Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (10 points each).

1. According to the Permanent Income Hypothesis, an increase in the saving rate should predict recessions.
   TRUE. If people expect their incomes to be lower in the future, they will start saving now, so that they don’t have to reduce their future consumption by as much as the decline in their future income. That is, they will smooth their consumption over time. See Lecture 9 slides for a picture.

2. An increase in government spending raises the real wage rate.
   FALSE. An increase in government spending is accompanied by an increase in taxes, which lowers disposable income. This produces a negative income effect, which causes both consumption and leisure to decline. That is, labor supply increases. The outward shift of the labor supply curve drives down wages. (Firms must be induced to hire more labor, so wages get bid down in response to the increased supply of labor).

3. In the Solow model, economies with faster population growth have lower per capita incomes.
   TRUE. Faster population growth rotates the breakeven investment line upwards, so that it intersects the savings function at a lower steady state capital/labor ratio, which means steady state per capita income will be lower when population growth is faster.

4. According to Ricardian Equivalence, government spending does not affect saving.
   FALSE. Ricardian Equivalence is concerned with the financing of a given path of government expenditures (e.g., whether it’s financed by taxes now, or by bonds (taxes later)). Temporary, or anticipated, government spending policies will generally cause private saving to respond, as households attempt to smooth out their effects over time.

5. Labor supply increases when the real wage increases.
   UNCERTAIN. It depends on the relative strength of income and substitution effects. A higher wage makes leisure more expensive (substitution effect). This would tend to increase labor supply. However, it also produces a positive income effect (assuming that the household was initially working, or close to working), and this tends to increase leisure and decrease labor supply. On net, labor supply increases only if the substitution effect dominates.

The following questions are short answer. Briefly explain your answer. Clarity will be rewarded.
6. (25 points). Consider a static one-period economy as discussed in Chapter 4. Output is produced using the Cobb-Douglas production function: \( Y = zK^{1/2}N^{1/2} \). The representative household’s preferences are given by

\[
U(C, \ell) = \beta C + \sqrt{\ell}
\]

where \( \beta > 0 \) is a fixed parameter. As discussed in class, the household’s time constraint is \( h = \ell + N \). For simplicity, suppose there is no government in this economy.

(a) Compute the competitive equilibrium wage rate for this economy.

The household’s optimality condition is:

\[
\frac{U_\ell}{U_c} = w
\]

Taking the derivatives we can write this as:

\[
\frac{1/2}{\beta \sqrt{\ell}} = w
\]

The firm’s optimality condition is:

\[(1/2)zK^{1/2}N^{-1/2} = w\]

Equating these two optimality conditions to eliminate \( w \) gives:

\[
\frac{1/2}{\beta \sqrt{\ell}} = (1/2)zK^{1/2}N^{-1/2}
\]

Now use the household’s time constraint, \( h = N + \ell \), to substitute out \( \ell \) and solve for \( N \) in using the previous equation:

\[
\sqrt{\frac{N}{h - N}} = \beta z K^{1/2}
\]

Solving for \( N \) then gives:

\[
N = h \left( \frac{\beta^2 z^2 K}{1 + \beta^2 z^2 K} \right)
\]

Finally, we can plug this into either optimality condition to get the market-clearing, competitive wage rate:

\[
w = (1/2)\sqrt{\frac{1 + \beta^2 z^2 K}{h \beta^2}}
\]

(b) What happens to the wage rate when \( \beta \) increases? Interpret your result using a Labor Supply/Labor Demand graph.

From the previous expression for the wage rate, it’s obvious that \( \beta \uparrow \Rightarrow w \downarrow \). Although not necessary, you can verify this by taking the derivative of \( w \) with respect to \( \beta \):

\[
\frac{dw}{d\beta} = -\frac{1}{2} \left( \frac{1 + z^2 \beta^2 K}{h \beta^2} \right)^{-1/2} \left( \frac{1}{h \beta^2} \right) < 0
\]

Intuitively, when \( \beta \) increases the household places more weight on consumption compared to leisure. This causes the labor supply curve to shift out. This then causes the wage rate to fall, and the equilibrium quantity of employment to increase.
7. (25 points). Consider an economy with the following characteristics: It produces output using the Cobb-Douglas production function \( Y = zK^\theta N^{1-\theta} \). It saves a constant fraction, \( s \), of its income each period. The labor force grows at rate \( n \), and the capital stock depreciates at rate \( \delta \).

(a) Derive an expression for the steady state capital/labor ratio and steady state per capita income. Illustrate your answer with a graph. What are the growth rates of total and per capita income in the steady state? Explain.

Expressed in per capita terms, the steady state condition is:

\[ sy = (n + \delta)k \]

Using the fact that \( y = zk^\theta \), we can write this as:

\[ szk^\theta = (n + \delta)k \]

Solving for \( k \) gives

\[ k^* = \left( \frac{zs}{n + \delta} \right)^{\frac{1}{1-\theta}} \]

Finally, substituting this back into the (per capita) production function gives us the following expression for steady state per capita income

\[ y^* = z \left( \frac{zs}{n + \delta} \right)^{\frac{\theta}{1-\theta}} = z \left( \frac{s}{n + \delta} \right)^{\frac{\theta}{1-\theta}} \]

In the steady state, growth in per capita income will be zero, and so the growth rate of total income will be \( n \).

(b) How does an increase in \( z \) affect output? Illustrate your answer with a graph. Be sure to compare and contrast the short-run and long-run effects.

The increase in \( z \) shifts up both the savings and output functions. In the short-run, when \( k \) is fixed, this causes output to increase in proportion to the increase in \( z \). However, greater productivity triggers additional investment, and over time the increased capital stock produces an additional increase in output. This is represented by a movement along the new higher savings function up to the new steady state capital/labor ratio.

Mathematically, notice that the exponent on \( z \) in the above expression for \( y^* \) exceeds one. That is, there is a more than proportional effect on output from an increase in \( z \).

(c) Does your answer to part (c) suggest any problems with traditional growth accounting procedures? In particular, how would a growth accounting decomposition interpret the increase in \( z \)?

The answer to the previous question suggests that, in practice, it can be very difficult to separate out the effects of capital accumulation and productivity when doing growth accounting decompositions. That’s because investment is endogenous. That is, it responds to changes in productivity. This suggests that traditional growth accounting decompositions likely underestimate the true importance of technology and productivity.