Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (10 points each).

1. The Solow model cannot explain growth.
   TRUE/UNCERTAIN. The Solow model cannot explain sustained growth in per capita income. However, it can explain temporary growth, which occurs while the economy converges to its steady state. (Some students might note that it can also “explain” why total output will grow even in the steady state, since total output grows at the (exogenous) rate of population growth. However, they do not need to mention this for full credit).

2. According to Friedman’s Permanent Income Hypothesis, high savings rates should predict recessions.
   TRUE. According to Friedman, households consume at a rate that is proportional to their permanent income, which is the expected present discounted value of their lifetime wages and asset income. If they expect their incomes to be lower in the future, due to a recession, then they will save in anticipation of it, so that they do not have to reduce their consumption by as much during the recession.

3. According to the DMP (Diamond-Mortensen-Pissarides) model of search unemployment, an increase in unemployment benefits increases the equilibrium unemployment rate.
   TRUE. Higher benefits raise the ‘outside option’ for households, and so via Nash bargaining, wages rise. Higher wages raise costs for firms. As a result, fewer vacancies are created. Labor market tightness, \( j \), declines until the search costs for firms decline enough to compensate for the higher wages, and the zero profit condition remains satisfied. Note that the effect on the size of the labor force is ambiguous, since higher benefits reduce the costs of search for households (since if they are unlucky and do not find a job, they get higher benefits), but the equilibrium decline in labor market tightness reduces the probability that they will find a job. However, the unemployment rate rises unambiguously, due to the decline in labor market tightness. (See Figure 6.11 in the text for a picture).

The following questions are short answer. Briefly explain your answer. Clarity will be rewarded.

4. (10 points). Using the National Income Accounting identity, derive the relationship between government budget deficits, current account deficits, investment, and private savings. Under what conditions will government budget deficits lead to current account deficits?
   Start with the expenditure-based National Income Accounting identity
   \[ Y = C + I + G + NX \]
   Add \( NFP \) to both sides
   \[ GNP = C + I + G + CA \]
   where \( CA = NX + NFP \) denotes the current account surplus. Now subtract \( C \) and \( T \) from both sides
   \[ GNP - C - T = S_p = I + (G - T) + CA \]
Subtracting $CA$ from both sides, and calling $-CA = CAD$ the current account deficit, we have

$$G - T = CAD + (S_p - I)$$

Hence, the government budget deficit is equal to the current account deficit plus the difference between private saving and investment. In other words, if the government runs a deficit, it must either borrow the money from foreigners (i.e., $CAD > 0$) or from domestic households (i.e., $S_p - I > 0$). So government budget deficits will produce current account deficits unless private saving rises, or private investment falls. (Note, if the budget deficit arises from a cut in lump-sum taxes, then Ricardian equivalence predicts that private saving will rise. However, they do not need to mention this for full credit).

5. (30 points). Consider a static one-period economy as discussed in Chapter 4. Output is produced using the production function: $Y = \phi N - \frac{1}{2} N^2$, where $N$ denotes hours of labor, and $\phi$ is a constant parameter. The representative household’s preferences are given by

$$U(C, \ell) = a \cdot \ell - \frac{1}{2} \ell^2 + b \cdot C$$

where $a$ and $b$ are fixed parameters. The household’s time constraint is $h = \ell + N$. For simplicity, suppose there is no government in this economy.

(a) Derive the labor supply and labor demand curves. How does labor supply depend on the wage rate? Explain intuitively.

*The household’s optimality condition is*

$$\frac{U_\ell}{U_c} = \frac{a - \ell}{b} = w$$

*Using the time constraint to sub out for $\ell$ in terms of $N^s$, and then solving for $N^s$ gives us the labor supply curve*

$$N^s = b \cdot w + h - a$$

*Note that labor supply rises with $w$, indicating that the substitution effect dominates the income effect. In fact, due to the quasi-linearity of the preferences here, there is no income effect on leisure, since the marginal utility of consumption is constant. Next, the firm’s optimality condition is*

$$MPL = \phi - N^d = w$$

*which gives us the following labor demand curve*

$$N^d = \phi - w$$

(b) Compute the competitive equilibrium wage rate and employment for this economy. How do $a$ and $\phi$ affect the equilibrium wage and output level? Illustrate your answers with a labor supply/labor demand graph. Setting $N^s = N^d$ gives us the following competitive equilibrium wage rate

$$\phi - w = b \cdot w + h - a \quad \Rightarrow \quad w = \frac{\phi + a - h}{b + 1}$$

Substituting the equilibrium wage into either the labor supply or demand curve then gives us the following equilibrium employment level

$$N = \frac{b\phi + h - a}{b + 1}$$

Notice that an increase in $a$ causes the wage to rise and employment to fall. This is because an increase in $a$ represents an increase in the marginal utility of leisure schedule. This produces an upward/left shift in the labor supply curve, which then causes $w$ to rise and $N$ to fall. On the other hand, note that an increase in $\phi$ causes both $w$ and $N$ to rise. This is because an increase in $\phi$ shifts out the $MPL$ schedule, which shifts the labor demand curve to the right.
6. (30 points). This question asks you to use the Solow model to analyze the effects of fiscal policy. Suppose output is produced with the production function \( Y = K^\alpha (E \cdot L)^{1-\alpha} \), where \( E \) denotes the efficiency of labor. Assume \( L \) is constant, and for simplicity normalize it so \( L = 1 \). The new twist here is that government spending (e.g., on infrastructure and education) is productive. In particular, suppose \( E = \phi G \), where \( G \) is government spending, and \( \phi \) is a constant parameter. Government spending is financed by a tax on output, so that \( G = \tau Y \), where \( \tau \) is the tax rate. As usual, suppose capital depreciates at the fixed rate \( \delta \) and that individuals save a constant fraction, \( s \), of their (after-tax) income.

(a) Write down the economy’s capital accumulation equation. Under what conditions will there be sustained growth?

The capital accumulation equation is as follows

\[
\Delta K = s(1 - \tau)Y - \delta K
\]

Using the facts that \( L = 1 \), \( G = \tau Y \) and \( E = \phi G \) allows us to derive the following expression for \( Y \)

\[
Y = K^\alpha [\phi \tau Y]^{1-\alpha}
\]

Solving for \( Y \) gives us

\[
Y = K[\phi \tau]^{(1-\alpha)/\alpha}
\]

Finally, subbing this into the capital accumulation equation give us

\[
\frac{\Delta K}{K} = s(1 - \tau)[\phi \tau]^{(1-\alpha)/\alpha} - \delta
\]

The economy will experience sustained growth when the first term on the left exceeds the second term (e.g., when \( s \) is high relative to \( \delta \)).

(b) What tax rate, \( \tau \), maximizes the economy’s long-run growth rate? Explain intuitively. (Hint: Your answer should only depend on \( \alpha \)).

Note that the growth rate depends nonlinearly on \( \tau \). For example, when the government taxes everything (\( \tau = 1 \)), nobody has any (after-tax) income to save, and so the economy does not grow (it shrinks at rate \( \delta \)). On the other hand, if \( \tau = 0 \), the government cannot afford to build roads or finance education, so there is no labor-augmenting technological progress, and again the economy cannot grow. The growth maximizing tax rate maximizes the term \((1 - \tau)\tau^{(1-\alpha)/\alpha} \). Taking the derivative with respect to \( \tau \) and setting it to zero gives us

\[
\left( \frac{1 - \alpha}{\alpha} \right) \tau^{\frac{1-\alpha}{\alpha} - 1} - \frac{1}{\alpha} \tau^{\frac{1}{\alpha} - 1} = 0 \quad \Rightarrow \quad \tau = 1 - \alpha
\]

Hence, the optimal tax rate is equal to labor’s share in GDP, which makes sense, since technological progress makes labor more productive. The more important labor is, the more you want the government to provide infrastructure and education.