models later in the course)
The Dynamics of Capital Accumulation

\[ K_{t+1} - K_t = I_t - \delta K_t \]

In a closed economy, \( S_t = I_t \), and we've assumed \( S_t = sY_t \). Therefore,

\[ K_{t+1} - K_t = sY_t - \delta K_t \]

\[ = sA K_t^{\frac{\alpha}{\alpha + 1}} L_t^{\frac{1}{\alpha + 1}} - \delta K_t \quad \text{(Assuming Cobb-Douglas)} \]

In a steady-state, \( K_{t+1} = K_t \), or \( K_{t+1} - K_t = 0 \).

We can visualize this as follows.

\[ S, I \]

\[ \delta K_t \]

\[ sY_t \]

\[ \text{Steady-state capital stock} \]
• Note, the steady state capital stock represents a balancing between the forces of depreciation and investment.

• The steady state is stable because of diminishing returns. When K increases, saving + investment increase, but at a diminishing rate. In contrast, depreciation is proportional to K. Eventually, depreciation will dominate.

• Stability of the steady state means that over time the economy will converge to the steady state. If you start below K*, then the capital stock will rise. If you start above K*, the capital stock will fall. These movements toward the steady state are called transition dynamics.
Solving for the Steady State

Steady State Condition

\[ sY^* = \int K^* \quad \Rightarrow \quad \frac{K^*}{Y^*} = \frac{s}{\delta} \]

Sub-in from Production Function,

\[ sA K^{\frac{4}{3}} L^{\frac{2}{3}} = \int K^* \]

\[ \Rightarrow \quad K^* = \left( \frac{sA}{\delta} \right)^{\frac{3}{2}} L \]

Find \( Y^* \) using Production Function

\[ Y^* = A \cdot \left( \frac{sA}{\delta} \right)^{\frac{3}{2}} L^{\frac{2}{3}} L^{\frac{1}{3}} \]

\[ = A^{\frac{3}{2}} \left( \frac{s}{\delta} \right)^{\frac{1}{2}} L \]

Or, in per capita terms,

\[ y^* = \frac{Y^*}{L} \]

\[ y^* = A^{\frac{3}{2}} \left( \frac{s}{\delta} \right)^{\frac{1}{2}} \]

Note \( y^* \) responds more "strongly" to \( A \) than to \( s \).
Implications for Cross-Country Income Differences

Can this model explain observed differences in per capita income? (assuming everyone is in their steady state!)

Suppose we compare two countries, a rich country, and a poor country. Our previous formula implies,

$$\frac{Y_{\text{rich}}}{Y_{\text{poor}}} = \left( \frac{A_{\text{rich}}}{A_{\text{poor}}} \right)^{3/2} \left( \frac{S_{\text{rich}}}{S_{\text{poor}}} \right)^{1/2} \quad \text{Assuming equal depreciation rates}$$

In the data, $Y_r/Y_p \approx 45$ and $S_r = 0.30$ and $S_p = 0.05$, so $S_r/S_p = 6$

Hence, saving investment can only account for about $\sqrt{6} \approx 2.5$ of the 45 $A$ ratio.

The above formula implies that productivity in rich countries must be about 7 times greater than productivity in poor countries. (That is, $7^{3/2} \approx 18$. Hence, most of cross-country income differences in due to productivity differences!
Implications for Rate of Return Differentials

Another way to see this is to suppose (counter factually) that all income differences are due to different savings rates. If so, there would be massive incentives for capital to flow from rich countries to poor countries.

- Start with a per capita production function
  \[ Y = A K^{1/3} L^{2/3} \Rightarrow y = A K^{1/3} \text{ where } k = K/L \]

- Therefore, \( k = \left( \frac{y}{A} \right)^{3} \)

- \( R = \text{Rate of Return on Capital} \)
  \[ = MPK = \alpha A K^{a-1} = \frac{1}{3} A \left( \frac{y}{A} \right)^{2} \]
Consider the USA and India

\[ y^\text{US} \approx 12 \cdot y^\text{India} \]

Now, if \( A^\text{US} = A^\text{India} \), then the previous formula implies

\[
\frac{R^\text{India}}{R^\text{US}} = \left( \frac{y^\text{India}}{y^\text{US}} \right)^2 = \left( \frac{y^\text{US}}{y^\text{India}} \right)^2
\]

\[ = (12)^2 \]

\[ = 144! \]

If per capita income differences between USA and India are solely due to differences in saving + investment, then the rate of return in India should be 144 times that in the USA. This just isn't plausible. Something else (i.e., productivity differences) must be going on.
Response to an Increased Savings Rate

- Suppose the economy is at an initial steady state, and then the savings rate suddenly increases permanently.

Over time, the capital stock gradually rises to a new higher steady state.
Note: An increased saving rate causes the (per capita) income level to rise permanently, but it only leads to a temporary increase in the growth rate of income.