Topics for Today

1.) Population Growth in the Solow Model

2.) The "Golden Rule"

3.) Some empirical evidence

4.) Convergence?
- Conditional vs. Unconditional Convergence

5.) Technological Progress in the Solow Model

6.) Endogenous Growth Theories
- Human Capital (Lucas)
- AK Production Functions
- Externalities (Romer)
- R&D (Romer)

Population Growth

- When the population (and labor force) is growing, it is convenient to express everything in per capita terms.

Per Capita Production Function

$$Y = A \cdot K^{\alpha} L^{1-\alpha}$$

$$Y/L = A K^{\alpha} L^{-\alpha}$$

$$\boxed{y = A k^{\alpha}} \qquad y = Y/L \qquad k = K/L$$

Per Capita Capital Accumulation

$$K_{t+1} - K_t = I_t - \delta K_t$$

$$\frac{K_{t+1}}{L_{t+1}} \cdot \frac{L_{t+1}}{L_t} - \frac{K_t}{L_t} = \frac{I_t}{L_t} - \delta \frac{K_t}{L_t}$$

$$\boxed{L_{t+1} = (1+n) L_t}$$

$$k_{t+1}(1+n) - k_t = i_t - \delta k_t$$

$$k_{t+1} - \frac{1}{1+n} k_t = \frac{1}{1+n}\left[i_t - \delta k_t \right]$$

For small n, $\frac{1}{1+n} \approx 1 - n$

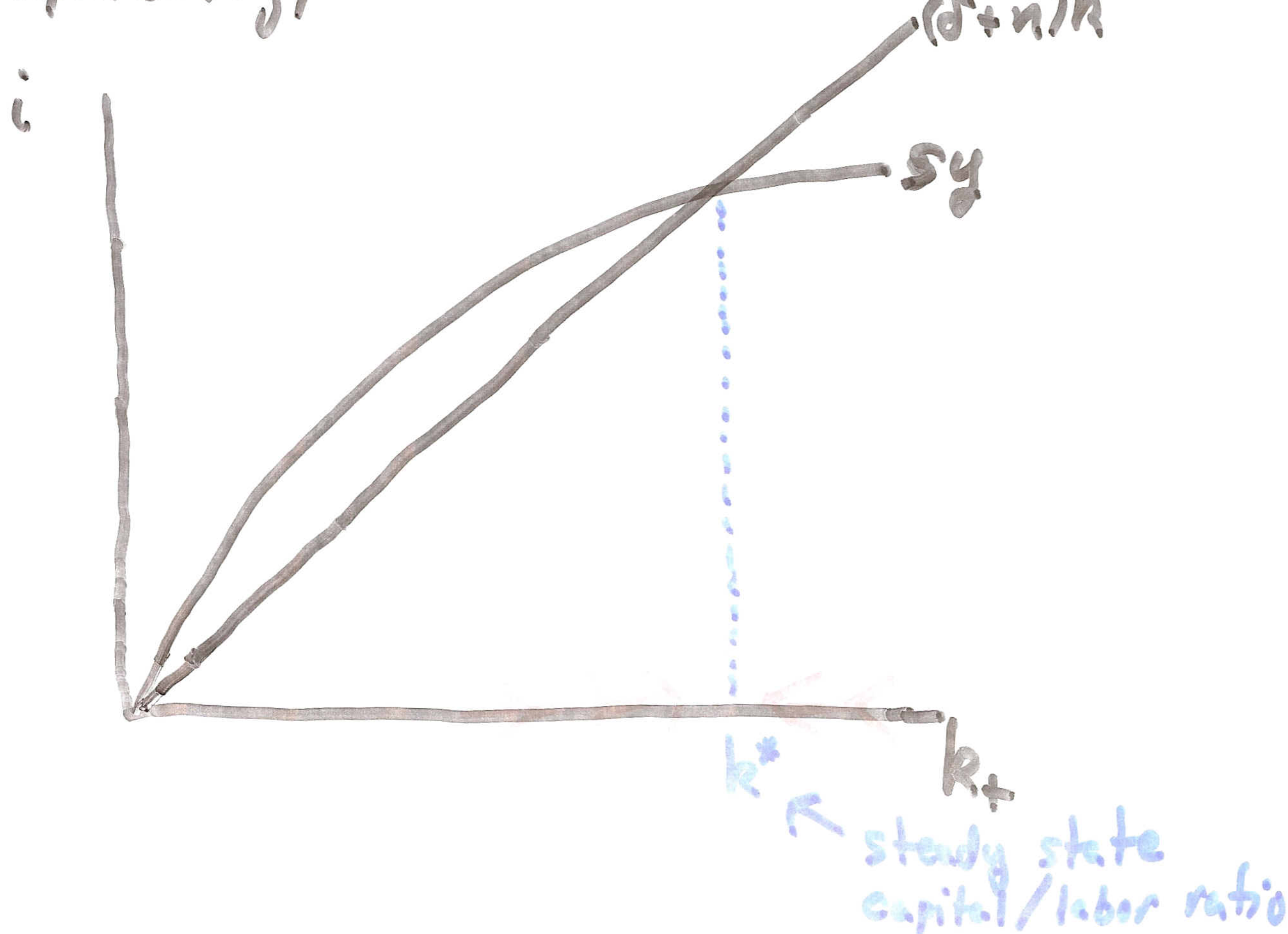
$$k_{t+1} - k_t = i_t - (\delta + n) k_t - \underbrace{n[i - \delta k_t]}_{\text{"2nd Order" Term can be ignored.}}$$

- Hence, in per capita terms, the model becomes

$$\boxed{\begin{aligned}\Delta k_{t+1} &= i_t - (\delta+n)k_t \\ &= sy_t - (\delta+n)k_t\end{aligned}}$$

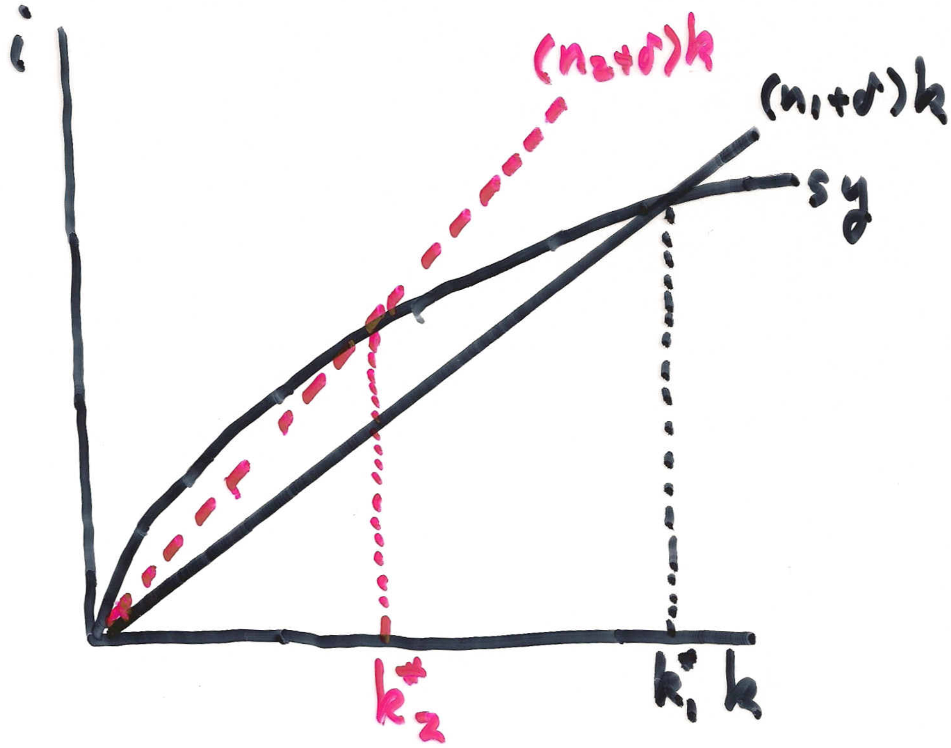
$\Delta k_{t+1} = k_{t+1} - k_t$

Graphically,



With population growth, the capital/labor ratio declines for two reasons: (1) The numerator declines because of depreciation, (2) The denominator rises because of population growth. To maintain the capital/labor ratio, investment must be sufficient to offset both.

What happens when $n \uparrow$?

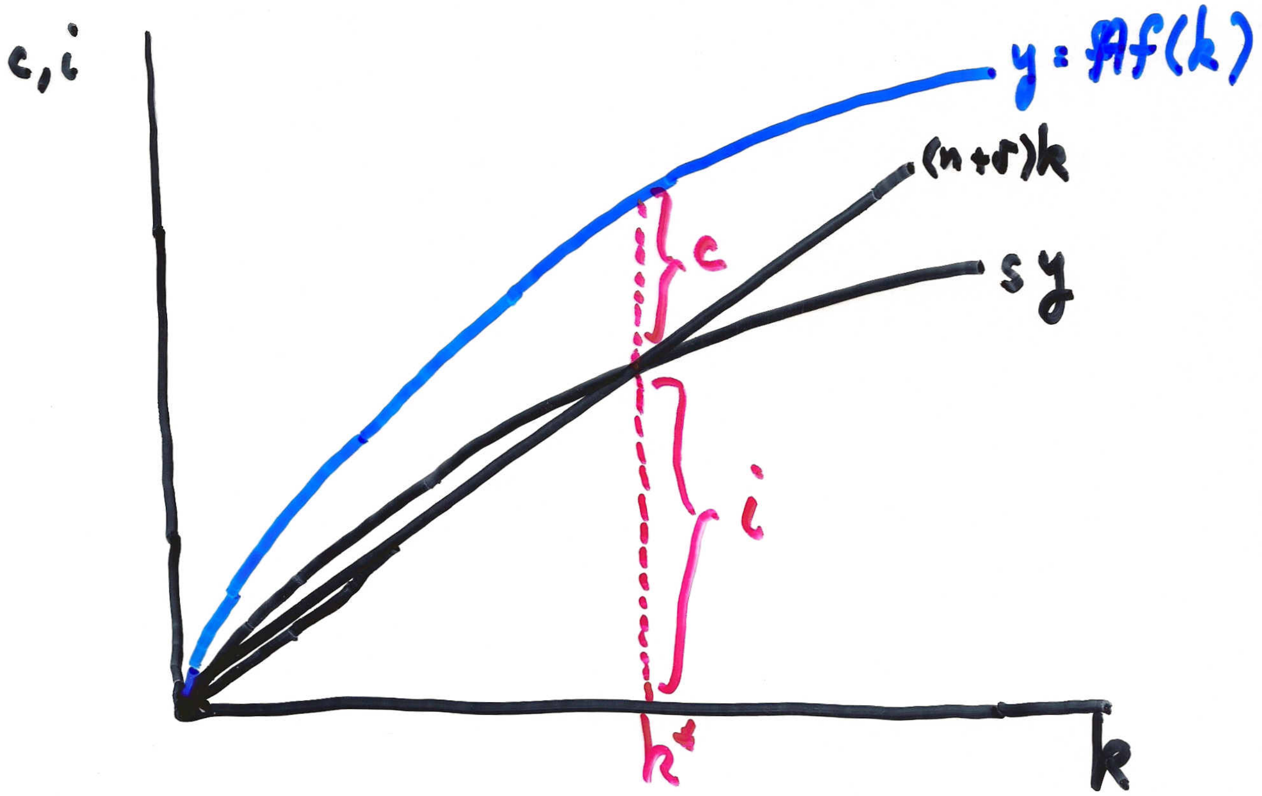


Conclusion: Higher population growth lowers the steady state capital/labor ratio, and therefore, lowers steady state per capita income. This prediction is consistent with the data.

What happens to the growth in total income?

Consumption

- So far, we've focused on saving + investment. What about consumption? We can infer what happens to consumption by just adding the (per capita) production function to our previous graph.



- Can we say anything about welfare in the Solow model? Is the resulting steady state "good" or "bad"?
- The answer depends on the savings rate. It turns out that it is possible to save too much!

The "Golden Rule"

- Given a savings rate, the economy converges to a steady state (per capita) income level.
- A natural question is: What savings rate **maximizes** steady state consumption?
- Let k^* be the steady state capital stock. Then steady state consumption is just,

$$\boxed{c^* = f(k^*) - (n+\delta)k^*}$$

} steady State consumption

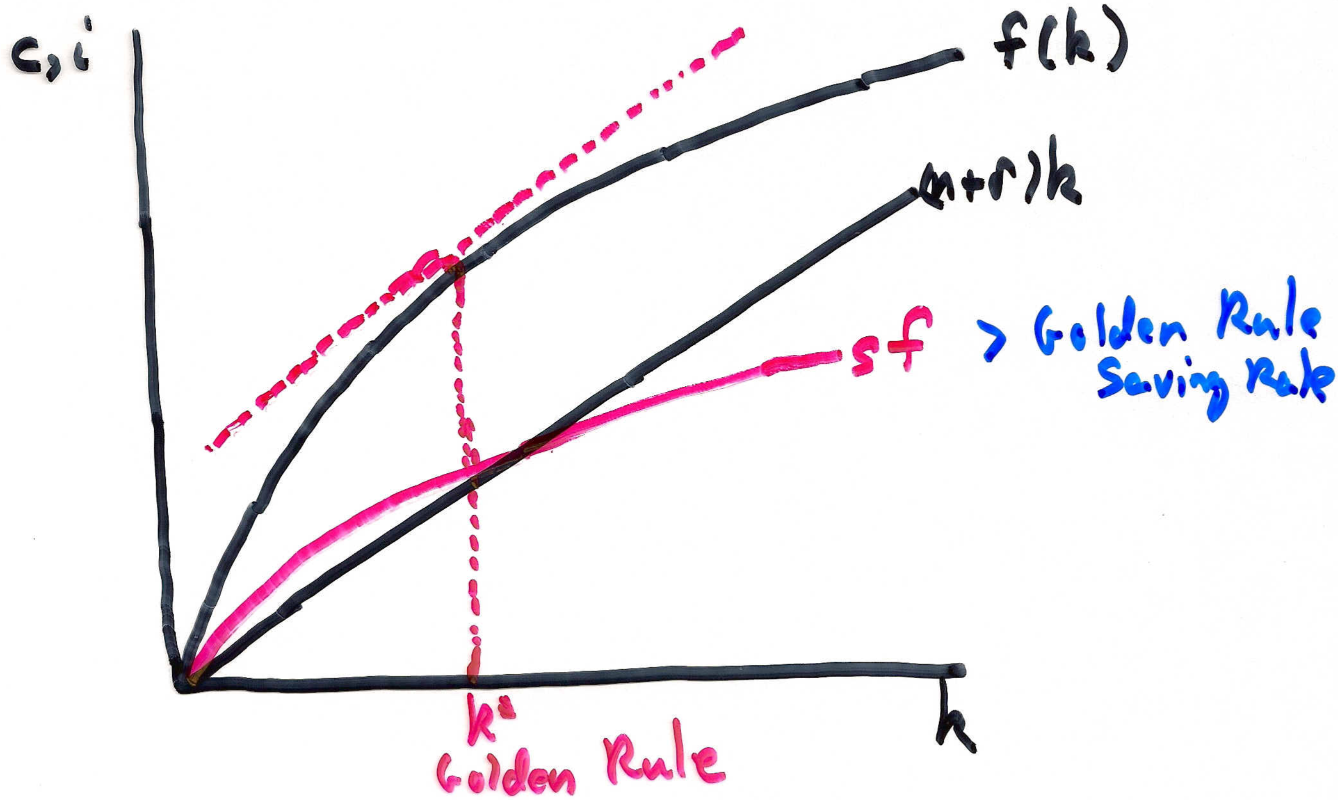
- Let's maximize this with respect to k^* (For you calculus fans, take the derivative and set it to zero!).

$$\boxed{f'(k^*) = n+\delta}$$

} Equation characterizing Golden Rule capital Stock

- Geometrically, this makes sense. From the previous graph, consumption is the vertical distance between the $f(k)$ line and the $(n+\delta)k$ line. This distance is maximized when they have the same slope!

Geometrically,



Mathematically,

$$f'(k^*) = n+\delta$$

$$\alpha A k^{*\alpha-1} = n+\delta$$

$$\boxed{k^* = \left(\frac{\alpha A}{n+\delta}\right)^{\frac{1}{1-\alpha}}}$$

- **Question**: What saving rate brings you to the Golden Rule?

- To find the answer, just go back to our earlier expression for the steady state capital/labor ratio,

$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

- Notice that when $s = \alpha$, we get the Golden Rule!

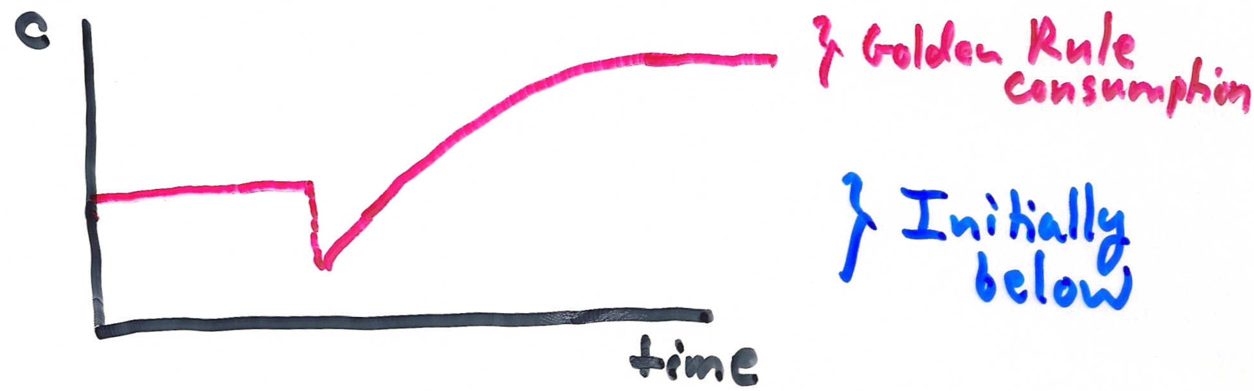
- How can we tell whether an economy is at the Golden Rule?

1.) Compare observed s with α, which is given by capital's share (about 33% in the data). Hence, if $s < .33$ we are below the Golden Rule.

2.) Notice that the Golden Rule condition can be written: $MPK = n + \delta$. In competitive factor markets, $MPK = r + \delta$. So another way of expressing the Golden Rule is $r = n$, i.e., the interest rate equals the growth rate. If [redacted] $r > n$ then you are below the Golden Rule.

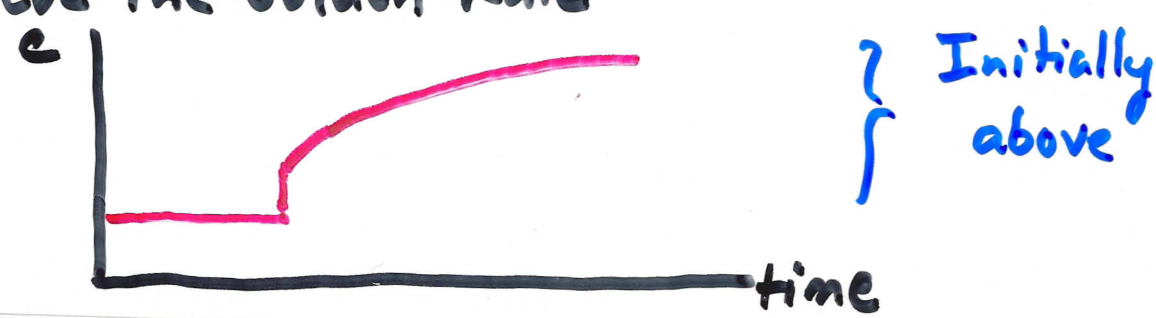
- Both conditions, $s < \alpha$ and $r > n$, describe the U.S. + Canada (and most other countries).

- Should the U.S. + Canada raise s to move to the Golden Rule? It's not so clear. It involves an intergenerational trade-off, since current consumption must be sacrificed in order to raise future consumption.



- Clearly, if all generations weighted equally, then you should do it, since there are <u>permanent</u> benefits.

- However, if you are above the Golden Rule then you should always cut the savings rate to achieve the Golden Rule



Some Empirical Evidence

- Last time I presented an example suggesting that differences in saving + investment rates aren't likely to explain observed differences in the per capita incomes of the very richest and very poorest countries.

- However, that's a very demanding test! Less ambitiously, how does the Solow model perform overall (i.e., on average)?

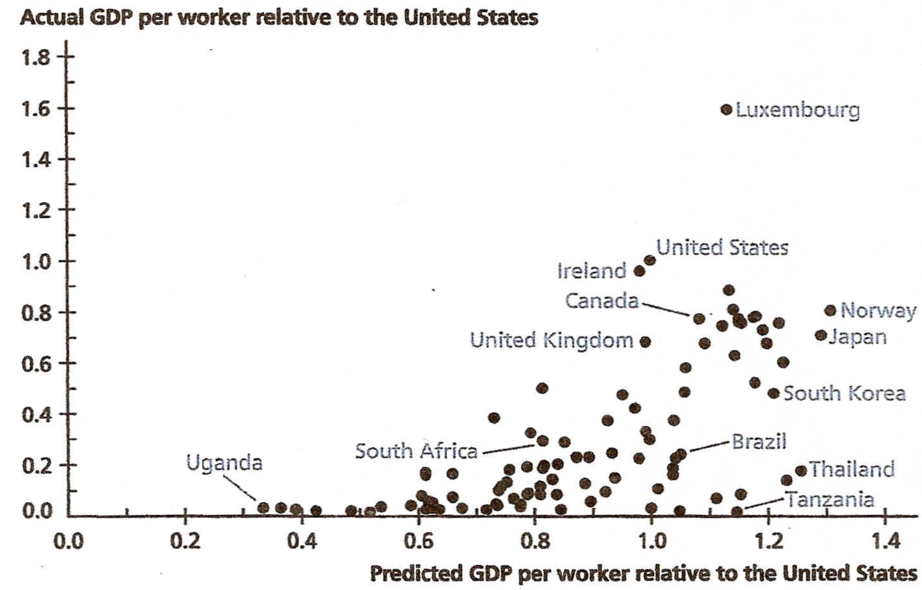
- How much of the variation in per capita incomes can be explained by variation in saving + investment rates?

- Suppose the only reason countries differ is in their savings rates. Also, suppose $\alpha = 1/3$, which is reasonably consistent with the data. Then,

$$\boxed{\frac{y_i}{y_j} = \left(\frac{s_i}{s_j}\right)^{1/2}}$$

<u>Example</u>: Suppose country 1 saves 20% and country 2 saves 5%. Then our model predicts that country 1's income will be twice as high as country 2's $\left(\frac{.20}{.05}\right)^{1/2} = 2$

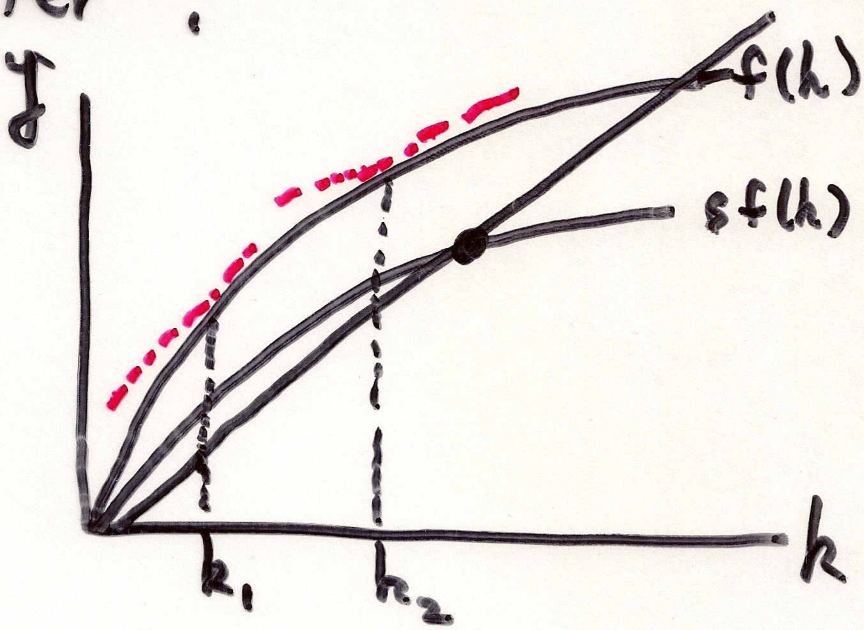
FIGURE 3.7
Predicted Versus Actual GDP per Worker

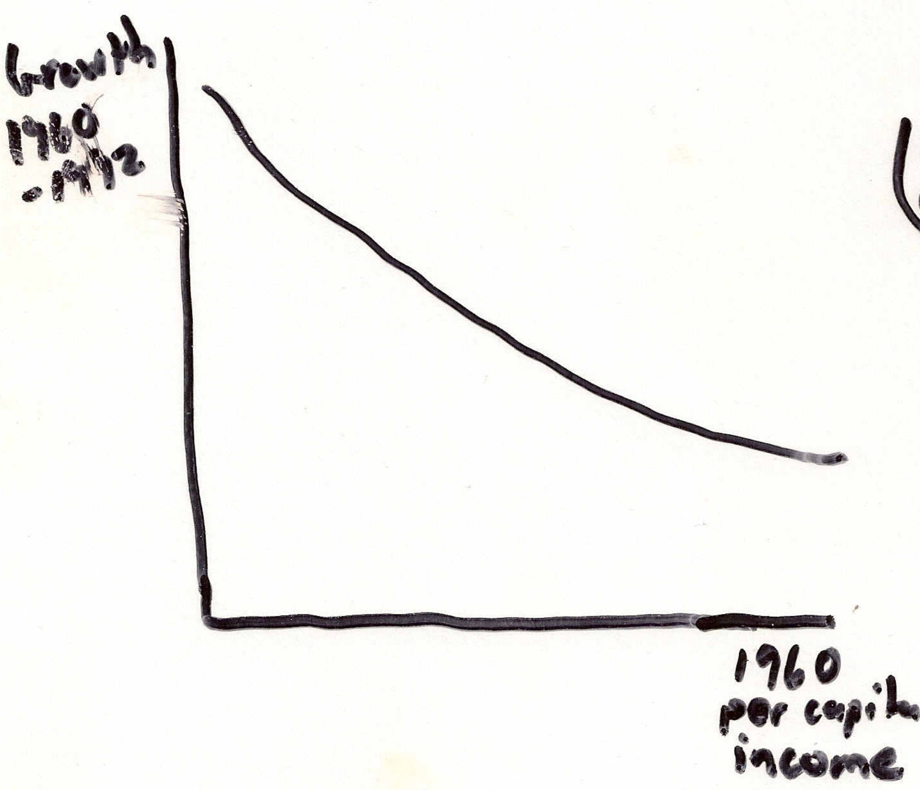


Source: Calculations based on Heston et al. (2002).

Convergence

Will poor countries catch-up?
That is, do poor countries grow faster?





$$\left(\begin{matrix}\text{Avg.}\\ \text{Growth}\end{matrix}\right)_i = \alpha + \beta\, Y_i^{1960}$$

convergence $\Rightarrow \beta < 0$

Convergence

FIGURE 3.3 PER CAPITA GDP, 1870–1994

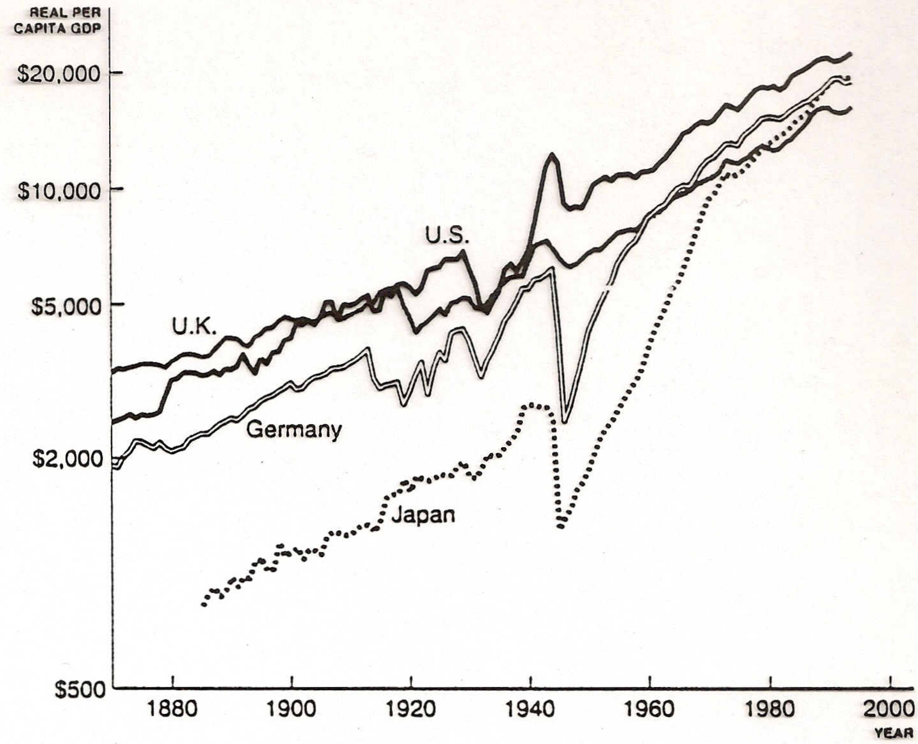
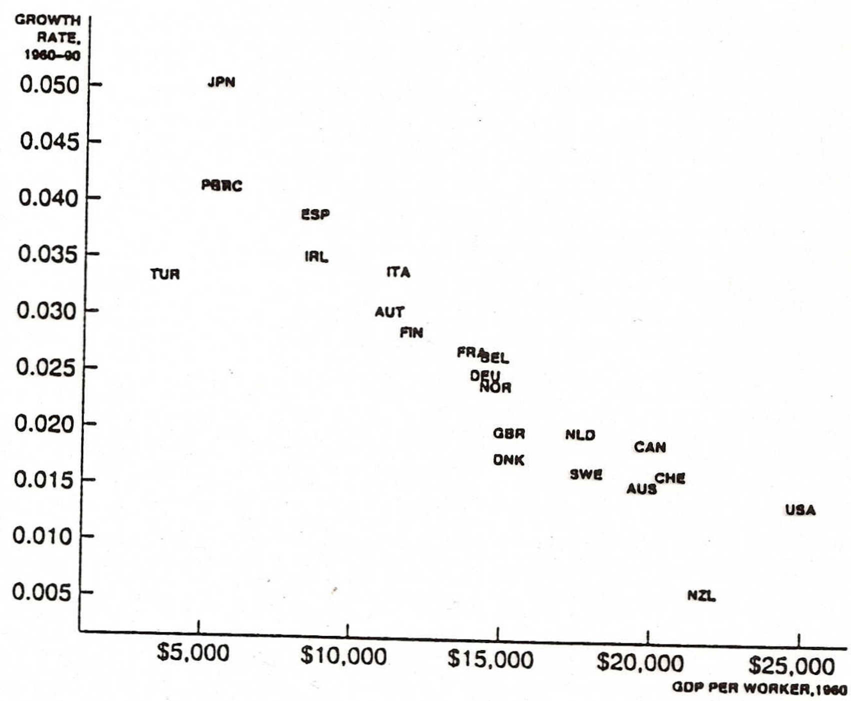


FIGURE 3.5 CONVERGENCE IN THE OECD, 1960–90



Conditional vs. Unconditional Convergence

FIGURE 3.6 THE LACK OF CONVERGENCE FOR THE WORLD, 1960–90

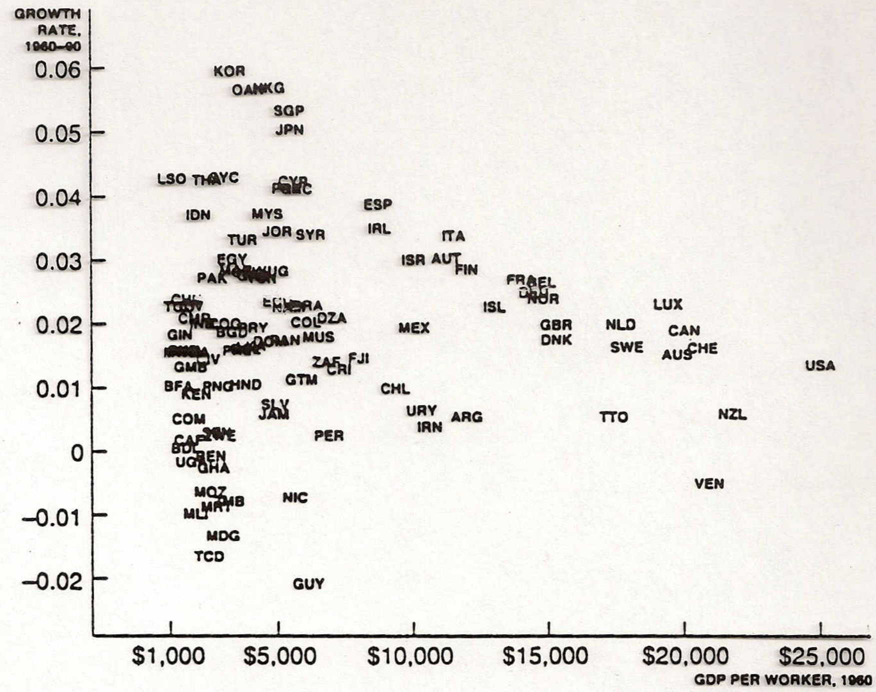
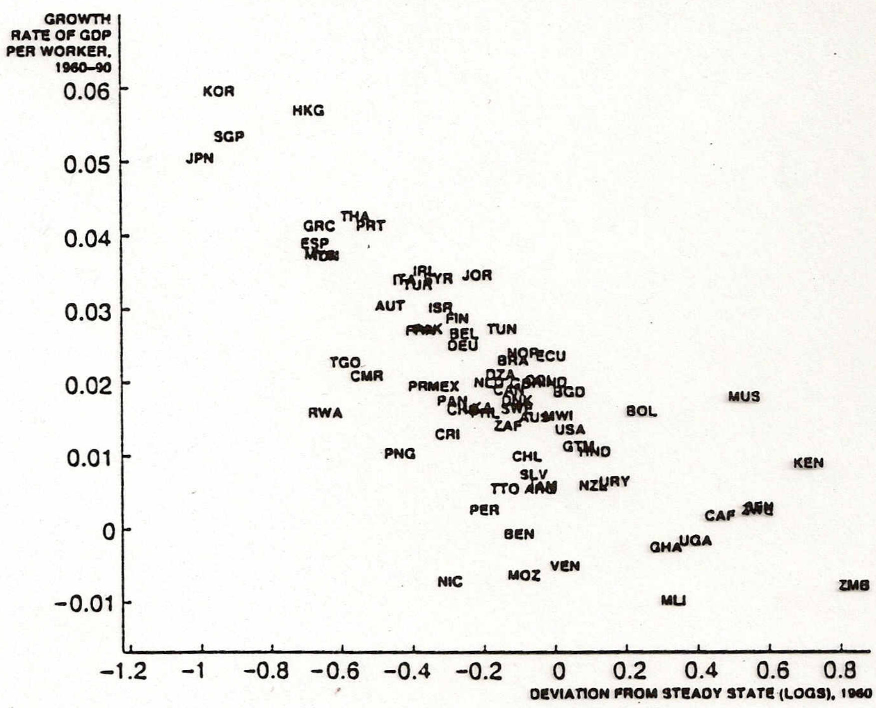


FIGURE 3.8 "CONDITIONAL" CONVERGENCE FOR THE WORLD, 1960–90



Note: The U.S. deviation (in logs) from steady state in 1960 is normalized to zero. Estimates of A for 1970 instead of 1990 are used to compute the steady state.

Conditional Convergence

$$\left(\begin{matrix}\text{Avg.}\\ \text{Growth}\end{matrix}\right)_i = \alpha + \gamma_1 s_i + \gamma_2 n_i + \beta Y_i^{1960}$$

For given saving and population growth rates, (i.e., <u>conditional</u> on saving + population growth) do poor countries grow faster?

Growth in the Solow Model

- Ironically, as a theory of <u>sustained</u> growth in <u>per capita</u> income, the Solow Model is a <u>failure</u>. The model can explain growth in <u>total</u> income (due to population growth), and <u>temporary</u> changes in growth (due to transition dynamics), but due to diminishing returns, the model predicts the economy inexorably converges to a long-run steady-state with <u>constant</u> per capita income.

- It turns out, however, that the model can be used to at least <u>describe</u> the process of sustained growth in per capita income. All we need to do is let productivity (our A parameter) grow over time.

Technological Progress

$Y = A \cdot F(K, L)$ A: Total Factor Productivity

$Y = F(K, E \cdot L)$ E: Labor Augmenting Tech. Progress

For Cobb-Douglas, they are proportional (in logs)

$$\left.\begin{aligned} Y &= A \cdot K^{\alpha} L^{1-\alpha} \\ Y &= K^{\alpha} (EL)^{1-\alpha} \end{aligned}\right\} \Rightarrow A = E^{1-\alpha}$$

$$\Rightarrow \frac{\Delta A}{A} = (1-\alpha) \frac{\Delta E}{E}$$

Now write, $y = \dfrac{Y}{EL}$ } output per "effective" worker

$k = \dfrac{K}{EL}$ } capital per "effective" worker

Then $y = f(k)$ as before

Assume $\dfrac{\Delta E}{E} = g$ } Growth Rate of Tech. Progress

Tech. Progress in the Solow Model

$$Y = F(K, E \cdot L)$$

E: Labor-Augmenting Tech. Progress

Define $y = \frac{Y}{EL}$ $k = \frac{K}{EL}$

y: output per effective worker

k: capital per effective worker

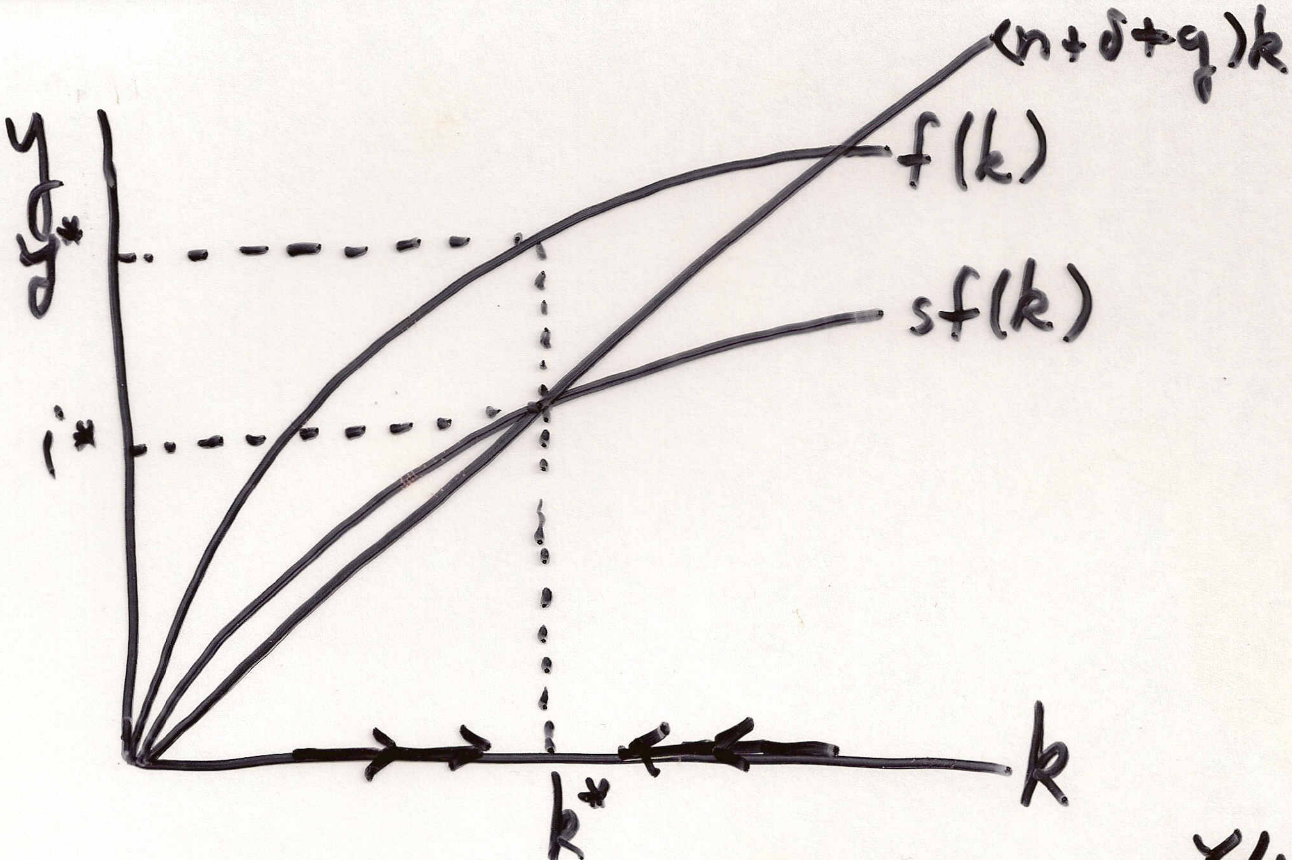
Then, $y = f(k)$ as before

Steady-State Condition,

$$\boxed{\Delta k = \underbrace{sf(k)}_{\text{investment}} - (n+\delta+g)k}$$

where $g = \frac{\Delta E}{E}$

= Rate of growth in Labor-Augmenting Tech. Progress



$$y = \frac{Y/L}{E}$$

Since $y = \frac{Y}{EL}$ converges to a steady state, and E grows at rate g, Output per capita must grow at rate g also.

Note: g is <u>exogenous</u>.

Endogenous Growth

- Unfortunately, simply assuming that A grows exogenously at some fixed rate g is not very satisfying. The real questions are: (1) **Why** does A grow over time, (2) Can govt. policy do something to **change** g?

- Recently, economists have developed models of **endogenous** growth, in which changes in A are **explained** by the model. There are several different versions, but they all share the common feature that, in one way or another, they overcome the forces of diminishing returns.

- We will discuss 4 basic versions:
  1.) Human Capital
  2.) "AK" production functions
  3.) Externalities
  4.) R & D

Human Capital

- So far, we've assumed the "quality" of capital and labor inputs is the same across countries (and the same over time within a country).

- This is probably a decent assumption for physical capital (although, of course, it's probably true that rich countries have better tractors than poor countries!)

- However, for labor it is highly dubious. For one thing, education levels differ significantly across countries (and over time within countries).

- There is abundant empirical evidence that education is productive. In particular, people with more education earn higher wages (on average).

- We can use this data to "correct" for differences across countries in "effective" labor supply.

Now we have

$$Y = AK^{\alpha}(hL)^{1-\alpha}$$

where

L = raw labor (e.g., # of workers)

h = effective labor input per worker

The idea is to construct measures of h for each country, using schooling data, and the rate of return to schooling

Before we had

$$y = A^{\frac{1}{1-\alpha}}\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Now we have

$$\boxed{\begin{aligned} y &= (h^{1-\alpha}A)^{\frac{1}{1-\alpha}}\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} \\ &= h \cdot A^{\frac{1}{1-\alpha}}\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} \end{aligned}}$$

Per capita income differences proportional to "human capital" differences

Table 6.1
Changes in the Level of Education, 1960–2000

		Average Years of Schooling	Percentage of the Adult Population with: No Schooling	Complete Primary Education	Complete Secondary Education	Complete Higher Education
Developing Countries	1960	2.05	64.1	17.1	2.5	0.4
	2000	5.13	34.4	43.0	14.8	3.0
Advanced Countries	1960	7.06	6.1	72.9	20.2	3.0
	2000	9.76	3.7	84.6	44.7	13.0
United States	1960	8.49	2.0	78.4	31.0	7.0
	2000	12.05	0.8	94.9	68.1	24.5

Source: Barro and Lee (2000). Data are for population aged 15 and over.

Figure 6.6
Effect of Education on Wages

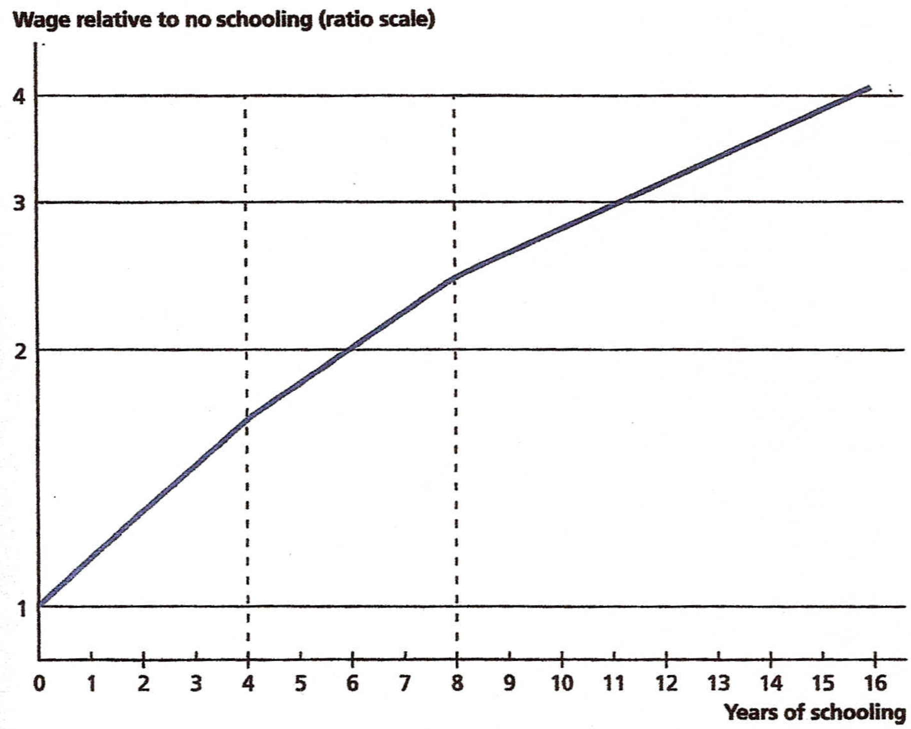
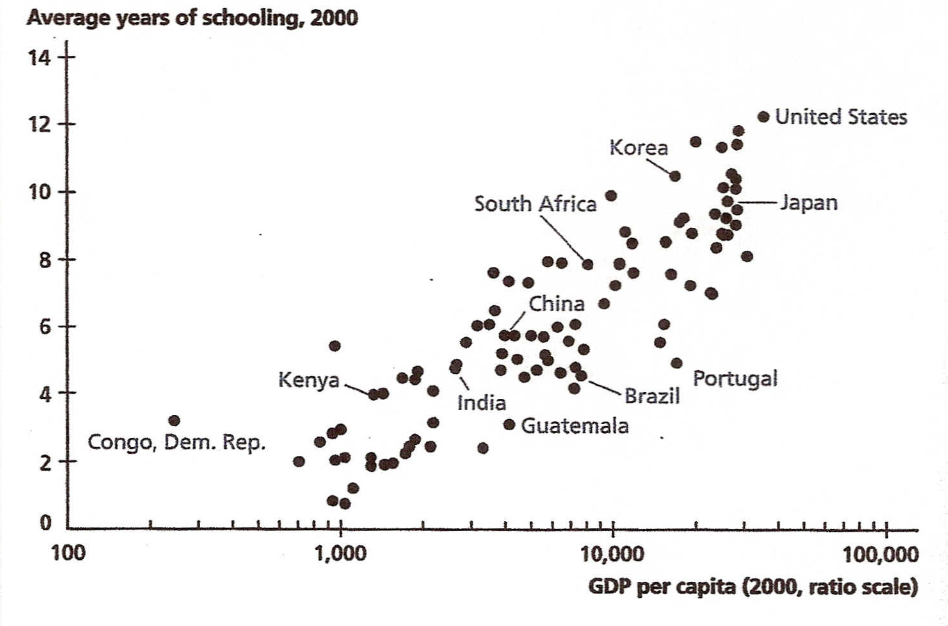
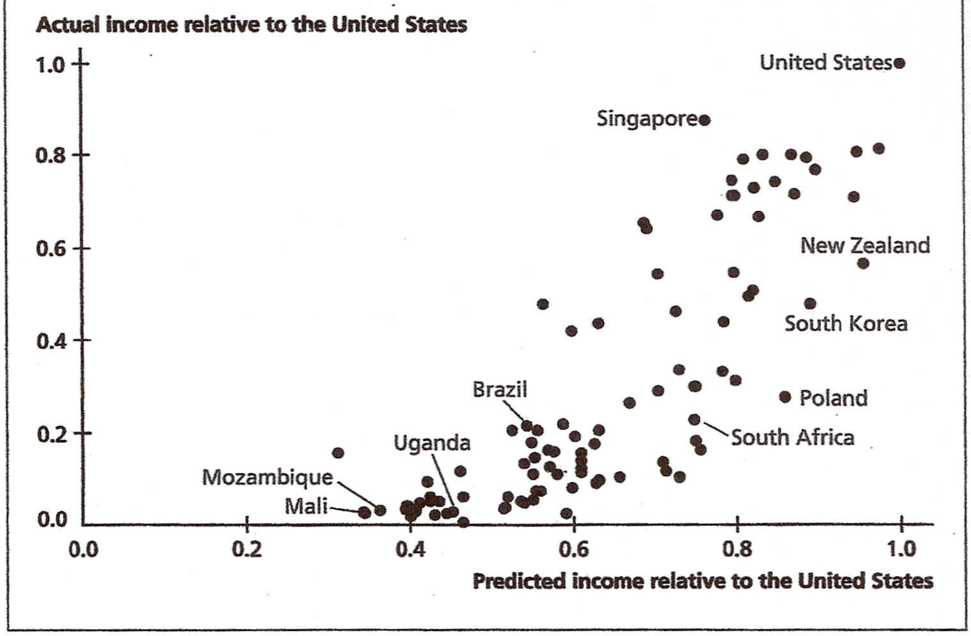


FIGURE 6.11
Average Years of Schooling Versus GDP per Capita



Source: Heston et al. (2002), Barro and Lee (2000).

FIGURE 6.12
Predicted Versus Actual Income Relative to the United States



The Lucas Human Capital Model

Suppose,

$$Y = K^{\alpha}\left(E \cdot (1-u)L\right)^{1-\alpha}$$ } Manufacturing Prod. Function

$$\Delta E = u \cdot E$$ } Human Capital Prod. Function

where,

E : human Capital per person

u : fraction of time spent at school

$1-u$: fraction of time spent working

Key Assumption: Accumulation of human capital is not subject to diminishing returns.

As before, define

$$y = \frac{Y}{E(1-u)L} \quad \Big\} \text{ output per effective worker}$$

$$k = \frac{K}{E(1-u)L} \quad \Big\} \text{ capital per effective worker}$$

$$\frac{Y}{E(1-u)L} = \left(\frac{K}{E(1-u)L}\right)^{\alpha} \Rightarrow \boxed{y = k^{\alpha}}$$

<u>Steady-State Condition</u>:

$$sy = (n + \delta + u)k$$

Note, per capita output grows at rate u, which is now a choice variable, i.e., growth is <u>endogenous</u>!

The Constant Returns "AK" Model

Suppose,

$$Y = AK$$ } Labor is like Capital!

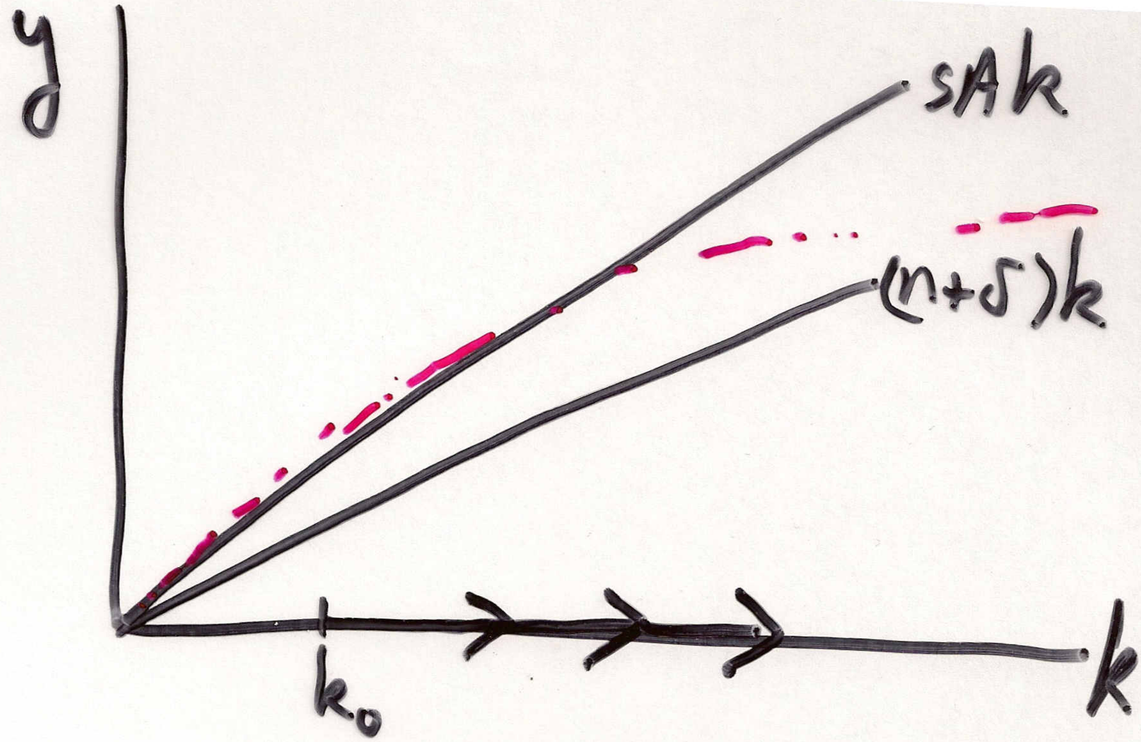
$$\Rightarrow y = Ak$$ } Assume A is constant

$$\Rightarrow \frac{\Delta y}{y} = \frac{\Delta k}{k}$$

$$\Delta k = sy - (n+\delta)k$$

$$= sAk - (n+\delta)k$$

$$\Rightarrow \boxed{\frac{\Delta k}{k} = sA - (n+\delta) = \frac{\Delta y}{y}}$$



As long as $sA > n+\delta$ there is perpetual growth!

1.) Now $s\uparrow \implies$ Growth rate $\uparrow$ permanently

2.) Also note, economies with similar characteristics have same growth rates, but there is no tendency toward conditional convergence. Initial differences persist.